QUESTION WITH SOLUTION

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H.O. : 394, Rajeev Gandhi Nagar, Kota
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1. If $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$ = (a + b +c) (x + a + b + c)^2 , x $\neq$ 0 and a + b + c $\neq$ 0, then x is equal to :

(A) 2(a + b + c) (B) –(a + b + c) (C) abc (D) –2(a + b + c)

Sol. **D**

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$(a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -b-c-a & 0 \\ 2c & 0 & -a-b-c \end{vmatrix}$$

$$(a+b+c)^3\begin{vmatrix} 1 & 0 & 0 \\ 2b & -1 & 0 \\ 2c & 0 & -1 \end{vmatrix}$$

Now

$(a+b+c)(a+b+c)^2 = (a+b+c)(x+a+b+c)^2$

x = 0, – 2(a+b+c)

2. Let K be the set of all real values of x where the fuction f(x) = sin |x| – |x| + 2 (x – π) cos |x| is not differentiable. Then the set K is equal to :

(A) ϕ (an empty set) (B) $\{\pi\}$ (C) {0} (D) $\{0 , \pi\}$

Sol. **A**

f(x) = sin |x| – |x| + 2(t – π) cos |x|

$$f(x) = \begin{cases} \sin x - x + 2(x-\pi)\cos x \ ; \ x \geq 0 \\ -\sin x + x + 2(x-\pi)\cos x \ ; \ x < 0 \end{cases}$$

$$f'(x) = \begin{cases} \cos x - 1 + 2\cos x - 2\sin x(x-\pi); \ x \geq 0 \\ -\cos x + 1 + 2\cos x - 2\sin x(x-\pi); \ x < 0 \end{cases}$$

$f'(0^+) = 2$

$f(0^-) = 2$

$f'(\pi) = -4$

$\Rightarrow (k \,\varepsilon\, \phi)$

3. Let a function f : $(0,\infty) \to (0,\infty)$ be defined by f(x) = $\left|1-\frac{1}{x}\right|$. Then f is :-

(A) Injective only (B) Not injective but it is surjective

(C) Both injective as well as surjective (D) Neither injective nor surjective

Sol. **D**

$f : (0, \infty) \to (0, \infty)$

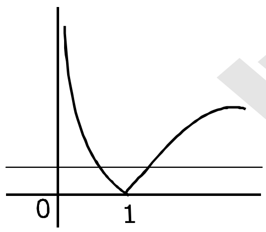
$$f(x) = \left|1 - \frac{1}{x}\right|$$

$$f(x) = \left|\frac{x-1}{x}\right|$$

$$\begin{array}{ll} \dfrac{x-1}{x} & ; x < 0 \\ \dfrac{-(x-1)}{x} & ; 0 < x \le 1 \\ \dfrac{(x-1)}{x} & ; x > 1 \end{array}$$

$$f(x) = \begin{cases} \dfrac{1-x}{x} & ; 0 < x \le 1 \\ \dfrac{x-1}{x} & ; x > 1 \end{cases}$$

$$f(x) = \begin{cases} -\dfrac{1}{x^2} & ;\ 0 < x \le 1 \ \to \text{decreasing} \\ \dfrac{1}{x^2} & ;\ x > 1 \ \to \text{increasing} \end{cases}$$



(Neither injective Nor surjective)

4. Let A and B be two invertible matrices of order 3×3. If $\det(ABA^T) = 8$ and $\det(AB^{-1}) = 8$, then $\det(BA^{-1}B^T)$ is equal to :

(A) 16 (B) 1 (C) $\frac{1}{16}$ (D) $\frac{1}{4}$

Sol. **C**

$|ABA^T| = 8 \Rightarrow |A|^2|B| = 8$

$|AB^{-1}| = 8 \Rightarrow \dfrac{|A|}{|B|} = 8$

Now, $\det(BA^{-1}B^T) = \dfrac{|B|^2}{|A|} =$

$|A|^3 = 64 \Rightarrow |A| = 4$ & $|B| = \dfrac{1}{2}$

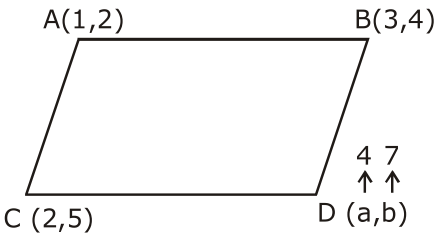
$\Rightarrow \dfrac{|B|^2}{|A|} = \dfrac{1}{16}$

5. If in a parallelogram ABDC, the coordinates of A, B and C are respectively (1, 2), (3, 4) and (2, 5) then the equation of the diagonal AD is :

(A) $3x + 5y - 13 = 0$ (B) $3x - 5y + 7 = 0$

(C) $5x + 3y - 11 = 0$ (D) $5x - 3y + 1 = 0$

Sol. **D**



$a + 1 = 5$ & $b + 2 = 9$

$a = 4$ $b = 7$

eq. of AD is :

$$y - 2 = (x - 1) \times \frac{5}{3}$$

$3y - 6 = 5x - 5$

$5x - 3y + 1 = 0$

6. The solution of the differential equation, $\frac{dy}{dx} = (x-y)^2$, when y(1) = 1, is :

(A) $\log_e\left|\frac{2-x}{2-y}\right| = x - y$ (B) $-\log_e\left|\frac{1-x+y}{1+x-y}\right| = 2(x-1)$

(C) $-\log_e\left|\frac{1+x-y}{1-x+y}\right| = x + y - 2$ (D) $\log_e\left|\frac{2-y}{2-x}\right| = 2(y-1)$

Sol. **B**

$$\frac{dy}{dx} = (x-y)^2$$

$x - y = t$

$$1 - \frac{dy}{dx} = \frac{dt}{dx}$$

$$1 - \frac{dt}{dx} = t^2 \Rightarrow \frac{dt}{dx} = 1 - t^2$$

$$\int \frac{dt}{1-t^2} = \int dx$$

$$-\frac{1}{2}\ln\left(\frac{1-t}{1+t}\right) = x + c$$

$$-\frac{1}{2}\ln\left(\frac{1-x+y}{1+x-y}\right) = x + c$$

$x = 1$, $y = 1$

$\ln(1) = 2 + \alpha + c \Rightarrow c = -2$

7. If $\int \frac{x+1}{\sqrt{2x-1}}dx = f(x)\sqrt{2x-1}+C$, where C is a constant of integration, then f(x) is equal to:

(A) $\frac{2}{3}(x-4)$ (B) $\frac{1}{3}(x+1)$ (C) $\frac{1}{3}(x+4)$ (D) $\frac{2}{3}(x+2)$

Sol. C

$$\int \frac{x+1}{\sqrt{2x-1}}dx = f(x)\sqrt{2x-1}+c$$

$$\int \frac{xdx}{\sqrt{2x-1}} + \int \frac{dx}{(2x-1)^{1/2}}$$

$$\frac{1}{2}\int \frac{(2x+1-1)}{\sqrt{2x-1}}dx + \int \frac{dx}{(2x-1)^{1/2}}$$

$$\frac{1}{2}\left[\int \sqrt{2x-1}dx + \int \frac{dx}{(2x-1)^{1/2}}\right] + \int \frac{dx}{(2t-1)^{1/2}}$$

$$\frac{1}{2}\left[\frac{2}{3}\frac{(2x-1)^{3/2}}{2}\right] + \frac{3}{2}\times 2\frac{(2x-1)^{1/2}}{2} + C$$

$$\sqrt{(2x-1)}\left\{\frac{1}{6}\times(2x-1)+\frac{3}{2}\right\}+C$$

$$\sqrt{2x-1}\left\{\frac{x+4}{3}\right\}+C$$

8. If 19th term of a non-zero A.P. is zero, then its (49th term) :(29th term) is:

(A) 3 : 1 (B) 2 : 1 (C) 1 : 3 (D) 4 : 1

Sol. A

$T_{19} = 0 \Rightarrow a + 18d = 0$

$$\frac{T_{49}}{T_{29}} = \frac{a+48d}{a+28d} \Rightarrow \frac{(48-18)d}{(28-18)d}$$

$$= \frac{30}{10} = \frac{3}{1}$$

9. Contrapositive of the statement " If two numbers are not equal, then their squares are not equal." Is:-

(A) If the squares of two numbers are equal, then the numebrs are equal.
(B) If the squares of two numbers are equal, then the numebrs are not equal.
(C) If the squares of two numbers are not equal, then the numebrs are equal.
(D) If the squares of two numbers are not equal, then the numebrs are not equal.

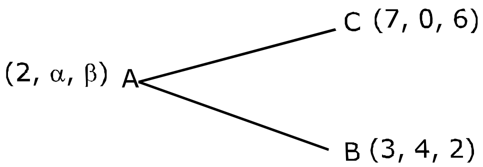
Sol. A

Contrapositive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$

10. If the point $(2, \alpha, \beta)$ lies on the plane which passes through the points (3, 4, 2) and (7, 0, 6) and is perpendicular to the plane $2x - 5y = 15$, then $2\alpha - 3\beta$ is equal to:

(A) 5 (B) 7
(C) 17 (D) 12

Sol. B



$\vec{n} = \overline{AB} \times \overline{AC} = (\alpha + \beta - 6)\hat{i} + (\beta - 1)\hat{j} + (5 - \alpha)\hat{k}$

Now, $2(\alpha + \beta - 6) - 5(\beta - 1) = 0$

$2\alpha - 3\beta = 7$

11. Let $(x + 10)^{50} + (x - 10)^{50} = a_0 + a_1x + a_2x^2 + ... + a_{50}x^{50}$, for all $x \in R$; then $\frac{a_2}{a_0}$ is equal to :

(A) 12.75 (B) 12.00 (C) 12.50 (D) 12.25

Sol. **D**

$(10 + x)^{50} = {}^{50}C_0(10)^{50} + {}^{50}C_1(10)^{49}x ..... + {}^{50}C_{50}x^{50}$

$(10 - x)^{50} = {}^{50}C_0(10)^{50} - {}^{50}C_1(10)^{49}x ..... + {}^{50}C_{50}x^{50}$

$(10 + x)^{50} + (10 - x)^{50} = 2\left[(10)^{50} + {}^{50}C_2(10)^{48}x^2 + ...\right]$

$\Rightarrow \frac{a_2}{a_0} = \frac{{}^{50}C_2 \times 10^{98}}{10^{56}} = \frac{{}^{50}C_2}{10^2}$

$= 12.25$

12. A circle cuts a chord of length 4a on the x-aixs and passes through a point on the y-axis, distant 2b from the origin. Then the locus of the centre of this circle, is:

(A) an ellipse (B) a parabola (C) a straight line (D) a hyperbola

Sol. **B**

Let eq. of circle in :

$x^2 + y^2 + 2gx + 2fy + C = 0$

given x - intercept = 4a

$2\sqrt{g^2 - C} = 4a$

$g^2 - C = 4a^2$

$c = g^2 - 4a^2$ ... (1)

also, if passed through (0, 2b)

$4b^2 + 4fb + c = 0$

from (1)

$4b^2 + 4fb + g^2 - 4a^2 = 0$

$x^2 - 4by + 4b^2 - 4a^2 = 0$

parabola

13. If the area of the triangle whose one vetex is at the vetex of the parabola, $y^2 + 4(x - a^2) = 0$ and the other two vertices and the points of intersection of the parabola and y-axis, is 250 sq. units then a value of 'a' is :

(A) $5\sqrt{5}$ (B) 5 (C) $(10)^{2/3}$ (D) $5\left(2^{1/3}\right)$

Sol. **B**

$y^2 + 4(x - a^2) = 0$

$v\ (a^2, 0)$

at y - axis, x = 0

$\Rightarrow y^2 = 4a^2 \Rightarrow y = \pm 2a$

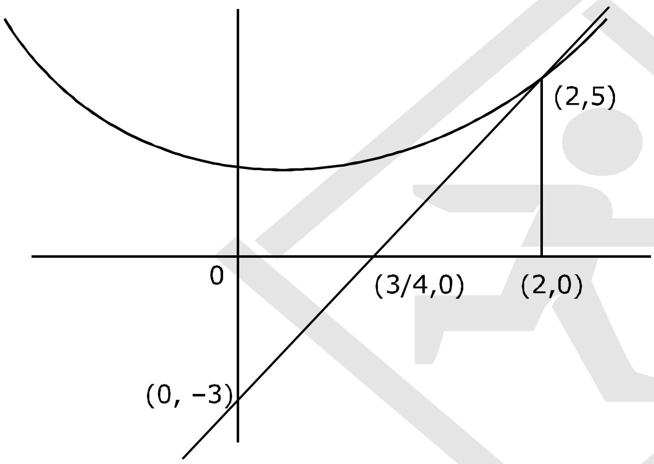
So, $\frac{1}{2}\begin{vmatrix} a^2 & 0 & 1 \\ 0 & 2a & 1 \\ 0 & -2a & 1 \end{vmatrix} = \pm 250$

$a^2 (4a) = \pm 500$
$a^3 = \pm 125$
$a = 5$ or -5

14. The area(in sq. units) in the first quadrant bounded by the parabola, $y = x^2 + 1$, the tangent to it at the point (2, 5) are the coordinate axes is :

(A) $\frac{187}{24}$ (B) $\frac{8}{3}$ (C) $\frac{37}{24}$ (D) $\frac{14}{3}$

Sol. **C**
$x^2 = y - 1$
$T = 0$
$2 \times 2x = y + 5 - 2$
$4x = y + 3$



$$\int_0^2 (x^2 + 1)\,dx - \frac{1}{2} \times (2 - 3/4) \times 5$$

$$\left(\frac{x^3}{3} + x\right)_0^2 - \frac{25}{8}$$

$$\frac{8}{3} + 2 - \frac{25}{8} \Rightarrow \frac{14}{3} - \frac{25}{8} \Rightarrow \frac{112 - 75}{24} \Rightarrow \frac{37}{24}$$

15. All x satisfying the inequality $(\cot^{-1}x)^2 - 7(\cot^{-1}x) + 10 > 0$, lie in the interval :

(A) $(-\infty, \cot 5) \cup (\cot 4, \cot 2)$ (B) $(\cot 5, \cot 4)$

(C) $(-\infty, \cot 5) \cup (\cot 2, \infty)$ (D) $(\cot 2, \infty)$

Sol. **D**
$t^2 - 7t + 10 > 0$
$t < 2$ or $t > 5$
$\cot^{-1}x < 2$ or $x > \cot 2$
$x \in (\cot 2, \infty)$

16. Let f(x) = $\dfrac{x}{\sqrt{a^2+x^2}} - \dfrac{d-x}{\sqrt{b^2+(d-x)^2}}, x \in R$ where a, b and d are non-zero real constants. Then:

(A) f is neither increasing nor decreasing function of x
(B) f is a decreasing function of x
(C) f' is not a continuous function of x
(D) f is an increasing function of x

Sol. **B**

$$f'(x) = \frac{\sqrt{a^2+x^2} - \dfrac{x^2}{\sqrt{a^2+x^2}}}{(a^2+x^2)}$$

$$- \frac{\sqrt{b^2+(d-x)^2} - (d-x)\left[\dfrac{-2(d-x)}{2\sqrt{b^2+(d-x)^2}}\right]}{b^2+(d-x)^2}$$

$$f'(x) = \frac{\sqrt{a^2+x^2} - \dfrac{x^2}{\sqrt{a^2+x^2}}}{(a^2+x^2)} + \frac{-(b^2+(d-x)^2)+(d-x)^2}{[b^2+(d-x)^2]^{3/2}}$$

$$\Rightarrow \quad \frac{a^2}{(a^2+x^2)^{3/2}} + \left\{\frac{b^2}{\left[b^2+(d-x)^2\right]^{3/2}}\right\}$$

$> 0 \Rightarrow$ Increasing function

17. Let $S_n = 1 + q + q^2 + ... + q^n$ and $T_n = 1 + \left(\frac{q+1}{2}\right) + \left(\frac{q+1}{2}\right)^2 + ... + \left(\frac{q+1}{2}\right)^n$ where q is a real number and $q \neq 1$. If ${}^{101}C_1 + {}^{101}C_2 \cdot S_1 + ... + {}^{101}C_{101} \cdot S_{100} = \alpha T_{100}$ then α is equal to:

(A) 200 (B) 2^{100} (C) 2^{99} (D) 202

Sol. **B**

$$S_n = 1 + q + q^2 + ... + q^n = \frac{q^{n+1}-1}{q-1}$$

$$T_n = \frac{\left(\dfrac{q+1}{2}\right)^{n+1} - 1}{\dfrac{q+1}{2} - 1} \Rightarrow \frac{(q+1)^{n+1} - 2^{n+1}}{(q-1)\times 2^n}$$

Let

$$S = {}^{101}C_1 + {}^{101}C_2S_1 + ...+ {}^{101}C_{101}S_{100}$$

$$S = \sum_{r=0}^{100} {}^{101}C_{r+1}S_r$$

$$= \sum_{r=0}^{100} {}^{101}C_{r+1} \times \left(\frac{q^{r+1}-1}{q-1}\right)$$

$$= \frac{1}{q-1}\sum_{r=0}^{100} {}^{101}C_{r+1}(q^{r+1}-1)$$

$$= \frac{1}{q-1}\left[(1+q)^{101}-1-(2^{101}-1)\right]$$

$$= \frac{(1+q)^{101}-2^{101}}{q-1}$$

Now, $T_{100} = \dfrac{(q+1)^{101}-2^{101}}{(q-1)\times 2^{100}}$

18. A bag contains 30 white balls and 10 red balls. 16 balls are drawn one by one randomly from the bag with replacement. If X be the number of white balls drawn; then $\left(\dfrac{\text{mean of X}}{\text{Standard deviation of X}}\right)$ is equal to:

(A) $3\sqrt{2}$ (B) 4 (C) $\dfrac{4\sqrt{3}}{3}$ (D) $4\sqrt{3}$

Sol. **D**

P (white ball) $= \dfrac{30}{40} = p$

$q = \dfrac{1}{4}$, n = 16

Mean = np = $16\times\dfrac{3}{4} = 12$

S.D. = $\sqrt{npq} = \sqrt{12\times 1/4} = \sqrt{3}$

$\Rightarrow \dfrac{\text{Mean}}{\text{S.D}} = \dfrac{12}{\sqrt{3}} = 4\sqrt{3}$

19. Let α and β be the roots of the quadratic equation $x^2 \sin\theta - x(\sin\theta\cos\theta + 1) + \cos\theta = 0$ $(0 < \theta < 45^o)$, and $\alpha < \beta$. Then $\sum_{n=0}^{\infty}\left(\alpha^n + \dfrac{(-1)^n}{\beta^n}\right)$ is equal to:

(A) $\dfrac{1}{1-\cos\theta}-\dfrac{1}{1+\sin\theta}$ (B) $\dfrac{1}{1+\cos\theta}+\dfrac{1}{1-\sin\theta}$

(C) $\dfrac{1}{1+\cos\theta}-\dfrac{1}{1-\sin\theta}$ (D) $\dfrac{1}{1-\cos\theta}+\dfrac{1}{1+\sin\theta}$

Sol. **D**

$x^2 \sin\theta - x(\sin\theta\cos\theta + 1) + \cos\theta = 0$

$$\sum_{n=0}^{\infty}\left(\alpha^n + \frac{(-1)^n}{\beta^n}\right) = ?$$

$$x = \frac{+(\sin\theta\cos\theta+1)\pm\sqrt{\sin^2\theta\cos^2\theta+2\sin\theta\cos\theta+1-4\sin\theta\cos\theta}}{2\sin\theta}$$

$$x = \frac{+(\sin\theta\cos\theta + 1) \pm |\sin\theta\cos\theta - 1|}{2\sin\theta}$$

$$x = \frac{+(\sin\theta\cos\theta + 1) \mp (\sin\theta\cos\theta - 1)}{2\sin\theta}$$

$$\frac{\sin\theta\cos\theta + 1 - \sin\theta\cos\theta + 1}{2\sin\theta} = \frac{1}{\sin\theta}$$

and $\frac{\sin\theta\cos\theta + 1 + \sin\theta\cos\theta - 1}{2\sin\theta} = \cos\theta$

$= \alpha = -\cos\theta$, $\beta = -\frac{1}{\sin\theta}$

$$\sum_{n=0}^{\infty}\left[(+\cos\theta)^n + \frac{(-1)^n}{(+1/\sin\theta)^n}\right]$$

$$\sum_{n=0}^{\infty}\left((+\cos\theta)^n + (-\sin\theta)^n\right)$$

$$\sum_{n=0}^{\infty}\left((-\cos\theta)^0 + (-\cos\theta)^1 + (-\cos\theta)^2 + ...\right)$$

$$+ \ [(\sin\theta)^0 + (\sin\theta)^1 + (\sin\theta)^2 + ...]$$

$$\Rightarrow \left(\frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}\right)$$

20. Let S = {1, 2, ..., 20}. A subset B of S is said to be "nice", if the sum of the elements of B is 203. Then the probability that a randomly chosen subset of S is "nice" is:

(A) $\frac{4}{2^{20}}$ (B) $\frac{6}{2^{20}}$ (C) $\frac{5}{2^{20}}$ (D) $\frac{7}{2^{20}}$

Sol. **C**

S = {1,2, ... 20}

sum of all elements (s) of 'S' = 210

$B \subset S$ & for "nice". sum of element(s) must be 203

$\Rightarrow$ T (1, 6), (2,5), (3,4), (1,2,4)

$\Rightarrow$ p = $\frac{5}{2^{20}}$

21. Let z be a complex number such that |z| + z = 3 + i (where i = $\sqrt{-1}$). Then |z| is equal to:

(A) $\frac{5}{3}$ (B) $\frac{5}{4}$ (C) $\frac{\sqrt{41}}{4}$ (D) $\frac{\sqrt{34}}{3}$

Sol. **A**

$|z| + z = 3 + i$

$\sqrt{x^2 + y^2} + x = 3$, y = 1

$\sqrt{x^2 + 1} = 3 - x$

$x^2 + 1 = 9 + x^2 - 6x$

6x = 8

$x = \frac{4}{3}$

$\Rightarrow\ z = \frac{4}{3} + i$

$\Rightarrow\ |z| = \sqrt{\frac{16}{9} + 1} = \frac{5}{3}$

22. $\lim\limits_{x \to 0} \frac{x \cot(4x)}{\sin^2 x \cot^2(2x)}$ is equal to:

(A) 4 (B) 1 (C) 0 (D) 2

Sol. **B**

$\lim\limits_{x \to 0} \frac{x \cot 4x}{\sin^2 x . \cot^2 2x}$

$\lim\limits_{x \to 0} \frac{x}{\frac{\sin^2 x}{x^2} \times x^2} \times \frac{\tan^2 2x}{\frac{\tan 4x}{4x}} \times \frac{1}{4x^2} \times 4x^2$

$\Rightarrow\ \frac{4x^3}{4x^3} = 1$

23. Given $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$ for a ΔABC with usual notation. If $\frac{\cos A}{\alpha} = \frac{\cos B}{\beta} = \frac{\cos C}{\gamma}$, then the ordered triad (α, β, γ) has a value :

(A) (19, 7, 25) (B) (7, 19, 25) (C) (5, 12, 13) (D) (3, 4, 5)

Sol. **B**

$\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$

$\frac{\cos A}{\alpha} = \frac{c+a}{\beta} = \frac{\cos c}{\gamma}$

$\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13} = \frac{2(a+b+c)}{36}$

$\frac{c}{5} = \frac{b}{6} = \frac{a}{7}$

$a = 7\lambda$, $b = 6\lambda$, $c = 5\lambda$

$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{61 - 49}{60} = \frac{12}{60} = \frac{1}{5}$

$\cos B = \frac{74 - 36}{70} = \frac{19}{35}$

$\cos C = \frac{60}{84} = \frac{5}{7}$

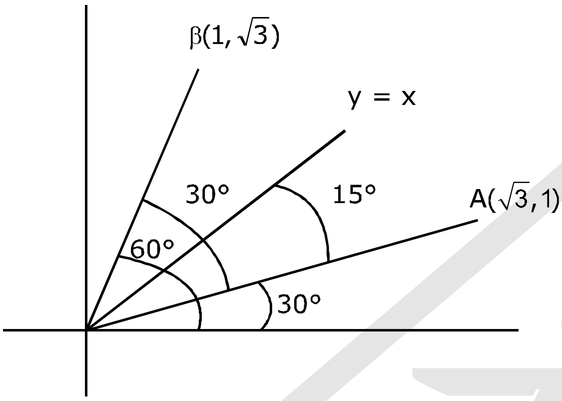
$\alpha : \beta : \gamma = \frac{1}{5} : \frac{19}{35} : \frac{5}{7}$

$\Rightarrow\ 7 : 19 : 25$

24. Let $\sqrt{3}\hat{i}+\hat{j}, \hat{i}+\sqrt{3}\hat{j}$ and $\beta\hat{i}+(1-\beta)\hat{j}$ respectively be the position vectors of the points A, B and C with repect to the origin O. If the distance of C from the bisector of the acute angle between OA and OB is $\frac{3}{\sqrt{2}}$ then the sum of all possible values of β is :

(A) 3 (B) 4 (C) 2 (D) 1

Sol.
D



Bisector is x − y = 0

$C(\beta, 1-\beta)$

$$\frac{|\beta+\beta-1|}{\sqrt{2}}=\frac{3}{\sqrt{2}}$$

$2\beta - 1 = 3$ or -3

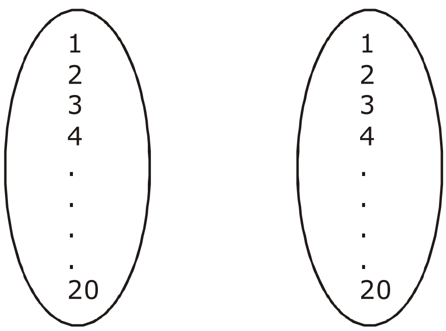
$\beta = 2, -1$

$2 + (-1) = 1$

25. The number of functions f from {1, 2, 3, ..., 20} onto {1, 2, 3, ..., 20} such that f(k) is a multiple of 3, whenever k is a multiple of 4, is :

(A) $5^6 \times 15$ (B) $6^5 \times (15)!$ (C) $(15)! \times 6!$ (D) $5! \times 6!$

Sol. **C**



for k = 4, 8, 12, 16, 20

No. of ways = 6!

for remaining = 15!

$\Rightarrow$ 6! × 15!

26. If a hyperbola has length of its conjugate axis equal to 5 and the distance between its foci is 13, then the eccentricity of the hyperbola is :

(A) 2 (B) $\frac{13}{8}$ (C) $\frac{13}{12}$ (D) $\frac{13}{6}$

Sol. **C**

$2b = 5,\ 2ac = 13$

$b^2 = \frac{25}{4} = a^2(c^2 - 1)$

$\frac{25}{4} = \frac{169}{4c^2}(e^2 - 1)$

$25e^2 = 169e^2 - 169$

$\frac{169}{144} = e^2 \Rightarrow e = \frac{13}{12}$

27. Let the length of the latus rectum of an ellipse with its major axis along x-axis and centre at the origin, be 8, If the distance between the foci of this ellipse is equal to the length of its minor axis, then which one of the following points lies on it?

(A) $\left(4\sqrt{2}, 2\sqrt{3}\right)$ (B) $\left(4\sqrt{3}, 2\sqrt{2}\right)$ (C) $\left(4\sqrt{3}, 2\sqrt{3}\right)$ (D) $\left(4\sqrt{2}, 2\sqrt{2}\right)$

Sol. **B**

$\frac{2b^2}{a} = 8 \Rightarrow b^2 = 4a$

$2ae = 2b$

$b = ae$

$b^2 = a^2 - a^2e^2$

$2a^2e^2 = a^2$

$\Rightarrow e^2 = \frac{1}{2}$

$e = \frac{1}{\sqrt{2}}$

$\Rightarrow b = \frac{a}{\sqrt{2}}$ & $\frac{a^2}{2} = 4a$

$a = 8,\ b = 4\sqrt{2}$

Now equation

$\frac{x^2}{64} + \frac{y^2}{32} = 1$

28. Two lines $\frac{x-3}{1} = \frac{y+1}{3} = \frac{z-6}{-1}$ and $\frac{x+5}{7} = \frac{y-2}{-6} = \frac{z-3}{4}$ intersect at the point R. The reflection of R in the xy-plane has coordinates:

(A) (2, –4, –7) (B) (2, – 4, 7) (C) (–2, 4, 7) (D) (2, 4, 7)

Sol. **A**

$\frac{x-3}{1} = \frac{y+1}{3} = \frac{3-6}{-1}$

& $\frac{x+5}{7} = -\frac{y-2}{6} = \frac{3-3}{4}$

$x = 3 + \lambda$ $\quad$ $x = 7\mu - 5$
$y = 3\lambda - 1$ $\quad$ $y = -6\mu + 2$
$z = 6 - \lambda$ $\quad$ $z = 4\mu + 3$

$6 - 7\mu = -8$
$\lambda + 4\mu = 3$

$-11\mu = -11$
$\mu = 1$
R (2, – 4, 7)
$\Downarrow$
Reflection is (2, – 4, –7)

29. The integral $\int_{\pi/6}^{\pi/4} \frac{dx}{\sin 2x\left(\tan^5 x + \cot^5 x\right)}$ equals :

(A) $\frac{1}{10}\left(\frac{\pi}{4} - \tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right)$

(B) $\frac{1}{20}\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)$

(C) $\frac{1}{5}\left(\frac{\pi}{4} - \tan^{-1}\left(\frac{1}{3\sqrt{3}}\right)\right)$

(D) $\frac{\pi}{40}$

Sol. **A**

$$I = \int_{\pi/6}^{\pi/4} \frac{dx}{\sin 2x(\tan^5 x + \cot^5 x)}$$

$$I = \int_{\pi/6}^{\pi/4} \frac{dx}{\sin 2x\left(\tan^5 x + \cot^5 x\right)}$$

$$I = \int_{\pi/6}^{\pi/4} \frac{\tan^5 x \, dx}{\frac{2\tan x}{1+\tan^2 x}\left(\tan^{10} x + 1\right)}$$

$$I = \int_{\pi/6}^{\pi/4} \frac{\tan^4 x \sec^2 x \, dx}{2\left(\tan^{10} x + 1\right)}$$

Let $\tan^5 x = t$
$5\tan^4 x \sec^2 x \, dx = dt$

$\tan^4 x \sec^2 x \, dx = \frac{dt}{5}$

$$I = \frac{1}{10}\int_{\left(\frac{1}{\sqrt{3}}\right)^5}^{1} \frac{dt}{t^2+1}$$

$$I = \frac{1}{10} \times \tan^{-1} t \Big|_{(1/\sqrt{3})^5}^{1}$$

$$= \frac{1}{10}\left(\frac{\pi}{4} - \tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right)$$

30. Let x, y be positive real numbers and m, n positive integers. The maximum value of the expression $\frac{x^m y^n}{\left(1+x^{2m}\right)\left(1+y^{2n}\right)}$ is :

(A) $\frac{1}{2}$ (B) 1 (C) $\frac{1}{4}$ (D) $\frac{m+n}{6mn}$

Sol. **C**

$$\frac{1+x^{2m}}{2} \geq x^m$$

$$\frac{1+y^{2n}}{2} \geq y^n$$

$$\frac{x^m}{(1+x^{2m})} \times \frac{y^n}{(1+y^{2n})} \leq \frac{1}{4}$$