IOQM 2021

PAPER WITH SOLUTION

1. Let ABCD be a trapezium in which AB||CD and AB = 3CD. Let E be the midpoint of the diagonal BD. If [ABCD] = n × [CDE], what is the value of n? [Here [Γ] denotes the area of the geometrical figure Γ).
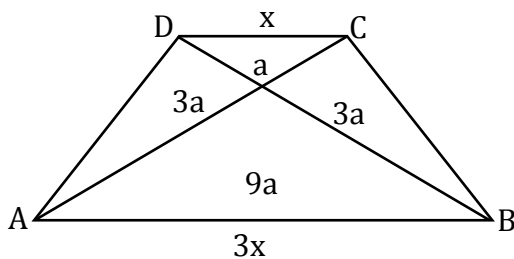
Sol. 8
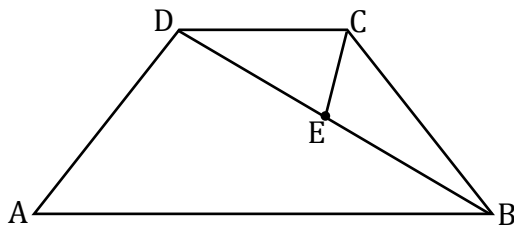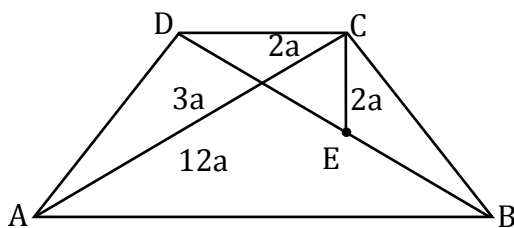
In trapezium ABCD

AB = 3CD

and E is mid-point of BD

or (ABCD) = n × or (CDE)

16 a = n × 2a

n = $\dfrac{16a}{2a}$

n = 8

2. A number N in base 10, is 503 in base b and 305 in base b + 2. What is the product of the digits of N?

Sol. **64**

Number N is base 10

Is 503 in base b and 305 in base (b + 2)

Convert in base 10

$5 \times b^2 + 3 = 3 (b + 2)^2 + 5$

$5b^2 + 3 = 3 (b^2 + ab + 4) + 5$

$5b^2 + 3 = 3b^2 + 12b + 12 + 5$

$2b^2 - 12b - 14 = 0$

$b^2 - 6b - 7 = 0$

$b^2 - 7b + b - 7 = 0$

$b(b - 7) + 1 (b - 7) = 0$

$(b - 7) (b + 1) = 0$

$b = 7, -1$ x rejected

$N = 5b^2 + 3$ or $3(b + 2)^2 + 5$

$N = 5(7)^2 + 3$

$= 248$

Product of digits of N = 2 × 4 × 8 = 64

3. If $\sum_{k=1}^{N} \dfrac{2k+1}{(k^2+k)^2} = 0.9999$ then determine the value of N.

Sol. **99**

$\sum_{K=1}^{N} \dfrac{2K+1}{(K^2+K)^2} = 0.9999$

$\sum_{K=1}^{N} \dfrac{2K+1}{K^2(K+1)^2}$

$\sum_{K=1}^{N} \dfrac{(K+1)^2 K^2}{K^2(K+1)^2}$

$\sum_{K=1}^{N} \left\{ \dfrac{1}{K^2} - \dfrac{1}{(K+1)^2} \right\}$

$$\Rightarrow \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{2^2} - \frac{1}{3^2} + \ldots + \frac{1}{n^2} - \frac{1}{(n+1)^2}$$

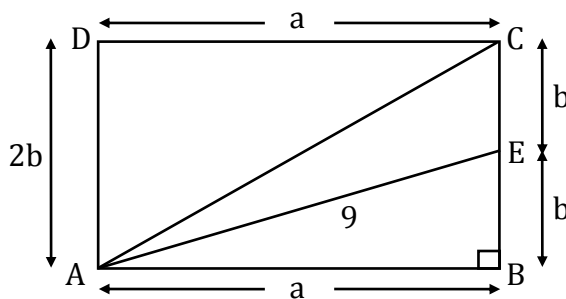$$= 1 - \frac{1}{(n+1)^2} = 0.9999$$

$$1 - 0.9999 = \frac{1}{(n+1)^2}$$

$$\frac{1}{10000} = \frac{1}{(n+1)^2}$$

n + 1 = 100

n = 99

4. Let ABCD be a rectangle in which AB + BC + CD = 20 and AE = 9, where E is the mid-point of the side BC. Find the area of the rectangle.

Sol. **19**

Let ar ABE=x, $= \frac{1}{2} \times a \times 2 = ab$

So ar. AEC = x

ar ADC = 2x.

or ABCD = ab + ab = 2ab

Now AB + BC + CD = 20

2a + 2b = 20

a + b = 10 ....(2)

in $\triangle$ABE

$a^2 + b^2 = 9^2$

$a^2 + b^2 = 81$ ....(3)

from (2)

a+b = 10

$a^2+b^2+2ab= 100$

from (3)

81 + 2ab = 100

2ab = 19

ar ABCD = 19.

5. Find the number of integer solutions to $\left\|x\right|-2020\right|<5$.

Sol. **18**

||x| – 2020| < 5

±(|x| – 2020) < 5

Here we have two cases

Case-I:

|x| – 2020 < 5

$\Rightarrow$ |x| < 2025

Case-II:

–|x| + 2020 < 5

$\Rightarrow$ –|x| < –2015

$\Rightarrow$ |x| > –2015

$\Rightarrow$ x> 2015

$\Rightarrow$ x< 2015

$\therefore$ 2015 < |x| < 2025

Here value of |x| can must be ±2016, ±2017, ..., ±2024.

So, total number of integer solutions = 9 + 9 = 18

6. What is the least positive integer by which $2^5.3^6.4^3.5^3.6^7$ should be multiplied so that, the product is a perfect square?

Sol. **15**

$2^5 \times 3^6 \times 4^3 \times 5^3 \times 6^7$

$2^5 \times 3^6 \times 2^3 \times 2^3 \times 5^3 \times 2^7 \times 3^7$
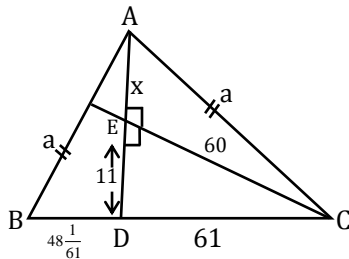
$2^{18} \times 3^{13} \times 5^3$

To make this product

Perfect square all powers of prime factor should be even

Number should be 3 & 5

So, required number is 3 × 5 = 15

7. Let ABC be a triangle with AB = AC. Let D be a point on the segment BC such that $BD = 48\frac{1}{61}$ and DC = 61. Let E be a point on AD such that CE is perpendicular to AD and DE = 11. Find AE.

Sol. **25**

$BD = 48\frac{1}{61}$

DC = 61

By Pythagoras theorem

CE = 60

$60^2 + x^2 = a^2$ by stewait theorem

$a^2 \times 61 + a^2 \times 48\frac{1}{61} = \left(48\frac{1}{61}+61\right)$ {48 × 61 + 1 + (x + 11)^2}

$a^2 = 2a2a + (x + 11)^2$

$3600 + x^2 = x^2 + 22x + 121 + 2929$

22x = 550

   x = 25

   AE = 25 cm

8.   A 5-digit number (in base 10) has digits k, k + 1, k + 2, 3k, k + 3 in that order, from left to right. If this number is m^2 for some natural number m, find the sum of the digits of m.

Sol.   **15**

Then according to questions.

$10^4(k) + 10^3(k + 1) + 10^2(k + 2) + 10(3k) + k + 3 = m^2$

10000 k + 1000 k + 1000 + 100k + 200 + 31 k + 3 = m^2

11131 k + 1203 = m^2 .....(1)

$\therefore$ given that 3k is digit.

3k is digit when $k \leq 3$

k = 1, 2, 3

If we put k = 1, 2 & 3 in equation (1)

We will get a perfect square number

Only for k = 3

11131(3) + 1203 = m^2

34596 = m^2

m = 186

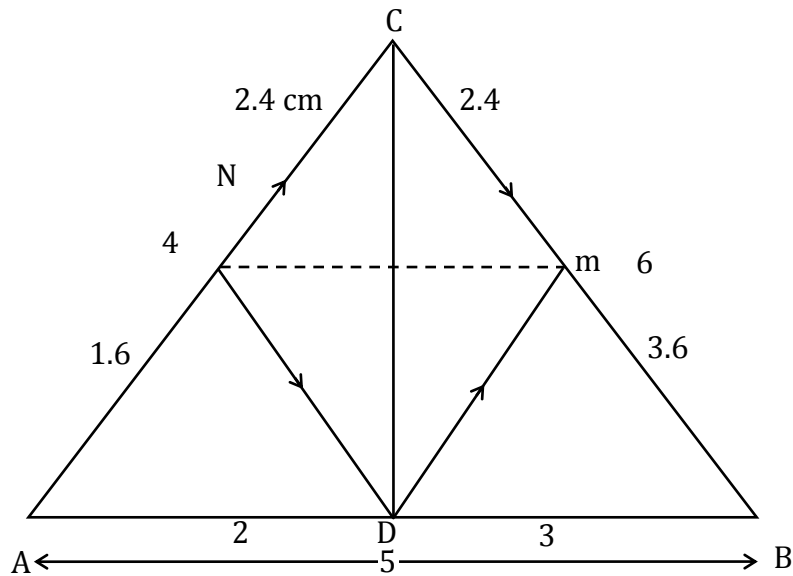Sum of digits of m is 1 + 8 +6 = 15

9.   Let ABC be a triangle with $AB = 5, AC = 4, BC = 6$. The internal angle bisector of C intersects the side AB at D. Points M and N are taken on sides BC and AC, respectively, such that $DM \parallel AC$ and $DN \parallel BC$. If $(MN)^2 = \dfrac{p}{q}$ where p and q are relatively prime positive integers then what is the sum of the digits of $|p - q|$?

Sol. 2

$$\frac{AB}{DB} = \frac{4}{6} = \frac{2}{3}$$

DM || AC

$$\frac{BD}{AB} = \frac{BM}{BC}$$

$$\frac{3}{5} = \frac{8M}{6}$$

$$8M = \frac{18}{5} = 3.6$$

CM = 6 – 3.6 = 2.4

DN || BC

$$\frac{AD}{AB} = \frac{AN}{AC}$$

$$\frac{2}{5} = \frac{AN}{4}$$

$$\frac{8}{5} = AN$$

$$\Rightarrow AN = 1.6$$

$$\cos C = \frac{6^4 + 4^4 - 5^2}{2 \times 6 \times 4}$$

$\cos C = \dfrac{36+16-25}{48}$

$\cos C = \dfrac{27}{48} = \dfrac{9}{16}$

$MN^2 = (2.4)^2 + (2.4)^2 - 2 \times 2.4 \times 2.4 \cos C$

$MN^2 = 5.76 + 5.76 - 2 \times 5.76 \times \dfrac{9}{18}$

$MN^2 = 11.52\left(1 - \dfrac{9}{16}\right)$

$11.52 \times \dfrac{7}{16}$

$MN^2 = 0.72 \times 7 = 5.04$

10. Five students take a test on which any integer score from 0 to 100 inclusive is possible. What is the largest possible difference between the median and the mean of the scores? (The median of a set of scores is the middlemost score when the data is arranged in increasing order. It is exactly the middle score when there are an odd number of scores and it is the average of the two middle scores when there are an even number of scores.)

Sol. **40**

Let number (marks)

0, 0, 100, 100, 100

Median = 100

Mean $= \dfrac{300}{5} = 60$

Difference of (median – mean)

= 100 – 60

= 40

11. Let $X = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$ and $S = \{(a,b) \in X \times X : x^2 + ax + b$ and

$x^3 + bx + a$ have at least a common real zero}.

How many elements are there in S?

Sol. **24**

(a, b) = (0, 0) (trivial case)

$\because x^2 + ax + b = 0$ and $x^3 + bx + 9 = 0$

have common root (let α) then

$\alpha^2 + a\alpha + b = 0 \qquad \Rightarrow \qquad x^2 + b \qquad \qquad$ ...(1)

$\alpha^3 + b\alpha + a = 0 \qquad \Rightarrow \qquad x^3 + bx \Rightarrow x(x^2 + b) \qquad$ ...(2)

From eq.(1) and (2)

Possible values of a = 0 and b = –5, –4, –3, –2, –1, 0

Number of values of order pair (a, b) = 6 $\qquad\qquad$ ...(a)

Now, take $\alpha = 1$

$\quad$ a + b + 1 = 0

$\Rightarrow$ a + b = –1 $\qquad\qquad$ ...(3)

Also, b – a + 1 = 0

$\Rightarrow$ a = b + 1 $\qquad\qquad$ ...(4)

From eq.(3) and (4)

Possible values of a = 5, 4, 3, ..., –3, –4 and b = 4, 3, ..., –4, –5

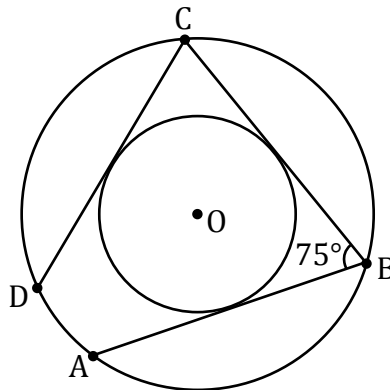Number of values of order pair (a, b) = 9 + 9 = 18 $\qquad$ ...(b)

From eq. (a) and (b)
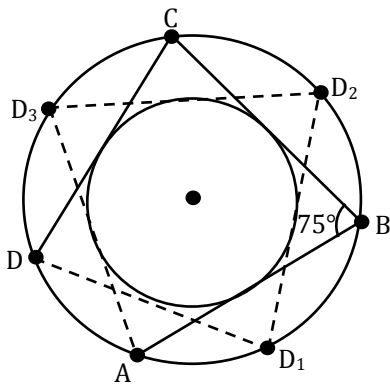
Number of values of order pair (a, b) = 6 + 18 = 24

12.   Given a pair of concentric circles, chords AB, BC, CD,...... of the outer circle are drawn such that they all touch the inner circle If $\angle ABC = 75°$. How many chords can he drawn before returning to the starting point?

Sol.   **24**

Given, AB, BC and CD are the chords of outer circle.

Since, AB, BC and CD are touches inner circle

So, length of chords must be equal i.e., AB = BC = CD.

Now, D, D_1, D_2, D_3 are possible case where we can draw a chord of outer circle, which touches the inner circle.

So, total possibility of chords = 4! = 24

13.   Find the sum of all positive integers n for which $\left| 2^n + 5^n - 65 \right|$ is a perfect square.

Sol.   **6**

$\left| 2^n + 5^n - 65 \right| = m^2$ (let)

$2^n \rightarrow$ 2, 4, 8, 6 (last digit of multiple of 2)

$5^n \rightarrow 5, 5, 5, 5$ (last digit of multiple of 5)

$2^n + 5^n \rightarrow 7, 9, 3, 1$ (last digit for $2^n + 5^n$)

$2^n + 5^n \rightarrow 7, \quad 9, \quad 3, \quad 1$

Now $\quad \underline{-\quad 65, 65, 65, 65}$

$\quad\quad\quad .....2,...4,..8,...6$ (last digit for (2^n+5^n-65))

Now, $\left|2^n + 5^n - 65\right| = m^2$

$\Rightarrow m^2 = \left|65 - \left(2^n + 5^n\right)\right|$

$n = 1 \rightarrow$ no perfect Square. i.e. $m^2 = 58$

$n = 2 \rightarrow$ perfect square i.e. $m^2 = 36 \Rightarrow m = \pm 6$

$n = 3 \rightarrow$ no perfect square i.e. $m^2 = 38$

$n = 4 \rightarrow$ perfect square i.e. $m^2 = 576 \Rightarrow m = \pm 24$

$n = 5 \rightarrow m^2 = 3092$ (Not Perfect Square)

$n = 6 \rightarrow m^2 = 15624$ (Not Perfect Square)

So, possible values for n = 2, 4

Sum of all values of n = 6.

14. The product $55 \times 60 \times 65$ is written as the product of five distinct positive integers. What is the least possible value of the largest of these integers?

Sol. **20**

$55 = 5 \times 11$

$60 = 5 \times 12 = 5 \times 2 \times 2 \times 3$

$65 = 5 \times 13$

$55 \times 60 \times 65$

$= 5 \times 11 \times 5 \times 2 \times 2 \times 3 \times 5 \times 13$

$= 11 \times 25 \times 12 \times 13 \times 5$

or $\ 2 \times 15 \times 10 \times 10 \times 13 \times 5$

or $\ 5 \times 11 \times \boxed{20} \times 15 \times 13$

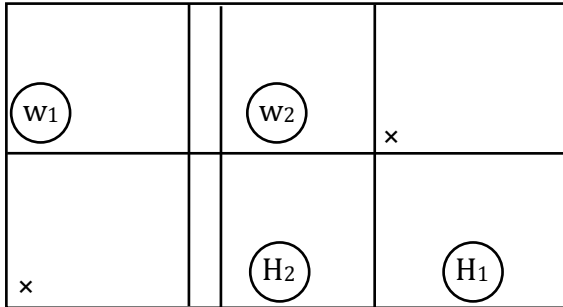So, least possible value of the largest of these integers is 20.

15. Three couples sit for a photograph in 2 rows of three people each such that no couple is sitting in the same row next to each other or in the same column one behind the other. How many arrangements are possible?

Sol. **96**

6 places

$$^6C_1 \times {}^2C_1 \times {}^4C_1 \times {}^1C_1 \times {}^2C_1 \times {}^1C_1$$

↓ ↓ ↓ ↓ ↓ ↓

for H_1 w_1 H_2 w_2 H_3 w_3

$6 \times 2 \times 4 \times 1 \times 2 \times 1 = 96$

16. The sides x and y of a scalene triangle satisfy $x + \dfrac{2\Delta}{x} = y + \dfrac{2\Delta}{y}$, where Δ is the area of triangle. If $x = 60. y = 63$. What is the length of the largest side of the triangle?

Sol. **87**

$$\frac{x^2 + 2\Delta}{x} = \frac{y^2 + 2\Delta}{y}$$

$x^2y + 2\Delta y = xy^2 + 2\Delta x$

$xy(x - y) = 2\Delta(x - y)$

$xy = 2\Delta$

Let z is side of triangle

Also, area(Δ) = $\dfrac{1}{2}$ xysinz

$\Delta = \dfrac{1}{2}.2\Delta.\text{sinz}$

$\Rightarrow$ sinz = 1

$\Rightarrow$ z = 90°

Right angle triangle

Hence, z = $\sqrt{x^2 + y^2}$

      = $\sqrt{60^2 + 63^2}$

z = 87

Where z is side of triangle.

17. How many two digit numbers have exactly 4 positive factors? (Here 1 and the number n are also considered as factors of n.)

Sol. **30**

It will be either of the form a³ or a × b (where a & b both are prime)

Now, two digits cubes = 27, (only)

And of the a × b form

2 and (5 to 47) = total primes = 13

3 and (5 to 31) = total primes = 9

5 and (7 to 19) = total primes = 5

And 7 and (11, 13) = total primes = 2

∴ Total number = 13 + 9 + 5 + 2 + 1 = 30

Ans. 30

18. If $\displaystyle\sum_{k=1}^{40}\left(\sqrt{1+\frac{1}{k^2}+\frac{1}{(k+1)^2}}\right)=a+\frac{b}{c}$, where a, b, c $\in \mathbb{N}$, b < c, gcd (b, c) = 1, then what

is the value of a + b?

Sol. **80**

$$\sum_{k=1}^{40}\sqrt{1+\frac{1}{k^2}+\frac{1}{(k+1)^2}}$$

$$\sum_{k=1}^{40}\sqrt{1+\frac{(k+1)^2+k^2}{k^2(k+1)^2}}$$

$$\sum_{k=1}^{40}\sqrt{1+\frac{2k^2+2k}{k^2(k+1)^2}+\frac{1}{k^2(k+1)^2}}$$

$$\sum_{k=1}^{40}\sqrt{1+\frac{2k(k+1)}{k^2(k+1)^2}+\frac{1}{k^2(k+1)^2}}$$

$$\sum_{k=1}^{40}\left(\sqrt{1+\frac{2}{k(k+1)}+\frac{1}{k^2(k+1)^2}}\right)$$

$$\sum_{k=1}^{40}\left(1+\frac{1}{k(k+1)}\right)$$

From this put k = 1, 2, 3 40

We get

$$\frac{(1+1+1+1......+1)}{40\,\text{times}} + \left\{ \frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + ... + \frac{1}{40\times 41} \right\}$$

$$40 + \left(\frac{2-1}{1\times 2} + \frac{3-2}{2\times 3} + \frac{4-3}{3\times 4} + ... + \frac{41-40}{40\times 41} \right]$$

$$40 + \left(1 - \frac{1}{41} \right)$$

$$= 40 + \frac{40}{41} \text{ by compare. From } a + \frac{b}{c} = 40 + \frac{40}{41}$$

a = 40, b = 40

a + b = 80

19. Let ABCD be a parallelogram. Let E and F be midpoints of AB and BC respectively. The lines EC and FD intersect in P and form four triangles APB, BPC, CPD and DPA. If the area of the parallelogram is 100 sq. units, what is the maximum area in sq. units of a triangle among these four triangles?

Sol. **50**

20. A group of women working together at the same rate can build a wall in 45 hours. When the work started, all the women did not start working together. They joined the work over a period of time, one by one, at equal intervals. Once at work, each one stayed till the work was complete. If the first woman worked 5 times as many hours as the last woman, for how many hours did the first woman work.

Sol. **75**

Let total women who work together is N

N woman can complete work in 45 hours

Amount of work completed by one woman in 1 hour = $\dfrac{1}{45N}$

Given:

First women worked 5 times as compared to last woman.

Since the last women would have worked equal interval total women worked is

4.

Output at x hours = $\dfrac{x}{45N}$

Substituting N = 4

Let second women joined after x hours

Output at x hours = $\dfrac{x}{180}$

When third women joined output = $\dfrac{2x}{180} + \dfrac{x}{180}$

When fourth women joined output = $\dfrac{3x}{180} + \dfrac{2x}{180} + \dfrac{x}{180}$

Since 4 women, after x hours since 4th women joined wall will be built.

Output = $\dfrac{5x}{180} + \dfrac{4x}{180} + \dfrac{2x}{180} + \dfrac{x}{180} = 1$ (completion of wall)

12x = 180

x = $\dfrac{180}{12}$ = 15 hours

Each of women joined after a time interval of 15 hours.

Time spent by first women = 5 × x = 75 hours.

21. A total fixed amount of N thousand rupees is given to three persons A, B, C, every year, each being given an amount proportional to her age. In the first year, A got half the total amount. When the sixth payment was made, A got six-seventh of the amount that she had in the first year; B got Rs. 1000 less than that she had in the first year; and C got twice of that she had in the first year. Find N.

Sol.   **35**

Let their age at beginning is A, B, C

Given : -

A received half of the total amount

$\Rightarrow \dfrac{A}{A+B+C} \times |x| = \dfrac{|x|}{2} \Rightarrow 2A = A + B + C \Rightarrow A = B + C$ ...(i)

In sixth payment

A got $\dfrac{6}{7}$ of amount she received in first year $\left(\dfrac{A+5}{A+B+C+15}\right) \times |x| = \dfrac{6}{7} \times \dfrac{|x|}{2}$

$\Rightarrow \dfrac{A+5}{A+B+C+15} = \dfrac{3}{7}$

7A + 35 = 3A + 3B + 3 C + 45

4A – 3 (B + C) = 10

From equation (1)

4A – 3 (A) = 10 $\Rightarrow$ A = 10 years

C received twice the amount the received in first year

$\dfrac{C+5}{A+B+C+15} = 2\left(\dfrac{C}{A+B+C}\right) \times N$

$\dfrac{C+5}{35} = \dfrac{2C}{20} = [A + B + C = 20]$

20C + 100 = 70 C$\Rightarrow$ 50C = 100 $\Rightarrow$ C = 2

B = 8

In sixth payment B received 1000 less than she got in first year

$\dfrac{B+5}{A+B+C+15} \times N = \left(\dfrac{B}{A+B+C}\right) N - 1000$

$\dfrac{13N}{35} = \dfrac{8N}{20} - 1000$

$\dfrac{13N}{35} = \dfrac{2N}{5} - 1000$

$\dfrac{13N}{35} = \dfrac{2N-5000}{5}$

65 N = 70 N – 35 × 5000

5 N = 35 × 5000

N = 35000

Therefore, the answer is 35.

22. In triangle ABC, let P and R be the feet of the perpendicular from A onto the external and internal bisectors of ∠ABC, respectively; and let Q and S be the feet of the perpendiculars from A onto the internal and external bisectors of ∠ACB, respectively. If PQ = 7, QR = 6 and RS = 8, what is the area of triangle ABC?

Sol. **84**

23. The incircle Γ of a scalene triangle ABC touches BC at D, CA at E and AB at F. Let r_A be the radius of the circle inside ABC which is tangent to Γ and the sides AB and AC. Define r_B and r_C similarly. If r_A = 16, r_B = 25 and r_C = 36, determine the radius of Γ.

Sol. **74**
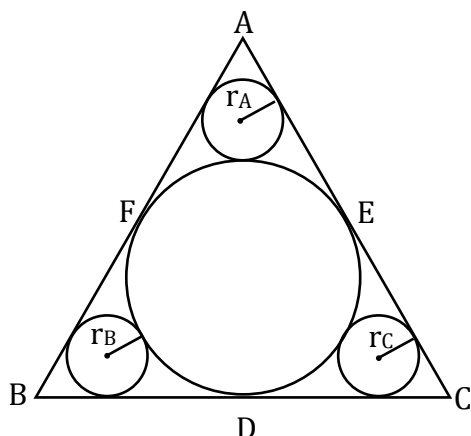
∵ r_A = 16

r_B = 25

r_C = 36

r is radius of incircle Γ

We know,

$r = \sqrt{r_A \cdot r_B} + \sqrt{r_B \cdot r_C} + \sqrt{r_C \cdot r_A}$

$ = \sqrt{16 \cdot 25} + \sqrt{25 \cdot 36} + \sqrt{36 \cdot 16}$

$ = 20 + 30 + 24$

$ = 74$

24. A light source at the point (0, 16) in the coordinate plane casts light in all directions. A disc (a circle along with its interior) of radius 2 with center at (6, 10) casts a shadow on the X axis. The length of the shadow can be written in the form $m\sqrt{n}$ where m, n are positive integers and n is square-free. Find m + n.

Sol. **21**
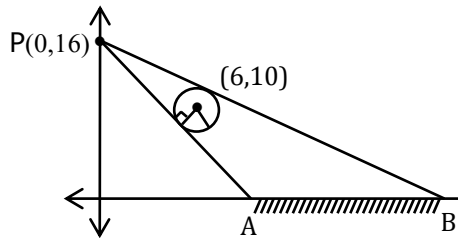
Equation of rays through P (0, 16) with slope m are

y – 16 = mx   ...(i)

Since, it is tangent to circle

$\therefore 2 = \dfrac{|10-16-m|}{\sqrt{1+m^2}}$ (condition of tangency)

4 (1+m²) = |6 + 6m²| = 36 (1 + m² + 2m)

9m² + 18m + 9 = m² + 1

8m² + 18m + 8 = 0

4m² + 9m + 4 = 0   ...(ii)

The line (i) cuts the x-axis at, $x = \dfrac{-16}{m}$

$\therefore$ Length of shadow, AB = $\left|\dfrac{-16}{m_1} + \dfrac{16}{m_2}\right|$

$= 16\,\dfrac{|m_1 - m_2|}{m_1 m_2}$

$= \dfrac{16\sqrt{81-64}}{4}$

$= 4\sqrt{17}$

Therefore, m + n = 21

25. For a positive integer n, let $\langle n \rangle$ denote the perfect square integer closest to n. For example. $\langle 74 \rangle = 81$, $\langle 18 \rangle = 16$. If N is the smallest positive integer such that

$$\langle 91 \rangle . \langle 120 \rangle . \langle 143 \rangle . \langle 180 \rangle . \langle N \rangle = 91.120.143.180.N$$

Find the sum of the squares of the digits of N.

Sol. **56**

$\langle 91 \rangle . \langle 120 \rangle . \langle 143 \rangle . \langle 180 \rangle = 91.120.143.18.N$

$\Rightarrow$ $100.121.144.169 \langle N \rangle = 91.120.143.180.N$

$\Rightarrow$ $\dfrac{\langle N \rangle}{N} = \dfrac{91.120.143.180}{100.121.144.169} = \dfrac{21}{22}$

$\Rightarrow$ $\dfrac{\langle N \rangle}{N} = \dfrac{21}{22}$

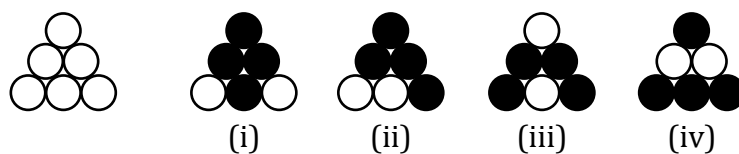$\Rightarrow$ $22 \langle N \rangle = 21.N$

$\Rightarrow$ $22 \times (441) = 21 \times (462)$

     $N = 462$

Sum of squares of digits of

$N = 4^2 + 6^2 + 2^2 = 56$

26. In the figure below, 4 of the 6 disks are to be colored black and 2 are to be colored white. Two colorings that can be obtained from one another by a rotation or a reflection of the entire figure are considered the same.

         (i)   (ii)   (iii)   (iv)

There are only four such colorings for the given two colors, as shown in figure 1. In how many ways can we color the 6 disks such that 2 are colored black, 2 are colored white, 2 are colored blue with the given identification condition?

Sol. **45**
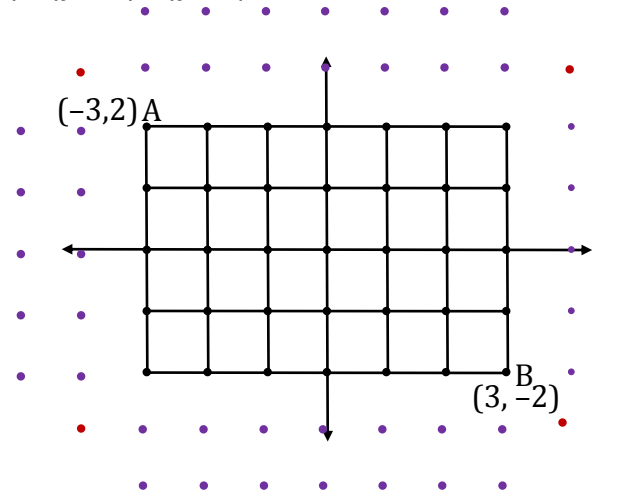
27. A bug travels in the coordinate plane moving only along the lines that are parallel to the x-axis or y-axis. Let A = (–3, 2) and B(3, –2). Consider all possible paths of the bug from A to B of length at most 14. How many points with integer coordinates lie on at least one of these paths.

Sol. **87**

$|x - 3| + |x + 3| + |y - 2| + |y +2| \leq 14$

The bug can travel from A to B by only taking horizontal and vertical steps in total 10 steps only. Therefore, the number of integer points = 35

But, since the maximum path length is 14, the bug can also go off track by maximum 2 units which gives rise to 28 + 20 more integer points.

There also 4 more integer points lying at the corners where the bug can go circular.

Therefore, the total number of points = 35 + 28 + 20 + 4 = 87

28. A natural number n is said to be good if n is the sum of r consecutive positive integers, for some $r \geq 2$. Find the number of good numbers in the set {1, 2, ..., 100}.

Sol. **93**

$n = k + k + 1 + ... + k + r - 1$

$n = rk + (1 + 2 + 3 ... + r - 1)$

$n = rk + \dfrac{r(r-1)}{2}$

$2n = r(2k + r - 1)$

When r is odd, 2k + r – 1 is even and when r is even 2k + r – 1 is odd.

The above equality can hold for n (taking $r \geq 2$) except numbers of the form 2^k i.e. n = 2, 4, 8, 16, 32, 64.

Also, n cannot be equal to 1.

Therefore, total number of good numbers n are 100 – 7 = 93.

29. Positive integers a, b, c satisfy $\dfrac{ab}{a-b} = c$. What is the largest possible value of a + b + c not exceeding 99?

Sol. **99**

$$\frac{ab}{a-b} = c \Rightarrow \frac{a-b}{ab} = \frac{1}{c}$$

$$\Rightarrow \quad \frac{1}{b} - \frac{1}{a} = \frac{1}{c}$$

$$\Rightarrow \quad \frac{1}{b} = \frac{1}{a} + \frac{1}{c}$$

$$\Rightarrow \quad \frac{1}{b} = \frac{a+c}{ac}$$

$\Rightarrow$ ac = ab + bc

$\Rightarrow$ (a – b) (c – b) = b^2 ...(1)

$\Rightarrow \quad \because$ a + b + c $\leq$ 99

We have

a > b and c > b

Possible value of a, b, c is

a = 27, b = 18, c = 54

Which satisfy (1).

30. Find the number of pairs (a, b) of natural numbers such that b is a 3 digit number, a + 1 divides b – 1 and b divides $a^2 + a + 2$.

Sol. **16**

Since, b divides $a^2 + a + 2$

$\therefore$ $a^2 + a + 2 = bk$ where, $k \in I$

$\because$ (a + 1) divides (b – 1)

  (a + 1)ℓ = b – 1   where, $\ell \in I$

$\therefore$ (a + 1)ℓ + 1 = b

  $a^2 + a + 2 = ((a + 1)\ell + 1)k$

$a^2 + a + 2 = \ell k((a + 1) + k$

$a(a + 1) + 2 = \ell k(a + 1) + k$

By compare

k = 2, kℓ = a $\Rightarrow$ 2ℓ = a $\Rightarrow$ b = $\dfrac{a(a+1)}{2}$ + 1

b is a three digit number when a = 14, 15, ..., 44

($\because$ a = 2ℓ, a must be an even number)

So, a = 14, 16, 18,....., 44

$\therefore$ Hence number of pairs (a, b) = 16.

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