IOQM KV 2021

PAPER WITH SOLUTION

MOTION™

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1. If a, b, c are real numbers and
$(a + b - 5)^2 + (b + 2c + 3)^2 + (c + 3a - 10)^2 = 0$
Find the integer nearest to $a^3 + b^3 + c^3$.

Sol. **57**

$a + b = 5$, $b + 2c + 3 = 0$, $c + 3a = 10$

$a + b = 5$

$b + 2c = -3$

$a - 2c = 8$

$2c + 6a = 20$

$7a = 28$

$a = 4$, $b = 1$, $c = -2$

$64 - 8 + 1 = 57$

2. If ABCD is a rectangle and P is a point inside it such that AP = 33, BP = 16, DP = 63. Find CP.

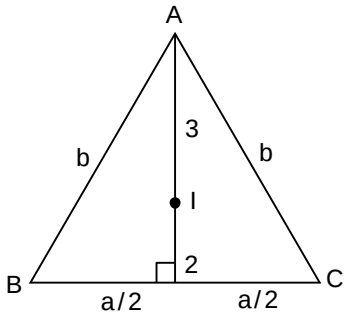
Sol. **56**

3. Sita and Geeta are two sisters. If Sita's age is written after Geeta's age a four digit perfect square (number) is obtained. If the same exercise is repeated after 13 years another four digit perfect sqaure (number) will be obtained. What is the sum of the present ages of Sita and Geeta?

Sol. **55**

4. Let ABC be an isosceles triangle with AB = AC and incentre I. If AI = 3 and the distance from I to BC is 2, what is the square of the length of BC?

Sol. **80**



$x = 2$

$5^2 + \frac{a^2}{4} = b^2$ ....(I)

$2 = \frac{\frac{1}{2} \times a \times 5}{b + \frac{a}{2}}$

$2b + a = \frac{5a}{2}$

$2b = \frac{3a}{2}$

$b = \frac{3a}{4}$

put in equation (i)

$25 + \frac{a^2}{4} = \frac{9a^2}{16}$

$25 = \frac{5a^2}{16}$

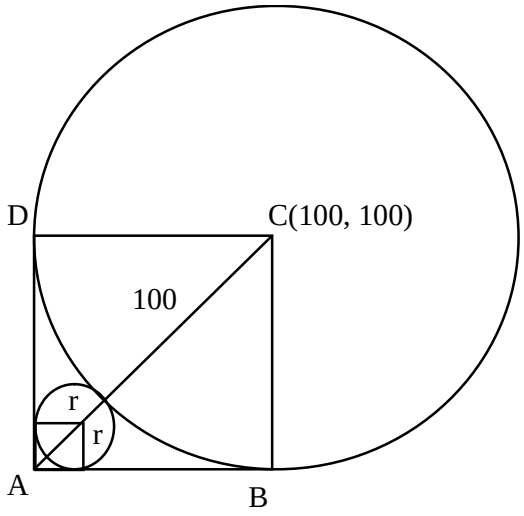
$a^2 = 80$

5. Find the number of positive integers n such that the highest power of 7 dividing n! is 8.

Sol. **7**

6. Let ABCD be a square with side length 100. A circle with centre C and radius CD is drawn. Another circle of radius r, lying inside ABCD, is drawn to touch this circle externally and such that the circle also touches AB and AD . If $r = m + n\sqrt{k}$, where m, n are integers and k is a prime number ,find the value of $\frac{m+n}{k}$.

Sol. **50**



$r + \sqrt{2}\,r + 100 = 100\sqrt{2}$

$r = \frac{100(\sqrt{2}-1)(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)}$

$= 100(2 + 1 - 2\sqrt{2})$

$= 300 - 200\sqrt{2}$

$\therefore \quad m = 300$

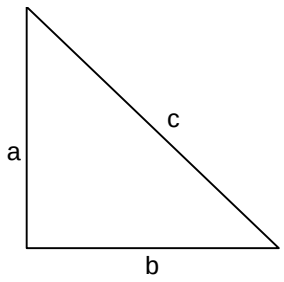
$n = -200$

$k = 2$

$\frac{m+n}{k} = 50.$

7. a, b, c are positive real numbers sich that $a^2 + b^2 = c^2$ and ab = c. Determine the value of $\left|\frac{(a+b+c)(a+b-c)(b+c-a)(c+a-b)}{c^2}\right|$

Sol. **4**



Let given expression be E

ab = c

$2\Delta = c$

$\frac{S(s-a)(s-b)(s-c)}{c^2} = \frac{E}{16}$

$16\ \frac{\Delta^2}{C^2} = E$

$4 \times \frac{C^2}{C^2} = E \Rightarrow E = 4$

8. Find the largest 2-digit number N which is divisible by 4, such that all integral powers of N end with N.

Sol. **76**

9. Find the number of ordered triples (x, y, z) of real numbers that satisfy the system of equation:

$X + y + z = 7, \quad x^2 + y^2 + z^2 = 27; \quad xyz = 5.$

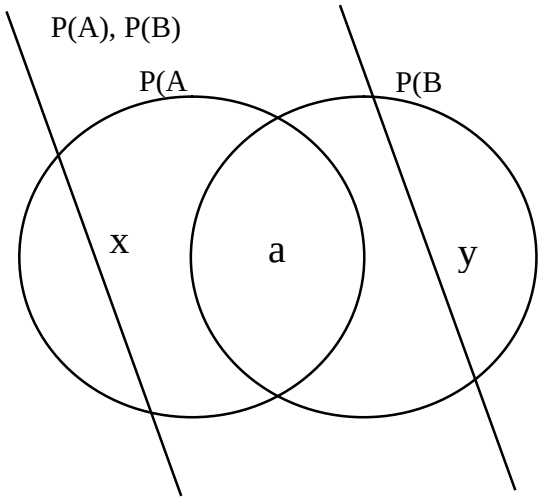
Sol. **3**

$x^3 - 7x^2 + 11x - 5 = 0$

$(x - 5)(x^2\ 2x + 1) = 0 \Rightarrow \boxed{x = 5, 1, 1}$.

10. Let A and B be two finite sets such that there are exactly 144 sets which are subsets of A or subsets of B. Find the number of elements in AUB.

Sol. **8**



X + a + y = 144

$z^x + z^y - z^c = 144$

a = 7, b = 5, c = 4

$\Rightarrow$ 4 elements in common

$n(A \cup B) = 7 + 5 - 4 = 8$

11. The prime numbers a, b and c are such that $a + b^2 = 4c^2$. Determine the sum of all possible values of a + b + c.

Sol. **31**

$a + b^2 = 4c^2$

$a = 4c^2 - b^2$

$a = (2c - b)(2c + b)$

$2c - b = 12$

$2c = b + 1$

$a = 2c + b$

$4c = a + 1$

$2c = b + 1$

$2b + 2 = a + 1$

$2b + 1 = a$

$b, 2b + 1, \frac{b+1}{2}$

5, 11, 3

12. Let A = {m : m an integer and the roots of $x^2 + mx + 2020 = 0$ are positive integers} and B = {n : n an integer and the roots of $x^2 + 2020x + n = 0$ are negative integers}. Suppose a is the largest element of A and b is the smallest element of B. Find the sum of digits of a + b.

Sol. **26**

A : $a + b = -m$, $a.b = 2020$

$\downarrow$ $\downarrow$

$m = -(2021)$ $\quad \frac{2 \times 2 \times 5 \times 101 \times 1}{2^2.5^1.101^1}$

$= -1012$ $\quad = (2 + 1)(1 + 1)(1 + 1)$

$= -509$ $\quad = 12$

$= -121$

B : $r : \delta = -2020$ $\quad 2018 \times 2$

$r.d = +n$ $\quad 2017 \times 3$

$n = 2019, \leftarrow b$ $\quad 2016 \times 4$

$\downarrow$

$\alpha + \beta = 2019 - 121$

$= 1898$

sum = 1 + 8 + 9 + 8

= 26

13. The sides of a triangle are x, 2x + 1 and x + 2 for some positive rational number x. If one angle of the triangle is 60°, what is the perimeter of the triangle?

Sol. **9**

$$\frac{1}{2} = \frac{x^2 + (2x+1)^2 - (x+1)^2}{2(x)(2x+1)}$$

$2x^2 + x = 4x^2 + 4x + 1 - (4x + 4)$

$2x^2 = x = 4x^2 - 3$

$\Rightarrow 2x^2 - x - 3 = 0$

$(2x - 3)(x + 1) = 0$

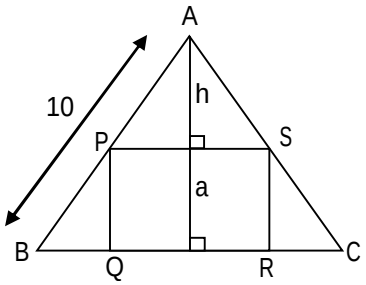
$x = \frac{3}{2}$

Perimeter = 4x + 3

$4 \times \frac{3}{2} + 3 = 9$

14. Let ABC be an equilateral triangle with side length 10. A square PQRS is inscibed in it, with P on AB, Q, R on BC and S on AC. If the area of the square PQRS is $m + n\sqrt{k}$ where m, n are integers and k is a prime number then determine the value of $\sqrt{\frac{m+n}{k^2}}$

Sol. 10



$$\frac{h}{5\sqrt{3}} = \frac{a}{10}$$

$$a = \frac{2h}{\sqrt{3}}$$

$$a + h = 5\sqrt{3}$$

$$a = \frac{2(5\sqrt{3} - a)}{\sqrt{3}}$$

$$a\sqrt{3} = 10\sqrt{3} - 29$$

$$a(2 + \sqrt{3}) = 10\sqrt{3}$$

$$a = 10\sqrt{3}(2 - \sqrt{3})$$

$$a^2 = 300(7 - 4\sqrt{3})$$

$$a^2 = 2100 - 1200\sqrt{3}$$

$$\sqrt{\frac{900}{9}} = 10$$

15. Ria has 4 greeen marbles and 8 red marbles. She arranges them in a circle randomly. If the probability that no two green marbles are adjacent is $\frac{p}{q}$, where the positive integers p, q have no common factors other than 1, what is p + q?

Sol. 40

$$P = \frac{7! \times {}^{8}C_{4} \times 4!}{11!}$$

$$= \frac{7! \times \frac{8!}{4! \times 4!} \times 4!}{11!}$$

$$= \frac{7 \times 6 \times 5 \times 4! \times 8!}{11 \times 10 \times 9 \times 8!}$$

$$= \frac{7}{33}$$

P + q = 7 + 33 = 40

16. If x and y are positive integers such such that $(x - 4)(x - 10) = 2^y$, find the maximum possible value of x + y.

Sol. **16**

$(x - 4)(x - 10) = 2^y$

$ab = 2^y$

$a - b = 6$ clearly $a = 2^p$

$b = 2^q$

$2^p - 2^q = 6$

$x_{max} = 18$ $y_{max} = 4$ $\Rightarrow p = 3, q = 1$

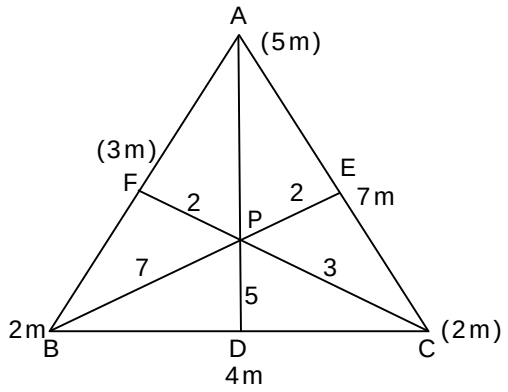
17. Two sides of a regular polygon with n sides, when extended, meet at an angle of 28°. What is the smallest possible value of n?

Sol. **45**

18. Let D, E, F be points on the sides BC, CA, AB of a triangle ABC, respectively. Suppose AD, BE, CF are concurrent at P.

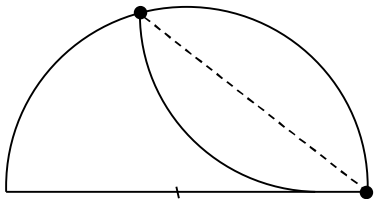
If $\frac{PE}{PC} = \frac{2}{3}, \frac{PE}{PB} = \frac{2}{7}$ and $\frac{PD}{PA} = \frac{m}{n}$, where m, n are positive integers with gcd(m, n) = 1, find m + n.

Sol. **45**



$\frac{PD}{PA} = \frac{4}{5}$

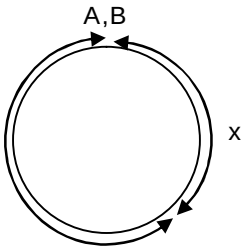
19. A semicircular paper is folded along a chord such that the folded circular arc is tangent to the diameter of the semicircle. The radius of the semicircle is 4 units and the point of tangency divides the diameter in the ratio 7 : 1. If the length of the crease (the dotted line segment in the figure) is l then determine l^2.



Sol. **39**

20. Two people A and B start from the same place at the same time to travel around a circular track of length 100m in opposite directions. First B goes more slowly than A until they meet, then by doubling his rate he next meets A at the starting point. Let d m be the distance travelled by B before he met A for the first time after leaving the starting point. Find the integer closest to d.

Sol. **41**



$$\frac{100-x}{20k} = \frac{x}{(100-x)k}$$

$(100 - x)^2 = 2x^2$

$X^2 - 200\,x + 10^4 = 2x^2$

$= x^2 + 200x - 10^4$

$$X = \frac{-200+200\sqrt{2}}{2}$$

$$X = 1009\left(\sqrt{2}-1\right)$$

$= 0.414 \times 100 = 41.4$

21. Let A = {1, 2, 3, 4, 5, 6, 7, 8}, B = {9, 10, 11, 12, 13, 15, 16} and C {17, 18, 19, 20, 21, 22, 23, 24}. Find the number of triaples (x, y, z) such that $x \in A$, $y \in B$, $z \in C$ and x + y + z = 36.

Sol. **46**

c = 24 x + y = 12 → 3

c = 23 x + y = 13 → 4

c = 22 x + y = 14 → 5

c = 21 x + y = 15 → 6

c = 20 x + y = 16 → 7

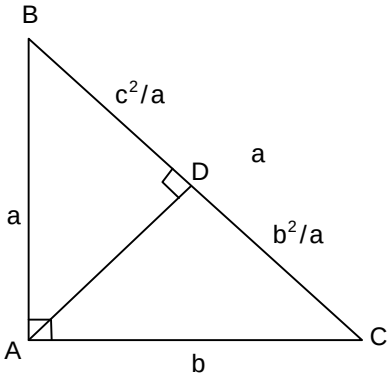
c = 19 x + y = 17 → 8

c = 18 x + y = 18 → 7

c = 17 x + y = 19 → 6

22. Let ABC be a triangle with ∠BAC = 90° and D be the point on the side BC such that AD⊥BC. Let r, r_1 and r_2 be the inradii of triangles ABC, ABD, and ACD, respectively. If r, r_1 and r_2 are positive integers and one of them is 5, find the largest possible value of $r + r_1 + r_2$.

Sol. **30**



$$r = \frac{b + c - a}{2}$$

$$r = \frac{b + c - a}{2}$$

$$r_1 = \frac{\frac{bc}{a} + \frac{c^2}{a} - c}{2}$$

$$r_2 = \frac{\frac{bc}{a} + \frac{b^2}{a} - b}{2}$$

$$r_1 \frac{c}{a} r \quad r_2 = \frac{b}{a} r$$

$$\frac{r}{a} = \frac{r_1}{c} = \frac{r_2}{b}$$

$$r_1^2 + r_2^2 = r^2$$

$$25 = r^2 - r_2^2$$

$$(r_2 - r_1)(r + r_2)$$

= 5,12,13

23. Find the largest positive integer N such that the number of integers in the set {1, 2, 3, …., N} which are divisible by 3 is equal to the number of integers which are divisible by 5 or 7 or (both).

Sol. **65**

$$\left[\frac{N}{3}\right] = \left[\frac{N}{5}\right] + \left[\frac{N}{7}\right] - \left[\frac{N}{35}\right]$$

By Observation

N = 35

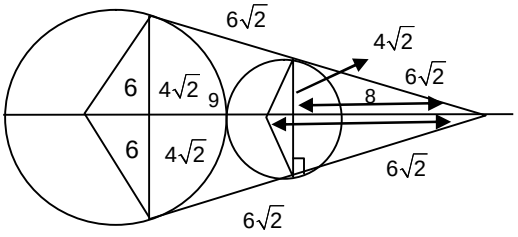
$$\left[\frac{35}{3}\right] = \left[\frac{35}{5}\right] + \left[\frac{35}{7}\right] - \left[\frac{35}{35}\right]$$

11 = 7 + 5 – 1

11 = 11

24. Two circles, S_1 and S_2, of radii 6 units and 3 units respectively, are tangent to each other, externally. Let AC and BD be their direct common tangents with A and B on S_1, and C and D on S_2. Find the area of quadrilateral ABDC to the nearest integer.

Sol. **68**



$\frac{1}{2} \times \left(8\sqrt{2} \times 4\sqrt{2}\right)$

$6\sqrt{2} \times 8$

$a \times a = 6\sqrt{2} \times 3$

$a = 2\sqrt{2}$

25. A fivive digit number $n = \overline{abcde}$ is such that when divided respectively by 2, 3, 4, 5, 6 the remainders are a, b, c, d, e. What is the remainder when n is divided by 100?

Sol. **11**

26. Let a, b, c be three distinct positive integers such that the sum of any two of them is a perfect square and having minimal sum, a + b + c. Find this sum.

Sol. **55**

$a = \frac{1^2 + 2^2 - 3^2}{2}$

$a = \frac{p^2 + q^2 - r^2}{2}$

$b = \frac{q^2 + r^2 - p^2}{2}$

$c = \frac{p^2 + r^2 - q^2}{2}$

$a + b + c = \frac{p^2 + q^2 + r^2}{2}$

$\frac{5^2 + 6^2 - 7^2}{2}$

$6 + 19 = 30 = 55$

27. Let ABC be an acute-angled triangle and P be a point in its interiror. Let P_A, P_B and P_C be the images of P under reflection in the sides BC, CA and AB, respectively. If P is the orthocentre of the triangle $P_A P_B P_C$ and if the largest angle of the traingle that can be formed by the line segments PA, PB, and PC is x°, determine the value of x.

Sol. **60**

28. For a natural number n, let n' denote the number obtained by deleting zero digits, if any. (For example, if n = 260, n' = 26; if n = 2020, n' = 22.) Find the number of 3-digit numbers n for which n' is a divisor of n, different from n.

Sol. **93**

29. Consider a permutation (a_1,a_2,a_3,a_4,a_5) of {1, 2, 3, 4, 5}. We say the 5-tuple (a_1,a_2,a_3,a_4,a_5) is flawless if for all $1 \le i < j < k \le 5$, the sequence (a_i,a_j,a_k). Is not an arithmetic progression (in that order). Find the number of flawless 5-tuples.

Sol. **20**

30. Ari chooses 7 balls at random from n balls numbered 1 to n. If the probability that no two of the drawn balls have consecutive numbers equals the probability of exactly one pair of consecutive numbers in the chosen balls, find n.

Sol. **54**

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Exam Date : **31st January 2021**

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