## ANSWER KEY

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 4 | 13 | 13 | 36 | 10 |
| **6** | **7** | **8** | **9** | **10** |
| 29 | 51 | 49 | 14 | 55 |
| **11** | **12** | **13** | **14** | **15** |
| 6 | 18 | 10 | 53 | 45 |
| **16** | **17** | **18** | **19** | **20** |
| 40 | 30 | 20 | 13 | Bonus |
| **21** | **22** | **23** | **24** | **25** |
| 17 | 78 | 55 | 37 | 48 |
| **26** | **27** | **28** | **29** | **30** |
| 50 | 84 | 15 | 47 | 64 |

# SOLUTIONS Pre-RMO 2019

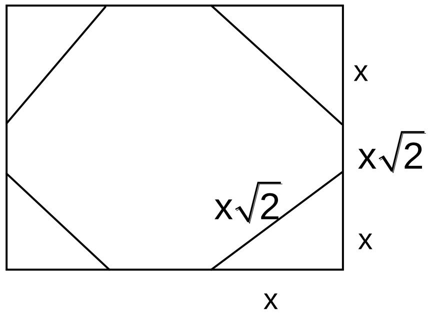
1. $2x + x\sqrt{2} = 5$

$$x = \frac{5}{2+\sqrt{2}} \Rightarrow x\sqrt{2} = \frac{5}{\sqrt{2}+1}$$

$$A = 2\left(1+\sqrt{2}\right)s^2 = 2\left(1+\sqrt{2}\right).\frac{25}{\left(\sqrt{2}+1\right)^2}$$

$$= 50\left(\sqrt{2}-1\right)$$

$$= 20.70$$




Area removed = 25 – 20.70 = 4.3
Hence answer is 4.

2. $$\left(x+\frac{1}{x}\right)^2 + a\left(x+\frac{1}{x}\right) + b = x^2 + ax + b + \frac{1}{x^2} + \frac{a}{x} + b$$

$$\left(x+\frac{1}{x}\right)^2 = x^2 + \frac{\;}{x^2} + b \Rightarrow b = 2$$

$$f(x) = x^2 + ax + 2$$

Since roots are integers.

$D = a^2 - 8$,

must be perfect square $\Rightarrow a^2 = 9$

$$\Rightarrow a^2 + b^2 = 9 + 4 = 13$$

3. $$x_5 = 1 + x_1x_2x_3x_4 \Rightarrow x_1x_2x_3x_4 = 42$$

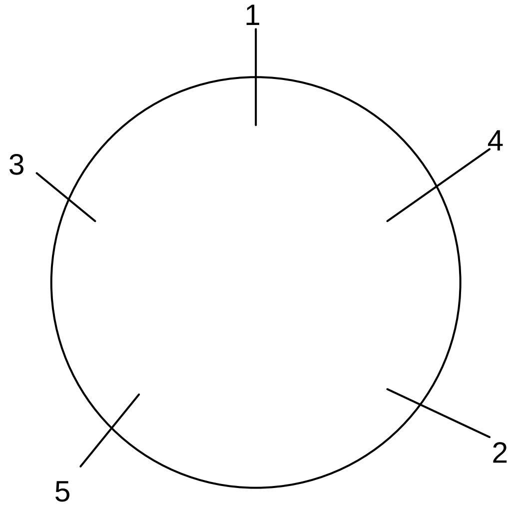
$$x_6 = 1 + x_1x_2x_3x_4x_5 = 1 + 42\times 43 = 1807 = 13\times 139$$

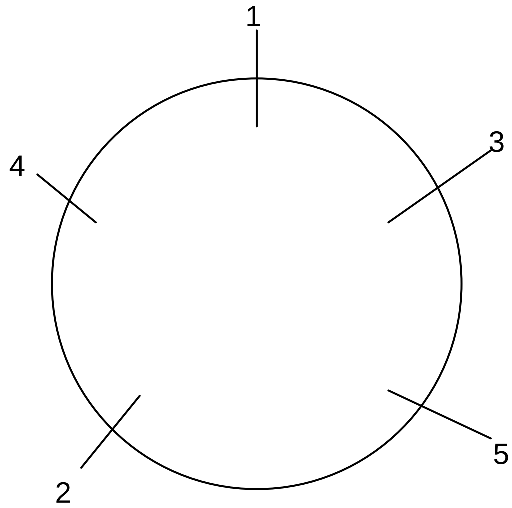
Largest prime factor = 139
Required sum = 13

4.

The situation clearly depicts a 9 – point star.
$\Rightarrow$ Distance traveled $= 4\times 9 = 36$ feet

5. Answer is 2 X 5=10 as can be seen below




6. $\overline{abc} = 100a + 10b + c$

$a^2 + b^2 = c^2$

a, b, c must be single digit numbers

$\Rightarrow a = 4; b = 3; c = 5$ is the only possibility

$\overline{abc} = 435$

$\Rightarrow$ Largest prime factor = 29

7. 2 times between 12 and 1, the arms make an angle of 90 degrees.At $\frac{180}{11}$ and at $\frac{540}{11}$ minutes

$\therefore$ So difference is $= \frac{360}{11} = 32 + \frac{8}{11}$

$\therefore\ a + b + c = 51$

Answer is 51

8. $x^{2^n} + x + 1$ should be divisible by $\omega$ and $\omega^2$

$n = 3, 5, 7, ......, 99$

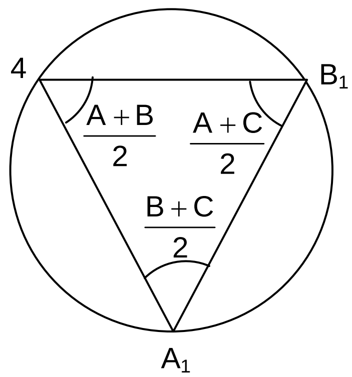
Satisfy

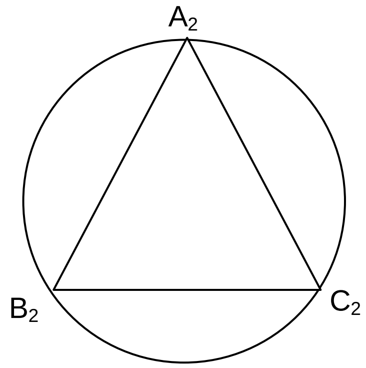
So number of values of $x = \frac{100}{2} - 1 = 49$

9. $\frac{311}{99}$

$p = 311; q = 99 \Rightarrow p - 3q = 311 - 297 = 14$

10.




$\therefore$ Smallest angle

$\therefore \frac{180+40}{4} = 55^o$

$$A_2 = \frac{\frac{A+B}{2} + \frac{A+C}{2}}{2} = \frac{180+A}{4}$$

$$B_2 = 180 + \frac{B}{4}$$

$$C_2 = 180 + \frac{C}{4}$$

11. $\sin kC = \frac{1 - \cos A \cos B}{\sin A \sin B} \le 1 \Rightarrow 1 \le \cos A \cos B + \sin A \sin B$

$\Rightarrow \cos(A - B) \ge 1 \Rightarrow \cos(A - B) = 1 \Rightarrow A = B \Rightarrow \sin kC = 1$

$\Rightarrow kC = 90^o \Rightarrow C = \frac{90^o}{k}$

$k = 1, 3, 5, 9, 15, 45$

Number of integer values = 6

12. The equation $\frac{1}{\sqrt{a_1}} + \frac{1}{\sqrt{a_2}} = 1$ has no solution for distinct $a_1$ and $a_2$

Now, $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$ i.e. $\frac{1}{\sqrt{4}} + \frac{1}{\sqrt{9}} + \frac{1}{\sqrt{36}} = 1$

Therefore 3 is a good number but 2 is not.

Now if k is a good number, then k+1 is also good because if

$$\frac{1}{\sqrt{a_1}}+\frac{1}{\sqrt{a_2}}+\frac{1}{\sqrt{a_3}}+...+\frac{1}{\sqrt{a_k}}=1$$

then $\frac{1}{\sqrt{4}}+\frac{1}{\sqrt{4a_1}}+\frac{1}{\sqrt{4a_2}}+\frac{1}{\sqrt{4a_3}}+....+\frac{1}{\sqrt{4a_k}}=1$

Hence $f(n)=3+4+...+(n+2)=\frac{n(n+5)}{2}$

If $f(n)|f(n+5)$

$n|n+10$

$n|10$

$\therefore n=1,2,5,10$

Hence required sum is 18

13. $(x_1+x_2+......+x_{101})^2=101+2S$

$2S=(x_1+x_2+.......+x_{101})^2-101$

$S=\frac{1}{2}\left\lfloor(x_1+x_2+.....+x_{101})^2-101\right]\geq 0$

$(x_1-x_2+......x_{101})^2\geq 101$

$|x_1+x_2+....+x_{101}|\geq 10....$

$\Rightarrow S_{min}=\frac{1}{2}\left\lfloor 11^2-101\right]=10$

14. $9N+7=P^2$

Or $N=\frac{P^2-7}{9}$

For P = 22 N = 53

Answer is 53

15. Let (x, y) be the pair of number of vertices of a diagonal.

If (x, y) is of type

even – even $\rightarrow$ 3 parallel diagonals

odd – odd $\rightarrow$ 3 parallel diagonals

odd – even $\rightarrow$ 2 parallel diagonals

Pairs of parallel diagonals $= \dfrac{2\left({}^5C_2 \times 3\right) + 5 \times 3 \times 2}{2}$

$= \dfrac{60+30}{2} = 45$

16. $13x + 17y = 10000$

$x = 17n + 16$

$\left.\begin{aligned} x &= 17n + 16 \\ y &= 576 - 13n \end{aligned}\right\}$ Use Bezout's identity

$x - y = 10(3n - 56)$

Since $|x - y|$ should be min. m $\Rightarrow n = 19$

$\Rightarrow x = 339;\ y = 329$

$\therefore$ In next year school paid a total of $17x + 13y = 10040$

Thus school paid Rs. 40/- more.

17. $C = 1 \to b = 4 \to a \to 2$ choices

$b = 3 \to a \to 4$ choices

$b = 2 \to a \to 5$ choices

$b = 1 \to a \to 7$ choices

18 triplets

$C = 2 \to b = 3 \to a \to 1$ choice

$b = 2 \to a \to 3$ choice

$b = 1 \to a \to 5$ choice

9 triplets

$C = 3 \to b = 2 \to a \to 1$ choice

$b = 1 \to a \to 2$ choices

3 triplets

Possible triplets = 18 + 9 + 3 = 30

18. Let $a = Hx$ $\quad x < y$

$b = Hy$ $\quad$ H = H. C. F. of a, b

L.C.M = Hxy

Now $495H = HXY$

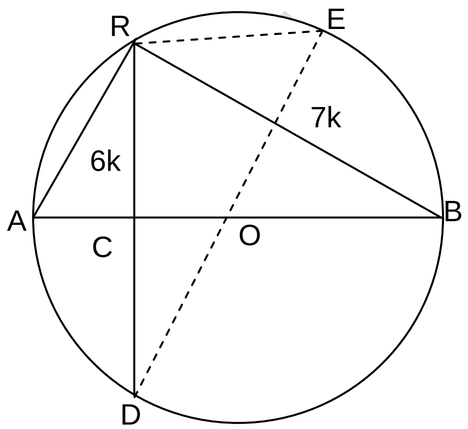
XY = 495

9 (55) $\quad H = 12, 13, ......18 \rightarrow 7$ values

11 (45) $\quad$ H = 10, 11, 12………..22 $\rightarrow$ 13 values

$\therefore$ **Total values = 7 + 13 = 20**

19. $CD = \sqrt{42}k, DE = 13k \Rightarrow RE = k$




$$\frac{[ABD]}{[CDE]} = \frac{\frac{1}{2} \times AB \times CD}{\frac{1}{2} \times CD \times RE} = \frac{AB}{RE} = \frac{13k}{k} = 13$$

Ans. 13

**20. Bonus as Question is wrong.**

21. $E = \{a, b, c, d, e\}$

A and B are of type $\{a\}, \{b, c, d, e\}$

ÒR

A and B are of type $\{a, b\}, \{c, d, e\}$

$ab + cde$ should be odd which can be obtained when a and b are 5 and 7 respectively and c, d, e are 6, 8, 9 respectively.

$\Rightarrow 35 + 432 = 467$

$\Rightarrow$ Answer 4 + 6 + 7 = 17

22. $\sqrt{n-1} < \sqrt{n} < \sqrt{n+1} \Rightarrow \sqrt{n+1} - \sqrt{n} < \frac{1}{2\sqrt{n}} < \sqrt{n} - \sqrt{n-1}$

$$\Rightarrow \sqrt{1600} - 1 < \sum_{n=1}^{1599} \frac{1}{2\sqrt{n}} < \sqrt{1599}$$

$$\Rightarrow 78 < \sum_{n=1}^{1599} < \frac{1}{2\sqrt{n}} < 79.97$$

Answer = 78

23. For the sum PA+PB+PC+PD to be minimum, P must be the point of intersection of diagonals. Moreover since it is a cyclic quadrilateral we have by Power of Point theorem,

$$DP \times PB = AP \times PC \Rightarrow 6 \times 4 = 3 \times 8 = 24.$$

So ABCD is cyclic

Area of quadrilateral $= \Delta APB + \Delta BPC + \Delta CPD + \Delta PDA = \frac{1}{2}(3\times4 + 4\times8 + 8\times6 + 6\times3) = 55$

24. $f(n) = {}^nC_0 2^n + {}^nC_2 2^{n-2} + {}^nC_4 2^{n\ \ 4}......$

$$= \frac{(2+1)^n + (2-1)^n}{2}$$

$$f(n) = \frac{3^n + 1}{2}$$

$$\frac{f(9)}{f(3)} = \frac{3^9+1}{3^3+1} = 703$$

Prime factors of 703 = 19 and 37

Largest prime factor = 37

25. $TB.TD = TD^2$

$16(16+d) = y^2$ .........(1)

Also, by similarly

$$\frac{16+r}{48+y} = \frac{r}{48}$$

$$ry = 48 \times 16$$

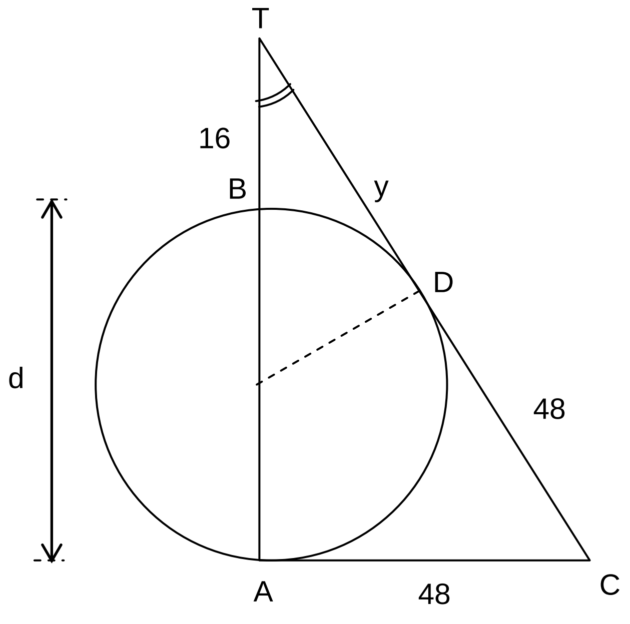
$dy = 48 \times 32$ ...........(2)

$16d^2(16+d) = d^2y^2$ (From (1))

$$16d^2(16+d) = 48^2.32^2$$

$$d^2(16+d) = 48^2 \times 64$$

Clearly, d = 48 satisfies



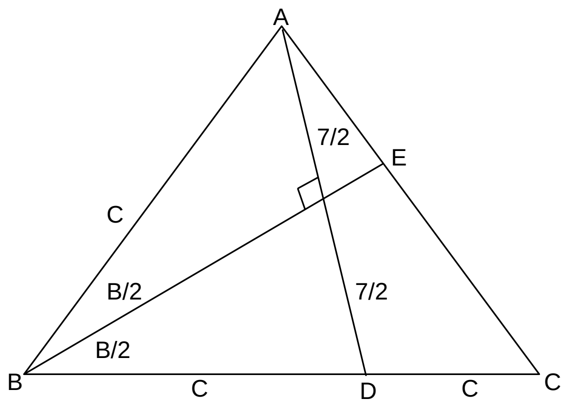
26. Min. value occurs for x = 26, y = 6, z = 4
$\therefore$ Answer = 50

27. BHARAT
1 2 3 4 5 6

Choose two positions for the A's out of $1, 2, 4, 6$ in ${}^4C_2$ ways.
To arrange the rest of four letters use Principle of Inclusion and Exclusion.
Required number of ways
$= {}^4C_2\left(4! - 3! - 3! + 2!\right) = 84$ ways

28. $r = \frac{\Delta}{S} = 15$
$r_1 + r_2 + r_3 = r = 15$ (Property)

29. a = 2c




$BE = \frac{2ac}{a+c}\cos\left(\frac{B}{2}\right)$ (Property)

$\Rightarrow \cos\frac{B}{2} = \frac{27}{4c} \Rightarrow c\cos\frac{B}{2} = \frac{27}{4} = BP$

$\Rightarrow$ Area $\left(\Delta ABC\right) = 4\,Ar\left(\Delta ABP\right)$

$= 4 \times \frac{1}{2} \times \frac{27}{4} \times \frac{7}{2} = \frac{63}{4} = 47.25$

**Answer 47**

30. Total number of values of n = 64 .