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MTA Pre-Regional Mathematical Olympiad (PRMO), 2019

Date: August 11, 2019 **Time: 10 Am to 1 PM**

Number of Questions 30: **Max Marks: 102**

INSTRUCTION

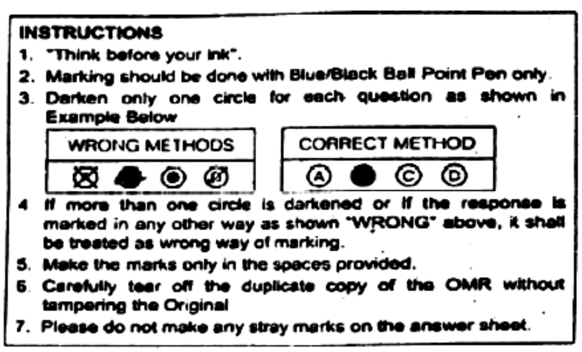
1. Use of mobile phones, smartphones, ipads, calculators, programmable wrist watches is **STRICTLY PROHIBITED**. Only ordinary pens and pencils are allowed inside the examination hall.

2. The correction is done by machine through scanning. On OMR sheet, darken bubbles completely with a black pencil or a black blue pen. Darken the bubbles completely only after you are sure of your answer : else, erasing lead to the OMR sheet getting damaged and the machine may not be able to read the answer.

3. The name, email address and date of birth entered on the OMR sheet will be your login credentials for accessing your PROM score.

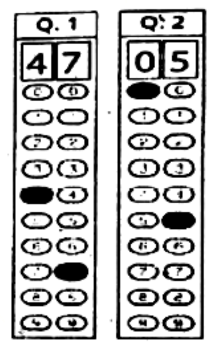
4. Incomplete /incorrectly and carelessly filled information may disqualify your candidature.

5. Each question has a one or two digit number as answer. The first diagram below shows improper and proper way of darkening the bubble with detailed instructions. The second diagram shows how to mark a 2-digit number and a 1-digit number.


INSTRUCTIONS
1. "Think before your ink".
2. Marking should be done with Blue/Black Ball Point Pen only.
3. Darken only one circle for each question as shown in Example Below

WRONG METHODS	CORRECT METHOD
	Ⓐ ● Ⓒ Ⓓ

4. If more than one circle is darkened or if the response is marked in any other way as shown "WRONG" above, it shall be treated as wrong way of marking.
5. Make the marks only in the spaces provided.
6. Carefully tear off the duplicate copy of the OMR without tampering the Original
7. Please do not make any stray marks on the answer sheet.



6. The answer you write on OMR sheet is irrelevant. The darken bubble will be considered as your final answer.

7. Questions 1 to 6 carry 2 marks each: Questions 7 to 21 carry 3 marks each : Questions 22 to 30 carry 5 marks each.

8. All questions are compulsory.

9. There are no negative marks.

10. Do all rough work in the space provided below for it. You also have pages at the end of the question paper to continue with rough work.

11. After the exam, you may take away the Candidate’s copy of the OMR sheet.

12. Preserve your copy of OMR sheet till the end of current olympiad season. You will need it further for verification purposes.

13. You may take away the question paper after the examination.

1. From a square with sides of length 5, triangular pieces from the four corners are removed to form a regular octagon. Find the area removed to the nearest integer?

2. Let $f(x) = x^2 + ax + b$. If for all non zero real x
$f\left(x + \frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$ and the roots of $f(x) = 0$ are integers, what is the value of $a^2 + b^2$?

3. Let x_1 be a positive real number and for every integer $n \geq 1$ let $x_{n+1} = 1 + x_1x_2 ..., x_{n-1}x_n$. If $x_5 = 43$, what is the sum of digits of the largest prime factor of x_6?

4. An ant leaves the anthill for its morning exercise. It walks 4 feet east and then makes a 160^o turn to the right and walks 4 more feet. It then makes another 160^o turn to the right and walks 4 more feet. If the ant continues this pattern until it reaches the anthill again. What is the distance in feet it would have walked?

5. Five persons wearing badges with numbers 1, 2, 3, 4, 5 are seated on 5 chairs around a circular table. In how many ways can they be seated so that no two persons whose badges have consecutive numbers are seated next to each other? (Two arrangements obtained by rotation around the table are considered different.)

6. Let $\overline{abc}$ be a three digit number with non zero digits such that $a^2 + b^2 = c^2$. What is the largest possible prime factor of abc?

7. On a clock, there are two instants between 12 noon and 1 PM, when the hour hand and the minute hand are at right angles. The difference in minutes between these two instants is written as $a + \frac{b}{c}$, where a, b, c are positive integers, with $b < c$ and $\frac{b}{c}$ in the reduced form. What is the value of $a + b + c$?

8. How many positive integers n are there such that $3 \leq n \leq 100$ and $x^{2^n} + x + 1$ is divisible by $x^2 + x + 1$?

9. Let the rational number $\frac{p}{q}$ be closest to but not equal to $\frac{22}{7}$ among all rational numbers with denominator < 100. What is the value of $p - 3q$?

10. Let ABC be a triangle and let be Ω its circumcircle. The internal bisectors of angles A, B and C intersect Ω at A_1, B_1 and C_1 respectively and the internal bisectors of angles A_1, B_1 and C_1 of the triangle $A_1B_1C_1$ intersect Ω at A_2, B_2 and C_2, respectively. If the smallest angle of triangle ABC is 40^o. What is the magnitude of the smallest angle of triangle $A_2B_2C_2$ in degrees?

11. How many distinct triangles ABC are there, up to similarity, such that the magnitudes of angles A, B and C in degrees are positive integers and satisfy $\cos A \cos B + \sin A \sin B \sin kC = 1$ for some positive integer k, where kC does not exceed 360°?

12. A natural number k > 1 is called good if there exist natural numbers $a_1 < a_2 < ... < a_k$ such that $\frac{1}{\sqrt{a_1}} + \frac{1}{\sqrt{a_2}} + ... + \frac{1}{\sqrt{a_k}} = 1$
Let f(n) be the sum of the first n good numbers $n \geq 1$. Find the sum of all values of n for which $f(n+5)/f(n)$ is an integer.

13. Each of the numbers $x_1, x_2 ..., x_{101}$ is ± 1. What is the smallest positive value of $\sum_{1 \leq i < j \leq 101} x_i x_j$?

14. Find the smallest positive integer $n \geq 10$ such that n + 6 is a prime and 9n + 7 is a perfect square.

15. In how many ways can a pair of parallel diagonals of a regular polygon of 10 sides be selected?

16. A pen costs Rs 13 and a note book costs Rs 17. A school spends exactly Rs 10000 in the year 2017 – 18 to buy x pens and y note books such that x and y are as close as possible (i.e., $|x - y|$ is minimum). Next year, in 2018-19, the school spends a little more than Rs 10000 and buys y pens and x note books. How much **more** did the school pay?

17. Find the number of ordered triples (a, b, c) of positive integers such that $30a + 50b + 70c \leq 343$.

18. How many ordered pairs (a, b) of positive integers with a < b and $100 \leq a, b \leq 1000$ satisfy GCD(a, b) : LCM(a, b) = 1 : 495?

19. Let AB be a diameter of a circle and let C be a point on the segment AB such that AC : CB = 6 : 7. Let D be a point on the circle such that DC is perpendicular to AB. Let DE be the diameter through D. If [XYZ] denotes the area of the triangle XYZ, find ABD / CDE to the nearest integer.

20. Consider the set E of all natural numbers n such that when divided by 11, 12, 13 respectively the remainders in that order are distinct prime numbers in an arithmetic progression. If N is the largest number in E, find the sum of digits of N.

21. Consider the set E= {5, 6, 7, 8, 9}. For any partition {A, B} of E with both A and B non-empty consider the number obtained by adding the product of elements of A to the product of elements of B. Let N be the largest prime number among these numbers. Find the sum of the digits of N.

22. What is the greatest integer not exceeding the sum $\sum_{n=1}^{1599} \frac{1}{\sqrt{n}}$?

23. Let ABCD be a convex cyclic quadrilateral. Suppose P is a point in the plane of the quadrilateral such that the sum of its distances from the vertices of ABCD is the least. If

$$\{PA, PB, PC, PD\} = \{3, 4, 6, 8\},$$

What is the maximum possible area of ABCD?

24. A $1 \times n$ rectangle ($n \geq 1$) is divided into n unit (1×1) squares. Each square of this rectangle is coloured red, blue or green. Let f(n) be the number of colourings of the rectangle in which there are an even number of red squares. What is the largest prime factor of f(9)/f(3)? (The number of red squares can be zero)

25. A village has circular wall around it, and the wall has four gates pointing north, south, east and west. A tree stands outside the village, 16 m north of the north gate and it can be just seen appearing on the horizon from a point 48 m east of the south gate. What is the diameter in meters of the wall that surrounds the village?

26. Positive integers x, y, z satisfy $xy + z = 160$. Compute the smallest possible value of $x + yz$.

27. We will say that a rearrangement of the letters of a word has no fixed letters if, when the rearrangement is placed directly below the word, no column has the same letter repeated. For instance, H B R A T A is a rearrangement with no fixed letters of B H A R A T. How many distinguishable rearrangements with no fixed letters does B H A R A T have? (The two As are considered identical)

28. Let ABC be a triangle with sides 51, 52, 53. Let Ω denote the incircle of ΔABC. Draw tangents to Ω which are parallel to the sides of ABC. Let r_1, r_2, r_3 be the inradii of the three corner triangles so formed. Find the largest integer that does not exceed $r_1 + r_2 + r_3$.

29. In triangle ABC, the median AD (with D on BC) and the angle bisector BE (with E on AC) are perpendicular to each other. If AD = 7 and BE = 9, find the integer nearest to the area of triangle ABC.

30. Let E denote the set of all natural number n such that $3 < n < 100$ and the set $\{1, 2, 3, \ldots, n\}$ can be partitioned in to 3 subsets with equal sums. Find the number of elements of E.