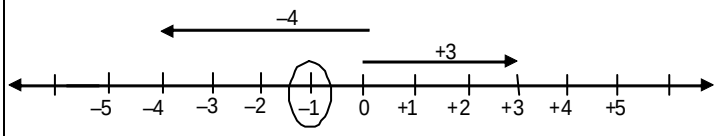
INTEGERS

Integers

Integers: All natural numbers, 0 and negative of natural numbers form the collection of all integers.

I or Z = { ..., – 3, – 2, – 1, 0, 1, 2, 3, ... }



- **Additive Inverse**

  For any integer a, we have

  a + (– a) = (– a) + a = 0

  The opposite of an integer a is (–a).

  The sum of an integer and its opposite is 0.

  Additive inverse of a is (–a).

  Similarly, additive inverse of (–a) is a.

- **Positive integers :**

  The set $\mathbf{I^+}$ **= {1, 2, 3, 4, … }**is the set of all positive integer, Clearly, **positive integers and natural numbers are synonyms**.

- **Negative integers :**

  The set $\mathbf{I^-}$ = {–1, –2,, – 3, … } is the set of all non-negative integers.

- **Non-negative integers :**

  The set **{0, 1, 2, 3 …..}** is the set of all non-negative integers.

  - **0 is neither positive nor negative.**
  - **All non-negative integers are whole numbers.**

Rules of Integers

- Sum of two positive integers is an integer.

  i.e., 1 + 2 = 3

- Sum of two negative integers is an integer.
  i.e., (– 1) + (– 2) = – 3

- Product of two positive integers is an positive integer.
  i.e., (+ 1) × (+ 2) = 2

- Product of two negative integers is an positive integer.
  i.e., (– 1) × (– 2) = 2

- Product of negative integer and positive integer is an negative integer.
  i.e., (– 1) × (+ 2) = – 2

- Sum of an integer and its additive inverse is equal to zero.
  i.e., (– 1) + (+ 1) = 0

- Product of an integer and its reciprocal is equal to 1.

  i.e., (2) × $\frac{1}{2}$ = 1 $\frac{1}{2}$

Absolute Value of Integers

The absolute value of an integer is the distance of that integer from 0 irrespective of the direction, i.e. negative or positive.

- **The absolute value of 5 is equal to 5.**
  |5| = 5

- **The absolute value of – 5 is equal to 5.**
  |–5| = 5

Rules for addition & Subtraction

- When **adding** integers with like signs **(both positive or both negative),** add their absolute values, and place the common sign before the sum.

  i.e., (+2) + (+5) = 7

  (–2) + (–5) = – 7

- When **adding** integers of **unlike signs**, find the difference of their absolute values and give the sign of larger integer.

  i.e., (+2) + (–5) = –3

  (–2) + (+5) = 3

- When subtracting two integer, change the sign of the second number which is being subtracted, and follow the rules of addition.

  i.e., (–7) – (+ 4) = (–7) + (–4) = –11

Multiplication & Division of Signed Integer Numbers

(i) If the integers have different signs, then the result is negative.

i.e., (+2) × (–3) = –6

(+6) ÷ (–3) = –2

(ii) If both the integers have same signs, then the result is negative.

i.e., (+2) × (+3) = +6

(–6) ÷ (–3) = –2

Ex.1 Find each of the following products:

(i) (– 115) × 8

(ii) {9 × (–3)} × (–6)

Sol. **(i)** We have,

(– 115) × 8 = – (115 × 8) == – 920

(ii) We have,

9 × (–3) × (–6) = {9 × (–3)} × (–6)

= – (9 × 3) × (–6)

= – 27 × (–6)

= 27 × 6 = 162.

Ex.2 Evaluate (–48) ÷ 12

Sol. $(-48) \div 12 = \frac{-48}{12} = -4$

Ex.3 Evaluate (–48) ÷ (–16).

Sol. $(-48) \div (-16) = \frac{-48}{-16} = 3.$

Properties of Integers

(i) Closure Property

a + b = integer

a × b = integer

(ii) Associative Property

a + (b + c) = (a + b) + c

a × (b × c) = (a × b) × c

(iii) Commutative Property

a + b = b + a

a × b = b × a

(iv) Distributive Property

a × (b + c) = a × b + a × c

(v) Additive Inverse Property

a + (–a) = 0

–a is additive inverse of integer a.

(vi) Multiplicative Inverse Property

$a \times (\frac{1}{a}) = 1$

Hence, $\frac{1}{a}$ is the multiplicative inverse of integer a.

(vii) Identity Property

a + 0 = a

a × 1 = a

Ex.4 Add the following

(a) 3 + 5

(b) 5 + (– 4)

(c) – 1 + 3

(d) – 11 + (– 9)

Ans. (a) 3 + 5 = (+ 3) + (+ 5) = 8

(b) 5 + (– 4) = – 5 + 4 = – 1

(c) – 1 + 3 = + (3 – 1) = + 2

(d) (–11) + (–9) = –11–9 = –20

WORKSHEET

1. The product of two integers is 12, if one integer is – 3 then the other one is :

(A) + 4 (B) – 4

(C) 3 (D) – 3

2. On subtracting (–6) from 0, we get :

(A) + 6 (B) 0

(C) – 5 (D) – 7

3. The additive inverse of – 6 is :

(A) 6 (B) 0

(C) – 5 (D) – 7

4. 30 × (– 23) + 30 × 14 = ?

(A) – 270 (B) 270

(C) 1110 (D) – 1110

5. (– 8) ÷ 0 = ?

(A) – 8 (B) 0

(C) 8 (D) Not defined

6. By how much does – 3 exceed – 5 ?

(A) – 2 (B) 2

(C) 8 (D) – 8

7. What must be subtracted from – 3 to get – 9 ?

(A) – 6 (B) 12

(C) 6 (D) – 12

8. How much less than – 8 is – 3 ?

(A) – 5 (B) 5

(C) 11 (D) – 11

9. The sum of two integers is 93. If one of them is – 59 the other one is :

(A) 34 (B) – 34

(C) 152 (D) – 152

10. Verify a – (–b) = a + b for the following values of a and b.

(i) a = 21, b = 18

(ii) a = 118, b = 125

(iii) a = 75, b = 84

(iv) a = 28, b = 11

11. Find each of the following products:

(i) (–18) × (–10) × 9

(ii) (–20) × (–2) × (–5) × 7

(iii) (–1) × (–5) × (– 4) × (– 6)

12. Use the sign of >, < or = in the box to make the statements true.

(i) (–8) + (-4) ☐ (–8) – (-4)

(ii) (–3) + 7 – (19) ☐ 15 – 8 + (–9)

(iii) 23 – 41 + 11 ☐ 23 – 41 – 11

(iv) 39 + (–24) – (15) ☐ 36 + (–52) – (–36)

(v) – 231 + 79 + 51 ☐ – 399 + 159 + 81

13. Compare: (–2 – 5) × (–6) and (–2) + (–5) × (–6).

14. Find the reciprocal of the follwing rational numbers :

(i) $\frac{-3}{4}$ (ii) 0

(iii) $\frac{6}{11}$ (iv) $\frac{5}{-9}$

15. Multiply $\frac{5}{8}$ by the reciprocal of $\frac{-3}{8}$

16. Find the value of the following using properties of multiplication.

$37 \times 865 + 18 \times 865 - 49 \times 865 - 6 \times 865$

17. Write the predecessor and successor of the following numbers 4, –4, 6, 1, b.

18. Simplify : $126 \times 55 + 126 \times 45$

19. Resolve the brackets and simplify: $(28 \div 2) \div (56 \div 8)$.

(A) 1 (B) 4

(C) 3 (D) 2

20. Write five pair of integers (m, n) such that $m \div n = -3$.

HINTS & SOLUTIONS

Sol.1 (B)

_____ × (–3) = 12

Here product has positive sign, so other number must be of negative sign.

(– 4) × (–3) = 12

Sol.2 (A)

0 – (–6) = 0 + 6 = 6.

Sol.3 (A)

Additive inverse of – 6 is 6.

Sol.4 (A) By distributive property

30 × (–23 + 14)

30 × (– 9) = – 270

Sol.5 (D) When any integer divide by 0 result is not define.

Sol.6 (B)

(– 3) – (–5) = – 3 + 5 = + 2

Sol.7 (C)

(–3) – _____ = – 9

(– 3) – (–9) = – 3 + 9 = 6

Sol.8 (A)

– 8 – (–3) = – 8 + 3 = – 5

Sol.9 (C)

____ + (–59) = 93

93 – (–59) = 93 + 59 = 152

Sol.10 (i) a = 21, b = 18

a – (–b) = 21 – (–18) = 21 + 18 = 39

a + b = 21 + 18 = 39

⇒ a – (–b) = a + b = 39

(ii) a = 118, b = 125

a – (–b) = 118 – (–125) = 118 + 125 = 243

a + b = 118 + 125 = 243

(iii) a = 75, b = 84

a + b = 75 –(–84) = 75 + 84 = 159

a + b = 75 + 84 = 159

⇒ a – (–b) = 159

(iv) a = 28, b = 11

a–(–b) = 28 –(–11) = 28 + 11 = 39

a + b = 28 + 11 = 39

⇒ a – (–b) = a + b = 39

Sol.11

(i) (–18) × (–10) × 9 = [(–18) × (–10)] × 9 = 180 × 9 = 1620

(ii) (–20) × (–2) × (–5) × 7 = – 20 × (–2 × –5) × 7 = [–20 × 10] × 7 = – 1400

(iii) (–1) × (–5) × (– 4) × (– 6) = [(–1) × (–5)] × [(– 4) × (– 6)] = 5 × 24 = 120

Sol.12

(i) (–8) + (–4) ☐ (–8) – (–4)

⇒ – 8 – 4 ☐ – 8 + 4

⇒ – 12 < – 4

(ii) (–3) + 7 – (19) ☐ 15 – 8 + (–9)

⇒ –3 + 7 – 19 ☐ 15 – 8 – 9

⇒ – 15 < – 4

(iii) 23 – 41 + 11 ☐ 23 – 41 – 11

⇒ – 7 > – 29

(iv) 39 + (– 24) – 15 ☐ 36 + (–52) – (–36)

⇒ 39 – 24 – 15 ☐ 36 – 52 + 36

⇒ 0 > 20

(v) – 231 + 79 + 51 ☐ – 399 + 159 + 81

⇒ – 101 > – 159

Sol.13 $(-2-5)\times(-6) = -7\times -6 = 42$

$-2+(-5)\times(-6) = -2+30 = 28$

Since $42 > 28$, So $(-2-5)\times(-6)$ is greater.

Sol.14 (i) Reciprocal of $\frac{-3}{4}$ is $\frac{-4}{3}$

(ii) Reciprocal of 0, i.e. $\frac{1}{0}$ is not defined.

(iii) Reciprocal of $\frac{6}{11}$ is $\frac{11}{6}$

(iv) Reciprocal of $\frac{5}{-9} = \frac{-9}{5}$.

Sol.15 Reciprocal of $\frac{-3}{8} = \frac{-8}{3}$

$$\frac{5}{\cancel{8}_1}\times\frac{\cancel{-8}_1}{3} = \frac{-5}{3}$$

Sol.16 $37\times865 + 18\times865 - 49\times865 - 6\times865$

$= 865\times(37+18-49-6)$

$= 865\times(55-55) = 865\times0 = 0$

Sol.17

Predecessor	3	–5	5	b – 1
Number	4	–4	6	n^2
Successor	5	–3	7	n^2+1

Sol.18 $126\times55 + 126\times45 = 126\times(55+45) =$

$126\times100 = 12600$

Sol.19. (D)

$28\div2 = 14$

$56\div8 = 7$

$(28\div2)\div(56\div8) = 14\div7 = 2$

Sol.20 (i) $(-3, 1) = (-3)\div1 = -3$

(ii) $(9, -3) = 9\div(-3) = -3$

(iii) $(6, -2) = 6\div(-2) = -3$

(iv) $(-24, 8) = (-24)\div8 = -3$

(v) $(18, -6) = 18\div(-6) = -3$