CLASSIFICATION OF NUMBERS

INTRODUCTION

Number System is a method of writing numerals to represent numbers.

- ❖ Ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are used to represent any number (however large it may be) in our number system.
- ❖ Each of the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 is called a digit or a figure.

(1) Natural numbers :

Counting numbers are known as natural numbers.

N = { 1, 2, 3, 4, ... }.

(2) Whole numbers :

All natural numbers together with 0 form the collection of all whole numbers.

W = { 0, 1, 2, 3, 4, ... }.

The four properties of whole numbers are as follows:

(a) Closure Property

(b) Associative Property

(c) Commutative Property

(d) Distributive Property

Let's explore them in details

(a) Closure Property

- Add any two whole numbers and you will see that the sum is again a whole number.

  For example: 0 + 2 = 2

  Here, 2 is a whole number.

- In the same way, multiply any two whole numbers and you will see that the product is again a whole number.

  For example: 3 × 5 = 15

  Here, 15 is a whole number.

Thus the set of whole numbers, W is closed under addition and multiplication.

The closure property of W is stated as follows:

For all a, b ∈ W

a + b ∈ W
and
a × b ∈ W

The sum and product are always whole numbers!

Interesting, isn't it ?

(b) Associative Property

The sum and the product of any three whole numbers remain the same though the grouping of numbers is changed.

Ex. (1 + 2) + 3 = 1 + (2 + 3)

because

(1 + 2) + 3 = 3 + 3 = 6

1 + (2 + 3) = 1 + 5 = 6

Ex. $(1\times2)\times3=1\times(2\times3)$

because

$(1\times2)\times3=2\times3=6$

$1\times(2\times3)=1\times6=6$

Thus the set of whole numbers, W is associative under addition and multiplication.

The associative property of W is stated as follows:

For all a, b, c ∈ W

$$a + (b + c) = (a + b) + c$$
and
$$a \times (b \times c) = (a \times b) \times c$$

The sum and product are NOT changed even when the grouping of numbers is changed.

Have you noticed this?

(c) Commutative Property

The sum and the product of two whole numbers remain the same even after interchanging the order of the numbers.

Ex. $2 + 3 = 3 + 2$

because

$2 + 3 = 5$

$3 + 2 = 5$

Ex. $2 \times 3 = 3 \times 2$

because

$2 \times 3 = 6$

$3 \times 2 = 6$

Thus the set of whole numbers, W is commutative under addition and multiplication.

The commutative property of W is stated as follows:

For all a, b ∈ W

$$a + b = b + a$$
and
$$a \times b = b \times a$$

The sum and product are NOT changed even when the numbers are interchanged.

Have you observed this ?

Let us summaries these **three properties of whole numbers in a table.**

Operation	Closure Property	Associative Property	Commutative Property
Addition	yes	yes	yes
Subtraction	no	no	no
Multiplication	yes	yes	yes
Division	no	no	no

(d) Distributive Property

The distributive property of multiplication over addition is

$$a \times (b + c) = a \times b + a \times c$$

Ex. $3 \times (2 + 5) = 3 \times 2 + 3 \times 5$

because

$3 \times (2 + 5) = 3 \times 7 = 21$

$3 \times 2 + 3 \times 5 = 6 + 15 = 21$

The distributive property of multiplication over subtraction is

$$a \times (b - c) = a \times b - a \times c$$

Ex. $3 \times (2 - 5) = 3 \times 2 - 3 \times 5$

because

$3 \times (2 - 5) = 3 \times (-3) = -9$

$3 \times 2 - 3 \times 5 = 6 - 15 = -9$

(3) **Integers:** All natural numbers, 0 and negative of natural numbers from the collection of all integers.

I or Z = { ..., –3, –2, –1, 0, 1, 2, 3, ... }.

(4) **Even Numbers:** All integers which are divisible by 2 are called even numbers. Even numbers are denoted by the expression 2n, where n is any integer. So, if E is a set of even numbers, then E = { ... , – 4, –2, 0, 2, 4,...}.

(5) **Odd Numbers:** All integers which are not divisible by 2 are called odd numbers. Odd numbers are denoted by the general expression 2n –1 where n is any integer. If O is a set of odd numbers, then O = {..., –5, –3, –1, 1, 3, 5,...}.

Properties of Even and Odd numbers

Addition

Even + even = even

Odd + odd = even

Odd + even = odd

Multiplication

The product of two even numbers is even.

The product of two odd numbers is odd.

The product of an even number and an odd number is even.

Ex.1 If a and b are odd integers, and c is even, which of the following must be an odd integer?

(a) abc (b) abb + a

(b) a + b + c (d) a (b + c)

Sol. **(d)** a (b + c)

Explanation:

Even numbers come in the form 2x, and odd numbers come in the form (2x + 1), where x is an integer. If this is confusing for you, simply plug in numbers such as 1, 2, 3, and 4 to find that:

Any odd number + any even number = odd number

Any odd number + any odd number = even number

Any even number x any number = even number

Any odd number x any odd number = odd number

a (b + c) = odd x (odd + even)

= odd x (odd) = odd

Ex.2 The sum of seven consecutive even integers is 0.

Col.A: The product of the seven integers

Col.B: 2

(a) Column B is greater.

(b) The two quantities are equal.

(c) The relationship cannot be determined from the given information.

(d) Column A is greater.

Sol. **(a)** Column B is greater.

Explanation: For the sum of 7 consecutive even integers to be zero, the only sequence possible is –6, –4, –2, 0, 2, 4, 6.

This can be determined algebraically by assigning the lowest number in the sequence to be "y" and adding 2 for each consecutive even integer, and then setting this equal to zero.

y, y + 2, y + 4, y + 6 . . .

The product of any number and 0 is 0.

Therefore, column B must be greater.

Ex.3 If m and n are both odd integers, which of the following is not necessarily odd?

(a) m^2n (b) 2m−n

(b) $m + n^2$ (d) m − 2n

Sol. **(c)** $m + n^2$

Explanation:

With many questions like this, it might be easier to plug in numbers rather than dealing with theoretical variables.

However, given that this question asks for the expression that is not always even or odd but only not necessarily odd, the theoretical route might be our only choice.

Therefore, our best approach is to simply analyze each answer choice.

m^2n : Since m is odd, m^2 is also odd, since and odd number multiplied by an odd number yields an odd product. Since n is also odd, multiplying it by m^2 will again yield an odd product, so this expression is always odd.

m−2n: Since n is odd, multiplying it by 2 will yield an even number. Subtracting this number from m will also give an odd result, since an odd number minus an even number gives an odd number. Therefore, this answer is also always odd.

2m−n: Since m is odd, multiplying it by 2 will give an even number. Since n is odd, subtracting it from our even number will give an odd number, since an even number minus and odd number is always odd. Therefore, this answer will always be odd.

m+n^2: Since both numbers are odd, there sum will be even. However, dividing an even number by another even number (2 in our case) does not always produce an even or an odd number.

For example, 5 and 7 are both odd. Their sum, 12, is even. Dividing by 2 gives 6, an even number. However, 5 and 9 are also both odd. Their sum, 14, is even, but dividing by 2 gives 7, an odd number. Therefore, this expression isn't necessarily always odd or always even, and is therefore our answer.

Ex.4 If a is even, and b is odd. Which of the following must be odd?

Possible Answers:

(a) abb + a (b) abb−1

(b) ab + ab (d) a^2b^3

Sol. **(b)** abb−1

Explanation: To solve, pick numbers to represent a and b. Let a = 2 and b = 3. Now try each of the equations given:

ab + ab = 2(3) + 2(3) = 6 + 6 = 12.

$a^2b^3 = (2)^2\,(3)^3 = 4*27 = 108$

abb + a = 2(3) (3) + 2 = 18 + 2 = 20

abb −1 = 2(3) (3) – 1 = 18 – 1 = 17

Only abb−1 works and is thus our answer.

(6) Rational numbers: These are real numbers which can be expressed in the form of p/q, where p and q are integers and $q \neq 0$. e.g. 2/3, 37/15, –17/19.

- ❖ All natural numbers, whole numbers and integers are rational numbers.
- ❖ Rational numbers include all Integers (without any decimal part to it), terminating fractions (fractions in which the decimal parts are terminating e.g. 0.75, – 0.02 etc.) and also non-terminating but recurring decimals e.g. 0.666....., – 2.333...., etc.

(7) Irrational Numbers: All real number which are not rational are irrational numbers. These are non-recurring as well as non-terminating type of decimal numbers. For Ex.: $\sqrt{2}$, $\sqrt[3]{4}$, $2 + \sqrt{3}$, $\sqrt{2+\sqrt{3}}$, $\sqrt[4]{\sqrt[7]{3}}$ etc.

(8) Real numbers: Numbers which can represent actual physical quantities in a meaningful way are known as real numbers. These can be represented on the number line. Number line is geometrical straight line with arbitrarily defined zero (origin).

(9) Prime numbers: All natural numbers that have one and itself only as their factors are called prime numbers i.e. prime numbers are exactly divisible by 1 and themselves. e.g. 2, 3, 5, 7, 11, 13, 17, 19,23, ...etc. If P is the set of prime number then P = {2, 3, 5, 7, ...}.

(10) Co-prime Numbers : If the H.C.F. of the given numbers (not necessarily prime) is 1 then they are known as co-prime numbers.

e.g. 4, 9 are co-prime as H.C.F. of (4, 9) = 1

- ❖ Any two consecutive numbers will always be co-prime.

(11) Twin primes : Pairs of prime numbers which have only one composite number between them are called Twin primes.3, 5 ; 5, 7 ; 11,13 ; 17, 19 ; 29, 31 ; 41, 43 ; 59, 61 and 71, 73 etc. are twin primes.

(12) Composite numbers : All natural numbers, which are not prime are composite numbers. If C is these of composite number then C = {4, 6, 8, 9, 10, 12,...}.

- ❖ 1 is neither prime nor composite number.

(13) Imaginary Numbers: All the numbers whose square is negative are called imaginary numbers.

e.g. 3i, –4i, i, ... ; where i = – 1 .

WORKSHEET

1. Which of the following numbers is an integer ?

(a) – 4 (b) 1/2

(c) 0.3 (d) 0.1

2. Name all the sets to which the following number belongs. $\sqrt{16}$

(a) Irrational

(b) Rational only

(c) Rational & Integer

(d) Rational, Integer, Whole & Natural

3. Natural numbers are also called _____ numbers.

(a) Counting (b) Elementary

(c) Decimal (d) Fractions

4. Which of the following is an example of an integer ?

(a) .35 (b) – 3

(c) $\frac{1}{2}$ (d) $\frac{-1}{2}$

5. 3.54 is known as which number type ?

(a) A rational number

(b) An irrational number

(c) An integer

(d) A natural number

6. Which set of numbers does not include-8 ?

(a) Whole numbers

(b) Integers

(c) Rational numbers

(d) Opposite of whole numbers

7. What are integers ?

(a) Positive and negative whole numbers and zero

(b) The point at which the x and y axes cross (0, 0)

(c) The line across the bottom of the graph

(d) None of these

8. Integers are _________ numbers plus negative numbers.

(a) Irrational (b) Whole

(c) Natural (d) All of these

9. Rational numbers include which types of numbers ?

(a) Integers (b) Natural

(c) Whole (d) All of above

10. Which is not a rational number ?

(a) $\frac{1}{3}$ (b) 0.25

(c) 0.6666666 (d) $\sqrt{2}$

11. Choose the correct number for seven hundred eighty-three.

(a) 70,083 (b) 7,803

(c) 783 (d) 70,803

12. This is another name for the ten numbers [0, 1, 2, 3, 4, 5, 6, 7, 8 and 9] that we use every day.

(a) Factors (b) Dividends

(c) Multiples (d) Digits

13. 0 is a natural number and a whole number.

(a) True (b) False

14. Classify the number – 4.

(a) Whole, integer, rational, real

(b) Integer, irrational, real

(c) Whole, rational, real

(d) Integer, rational, real

15. Classify the number – 7.23.

(a) Integer, rational, real

(b) Integer, irrational, real

(c) Rational, real

(d) Irrational, real

16. Classify the number –1.

(a) Natural, whole, integer, rational, real

(b) Whole, integer, rational, real

(c) Integer, rational, real

(d) Rational, real

17. Identify the type of number. 0.29

(a) Rational (b) Irrational

(c) Whole (d) Real

18. Which is an irrational number ?

(a) $\sqrt{16}$ (b) $\sqrt{100}$

(c) $\sqrt{50}$ (d) $\sqrt{81}$

19. List all the classification for $\frac{1}{-7}$.

(a) Irrational, real

(b) Integer, irrational, real

(c) Integer, rational, real

(d) Rational, real

20. Identify the type of number- $\frac{4}{9}$

(a) Rational (b) Irrational

(c) Real (d) Whole

SOLUTION SHEET

1.	(a)	2	(d)	3.	(a)
4.	(b)	5.	(a)	6.	(a)
7.	(a)	8.	(b)	9.	(d)
10.	(d)	11.	(c)	12.	(d)
13.	(b)	14.	(d)	15.	(c)
16.	(c)	17.	(d)	18.	(c)
19.	(d)	20.	(a)		