OPERATION ON FRACTIONS

FRACTIONS

(1) What is a fraction?

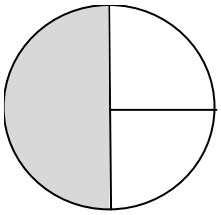
A fraction consists of a numerator (part) on top of a denominator (total) separated by a horizontal line.

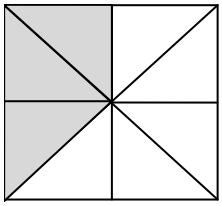
$$\text{Fraction} = \frac{\text{numerator}}{\text{denominator}}$$

For example, the fraction of the circle which is shaded is:

$$\frac{2(\text{parts shaded})}{4(\text{total parts})}$$

In the square on the right, the fraction shaded is $\frac{3}{8}$ and the fraction unshaded is $\frac{5}{8}$.





(2) Equivalent Fractions – Multiplying?

The three circles on the right each have equal parts shaded, yet are represented by different but equal fractions. These fractions, because they are equal, are called equivalent fractions.

Any fraction can be changed into an equivalent fraction by multiplying both the numerator and denominator by the same number

$\frac{1\times2}{2\times2}=\frac{2}{4}$ or $\frac{1\times4}{2\times4}=\frac{4}{8}$ so $\frac{1}{2}=\frac{2}{4}=\frac{4}{8}$

Similarly

$\frac{5\times2}{9\times2}=\frac{10}{18}$ or $\frac{5\times3}{9\times3}=\frac{15}{27}$ so $\frac{5}{9}=\frac{10}{18}=\frac{15}{27}$

You can see from the above examples that each fraction has an infinite number of fractions that are equivalent to it.

(3) Equivalent Fractions – Dividing (Reducing)

Equivalent fractions can also be created if both the numerator and denominator can be divided by the same number (a factor) evenly.

This process is called **"reducing a fraction"** by dividing a common **factor** (a number which divides into both the numerator and denominator evenly).

$\frac{4}{8}\div\frac{4}{4}=\frac{1}{2}$

$\frac{27}{81}\div\frac{9}{9}=\frac{3}{9}$

$\frac{5}{30}\div\frac{5}{5}=\frac{1}{6}$

$\frac{6}{10}\div\frac{2}{2}=\frac{3}{5}$

(4) Simplifying a Fraction (Reducing to its Lowest Terms)

It is usual to reduce a fraction until it can't be reduced any further.

A simplified fraction has no common factors which will divide into both numerator and denominator.

$\frac{27}{81} \div \frac{9}{9} = \frac{3}{9}$

$\frac{3}{9} \div \frac{3}{3} = \frac{1}{3}$

$\frac{27}{81} \div \frac{27}{27} = \frac{1}{3}$

Notice that, since 27 and 81 have a common factor of 9, we find that $\frac{3}{9}$ is an equivalent fraction.

But this fraction has a factor of 3 common to both numerator and denominator.

So, we must reduce this fraction again. It is difficult to see, but if we had known that 27 was a factor (divides into both parts of the fraction evenly), we could have arrived at the answer in one step

e.g. $\frac{8}{24} \div \frac{8}{8} = \frac{1}{3}$, $\frac{45}{60} \div \frac{15}{15} = \frac{3}{4}$

TYPES OF FRACTIONS

(1) Proper Fractions

A Proper fraction is one in which the numerator is less than the denominator (or a fraction which is less than the number 1).

e.g. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{88}{93}$, $\frac{8}{15}$ are all Proper fractions.

(2) Fractions that are Whole Numbers

Some fractions, when reduced, are really whole numbers (1, 2, 3, 4… etc).

Whole numbers occur if the denominator divides into the numerator evenly.

e.g. $\frac{8}{4}$ is the same as $\frac{8}{4} \div \frac{4}{4} = \frac{2}{1}$ or 2

$\frac{30}{5}$ is the same as $\frac{30}{5} \div \frac{5}{5} = \frac{6}{1}$ or 6

So, the fraction $\frac{30}{5}$ is really the whole number 6. Notice that a whole number can always be written as a fraction with a denominator of 1. e.g. $10 = \frac{10}{1}$

(3) Mixed Numbers

A mixed number is a combination of a whole number and a Proper fraction.

e.g. $2\frac{3}{5}$ (two and three-fifths)

$27\frac{2}{9}$ (twenty-seven and two-ninths)

$9\frac{3}{6} = 9\frac{1}{2}$ (always reduce fractions)

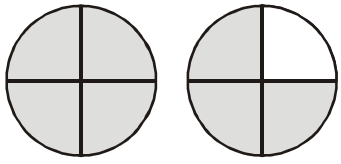
(4) Improper Fractions

An improper fraction is one in which the numerator is larger than the denominator.

From the circles on the right, we see that $1\frac{3}{4}$ (mixed number) is the same as $\frac{7}{4}$ (improper fraction).

An improper fraction, like, can be changed to a mixed number by dividing the denominator into the numerator and expressing the remainder (3) as the numerator.

e.g. $\frac{16}{5} = 3\frac{1}{5}$, $\frac{29}{8} = 3\frac{5}{8}$, $\frac{14}{3} = 4\frac{2}{3}$



$1\frac{3}{4} = \frac{7}{4}$

$\frac{7}{4} = 4\overline{)7} = 1\frac{3}{4}$

$\frac{-4}{3}$

A mixed number can be changed to an improper fraction by changing the whole number to a fraction with the same denominator as the common fraction.

$2\frac{3}{5} = \frac{10}{5}$ and $\frac{3}{5}$ $\quad$ $10\frac{1}{9} = \frac{90}{9}$ and $\frac{1}{9}$

A simple way to do this is to multiply the whole number by the denominator, and then add the numerator.

e.g. $4\frac{5}{9} = \frac{4\times 9+5}{9} = \frac{36+5}{8} = \frac{41}{9}$

$10\frac{2}{7} = \frac{10\times 7+2}{7} = \frac{70+2}{7} = \frac{72}{7}$

(5) Like Fractions

The fractions which have the same denominators are called like fractions. For example $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$ are like fractions.

The simplification of such fractions is easy, as all the denominators here are the same. Suppose we need to add all the above like fractions, then;

$\frac{1}{2}+\frac{3}{2}+\frac{5}{2}+\frac{7}{2} = \frac{(1+3+5+7)}{2} = \frac{16}{2} = 8.$

(6) Unlike Fractions

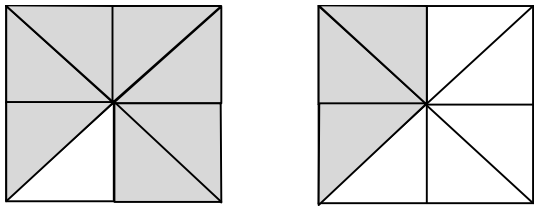
The fractions which have unequal denominators or different denominators are called, unlike fractions.

For example $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$.

COMPARING FRACTIONS

(1) Proper Fractions

In the diagram on the right, it is easy to see that $\frac{7}{8}$ is larger than $\frac{3}{8}$ (since 7 is larger than 3). However, it is not as easy to tell that $\frac{7}{8}$ is larger than $\frac{5}{6}$.



$\frac{7}{8}$ $\quad = \quad$ $\frac{3}{8}$

In order to compare fractions, we must have the same (common) denominators. This process is called **"Finding the Least Common Denominator"** and is usually abbreviated as finding the LCM (lowest common multiple).

Which is larger : $\frac{7}{8}$ or $\frac{5}{6}$?

In order to compare these fractions, we must change both fractions to equivalent fractions with a common denominator.

To do this, **take the largest denominator (8) and examine multiples of it**, until the other denominator (6) divides into it.

Notice that, when we multiply 8 x 3, we get 24, which 6 divides into.

Now change the fractions to 24^{th} s.

When we change these fractions to equivalent fractions with an LCM of 24, we can easily see that $\frac{7}{8}$ is larger than $\frac{5}{6}$ since $\frac{21}{24}$ is greater than.

Which is larger : $\frac{4}{9}$ or $\frac{5}{12}$?

Examine multiples of the larger denominator (12) until the smaller denominator divides into it. This tells us that the LCM is 36.

Now, we change each fraction to equivalent fractions with the LCM of 36.

$12 \times 1 = 12$

$12 \times 2 = 24$

$12 \times 3 = 36$

(LCM)

$\frac{4}{9} \overset{\times 4}{=} \frac{16}{36}$ $\quad$ $\frac{5}{12} \overset{\times 3}{=} \frac{15}{36}$

So $\frac{4}{9}$ is larger than $\frac{5}{12}$.

Which is larger : $\frac{4}{5}$ or $\frac{13}{15}$ or $\frac{11}{12}$?

Find the LCM by examining multiples of 15. Notice that, when we multiply 15×4, we find that 60 is the number that all denominators divide into.

$15 \times 1 = 15$

$15 \times 2 = 30$

$15 \times 3 = 45$

$15 \times 4 = 60$

(LCM)

$\frac{4}{9} \overset{\times 12}{=} \frac{48}{60}$ $\quad$ $\frac{13}{12} \overset{\times 4}{=} \frac{52}{60}$ $\quad$ $\frac{11}{12} \overset{\times 5}{=} \frac{55}{60}$

So $\frac{11}{12}$ is largest fraction.

Notice that one denominator (9) divides into the other denominator (18).

This means that the LCM = 18 and we only have to change one fraction $\frac{7}{9}$ to an equivalent fraction.

$\frac{7}{9} \overset{\times 2}{=} \frac{14}{18}$

So, $\frac{7}{9}$ is larger than $\frac{13}{18}$.

ADDING FRACTIONS

There are four main operations that we can do with numbers: addition (+), subtraction (–), multiplication (x), and division (÷).

In order to add or subtract, fractions must have common denominators.

This is not required for multiplication or division.

(1) Adding with Common Denominators

To add fractions, if the denominators are the same, we simply add the numerators and keep the same denominators.

$\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$

e.g. Add $\frac{1}{12}$ and $\frac{5}{12}$.

Since the denominators are common, simply add the numerators. Notice that we must reduce the answer, if possible.

$\frac{1}{12} + \frac{5}{12} = \frac{6}{12} = \frac{1}{2}$

(2) Adding When One Denominator is a Multiple of the Other:

$\frac{2}{9} \overset{\times 3}{=} \frac{6}{27}$

Add $\frac{2}{9}$ and $\frac{5}{27}$.

Notice that the denominators are not common. Also notice that 27 is a multiple of 9 (since 9 × 3 = 27). This means that the LCM = 27 (see the last example in "Comparing Fractions").

$$\frac{2}{9}+\frac{5}{27}=\frac{6}{27}+\frac{5}{27}=\frac{11}{27}$$

(3) Adding any Fraction:

Add $\frac{7}{12}$ and $\frac{13}{15}$.

$$\frac{7}{12}+\frac{13}{15}=\frac{35}{60}+\frac{52}{60}=\frac{87}{60}=1\frac{9}{20}$$

We must find a common denominator by examining multiples of the largest denominator. We find that the LCM = 60.

Add $1\frac{5}{6}$ and $2\frac{3}{8}$.

When adding mixed numbers, add the whole numbers and the fractions separately. Find common denominators and add.

$$\begin{array}{r} 1\frac{5}{6}=1\frac{20}{24} \\ +\quad 2\frac{3}{8}=2\frac{9}{24} \\ \hline \text{total equals}\quad 3\frac{29}{24} \end{array}$$

$$3\frac{29}{24}=3+1\frac{5}{24}=4\frac{5}{24}$$

If an improper fraction occurs in the answer, change it to a common fraction by doing the following.

(4) The Language of Addition

$$\frac{1}{12}+\frac{5}{12}=\frac{6}{12}=\frac{1}{2}$$

$\frac{1}{2}$ plus $\frac{2}{3}$ $\qquad$ $\frac{1}{2}$ and $\frac{2}{3}$

total of $\frac{1}{2}$ and $\frac{2}{3}$ $\qquad$ sum of $\frac{1}{2}$ and $\frac{2}{3}$

addition of $\frac{1}{2}$ and $\frac{2}{3}$ $\qquad$ $\frac{1}{2}$ combined with $\frac{2}{3}$

$\frac{1}{2}$ more than (or greater than) $\frac{2}{3}$

Note: All of these can be worded with the fractions in reverse order:

e.g. $\frac{2}{3}$ plus $\frac{1}{2}$ is the same as $\frac{1}{2}$ plus $\frac{2}{3}$

SUBTRACTING FRACTIONS

(1) Common Fractions

As in addition, we must have common denominators in order to subtract. Find the LCM; change the fractions to equivalent fraction with the LCM as the denominator. Then subtract the numerators, but keep the same denominator.

$$\frac{5}{8}-\frac{3}{8}=\frac{2}{8}\text{ or }\frac{1}{4}$$

$$\frac{2}{3}-\frac{3}{8}=\frac{16}{24}-\frac{9}{24}=\frac{7}{24}$$

(2) Mixed Numbers

When subtracting whole numbers, subtract the whole numbers, and then subtract the fractions separately. However, if the common fraction we are subtracting is smaller than the other common fraction, we must borrow the number "1" from the large whole number.

$$3\frac{5}{9}-1\frac{3}{9}=2\frac{2}{9}$$

i.e. $4\frac{2}{7}=3+\frac{7}{7}+\frac{2}{7}$, or $3\frac{9}{7}$

$$4\frac{2}{7}-2\frac{5}{7}=3\frac{9}{7}-2\frac{5}{7}=1\frac{4}{7}$$

To subtract $1\frac{3}{4}$ from $6\frac{2}{3}$, first change the common fractions to equivalent fractions with the LCM. Since $\frac{8}{12}$ is smaller than $\frac{9}{12}$, borrow from 6.

$$6\frac{2}{3}-1\frac{3}{4}=6\frac{8}{12}-1\frac{9}{12}=5\frac{20}{12}-1\frac{9}{12}=4\frac{11}{12}$$

$$6\frac{8}{12}=5+\frac{12}{12}+\frac{8}{12}=5\frac{20}{12}$$

(3) The Language of Subtraction

$\frac{5}{6}-\frac{2}{3}$ CAN BE WORDED

$\frac{5}{6}$ min us $\frac{2}{3}$ (NOT $\frac{2}{3}$ min us)

$\frac{2}{3}$ subtracted from $\frac{5}{6}$

$\frac{2}{3}$ from $\frac{5}{6}$

$\frac{2}{3}$ less than $\frac{5}{6}$

$\frac{5}{6}$ decreased by or lowered by $\frac{2}{3}$

the difference of $\frac{5}{3}$ and $\frac{2}{3}$

NOTE: Unlike addition, we cannot reword the above with the fractions in reverse order:

i.e. $\frac{1}{2}-\frac{2}{3}$ is NOT the same as $\frac{2}{3}-\frac{1}{2}$

MULTIPLYING FRACTIONS

(1) When multiplying fractions, a common denominator is not needed. Simply multiply the numerators and multiply the denominators separately.

$$\frac{2}{5}\times\frac{5}{9}=\frac{2\times5}{3\times9}=\frac{10}{27}$$

Sometimes, we can reduce the fractions before multiplying.

$$\frac{3}{5}\times\frac{5}{7}=\frac{2\times\cancel{5}}{\cancel{5}\times9}=\frac{3}{7}$$

Any common factor in either numerator can cancel with the same factor in the denominator. Multiply after cancelling (reducing).

$$\frac{4}{9}\times\frac{3}{8}=\frac{\cancel{4}\times\cancel{3}}{\cancel{9}\times\cancel{8}}=\frac{1}{6}$$

Note that any whole number (16) has the number "1" understood in its denominator.

$$\frac{5}{8}\times16=\frac{5\times\cancel{16}}{\cancel{8}\times1}=10$$

If more than two fractions are multiplied, the same principles apply.

$$\frac{2}{9}\times\frac{3}{20}\times\frac{1}{4}=\frac{2\times3\times1}{9\times20\times4}=\frac{1}{120}$$

(2) Mixed Numbers

Mixed numbers must be changed to improper fractions before multiplying. Remember that a mixed number (like) can be changed to an improper fraction by multiplying the whole number (2) by the denominator (4) and then adding the numerator.

$$2\frac{3}{4}\times 4\frac{8}{11}=\frac{11}{4}\times\frac{52}{11}=13$$

$$5\frac{4}{9}\times 2\frac{3}{7}=\frac{49}{9}\times\frac{17}{7}=\frac{119}{9}\ \text{ or }\ 13\frac{2}{9}$$

(3) The Language of Multiplication

$$\frac{1}{2}\times\frac{2}{3}\ \text{CAN BE WORDED}$$

$\frac{1}{2}$ multiplied by $\frac{2}{3}$ $\qquad$ $\frac{1}{2}$ by $\frac{2}{3}$

$\frac{1}{2}$ of $\frac{2}{3}$ $\qquad$ the product of $\frac{1}{2}$ and $\frac{2}{3}$

NOTE: When multiplying, it doesn't matter which fraction is first.

i.e. $\frac{1}{2}\times\frac{2}{3}$ is the same as $\frac{2}{3}\times\frac{1}{2}$.

DIVIDING FRACTIONS

To divide fractions, we invert (take the reciprocal of) the fraction that we are dividing by, then cancel (reduce), and then multiply.

$$\frac{2}{3}\ \text{reciprocal}\ \frac{3}{2}$$

Taking the reciprocal of a fraction involves "flipping" the fraction so that the numerator and denominator switch places.

$$\frac{8}{19}\ \text{reciprocal}\ \frac{19}{8}$$

Note that a whole number is really a fraction (e.g. $4=\frac{4}{1}$).

$$4\ \text{reciprocal}\ \frac{1}{4}$$

(1) Common Fractions

Simply invert (take the reciprocal of) the fractions that we are dividing by ($\frac{8}{9}$). Then cancel and multiply.

$$\frac{5}{7}\div\frac{8}{9}=\frac{5}{7}\times\frac{9}{8}=\frac{45}{56}$$

Note: you can only cancel after the division is changed to a multiplication.

$$\frac{9}{16}\div\frac{3}{32}=\frac{9}{16}\times\frac{32}{3}=6$$

(2) Mixed Numbers

As in multiplication, mixed numbers must be changed to improper fractions.

$$1\frac{2}{3}\div 2\frac{1}{7}=\frac{5}{3}\div\frac{15}{7}=\frac{5}{3}\times\frac{7}{15}=\frac{7}{9}$$

(3) The Language of Division

$$\frac{1}{2}\div\frac{2}{3}\ \text{CAN BE WORDED}$$

$\frac{1}{2}$ divided by $\frac{2}{3}$ $\qquad$ $\frac{2}{3}$ into $\frac{1}{2}$ $\qquad$ divide $\frac{1}{2}$ by $\frac{2}{3}$

NOTE: In multiplication, the order of the fractions was not important.

i.e. $\frac{1}{2}\times\frac{2}{3}$ is the same as $\frac{2}{3}\times\frac{1}{2}$.

In division, this is not the case. The order of the fractions is important.

Consider the following:

$$\frac{1}{2}\div\frac{2}{3}=\frac{1}{2}\times\frac{3}{2}=\frac{3}{4}$$

But $$\frac{2}{3}\div\frac{1}{2}=\frac{2}{3}\times\frac{2}{1}=\frac{4}{3}=1\frac{1}{3}$$

WORKSHEET

1. Find the value of $\dfrac{2}{1+\dfrac{1}{1-\dfrac{1}{2}}} \times \dfrac{3}{\dfrac{5}{6}\text{ of }\dfrac{3}{2} \div 1\dfrac{1}{4}}$

2. Find the value of

$$\frac{1}{1+\dfrac{1}{3-\dfrac{4}{2+\dfrac{1}{3-\dfrac{1}{2}}}}} + \frac{3}{3-\dfrac{4}{3+\dfrac{1}{1-\dfrac{1}{2}}}}$$

3. If $\frac{1}{4} \times \frac{2}{6} \times \frac{3}{8} \times \frac{4}{10} \times \frac{5}{12} \times \ldots\ldots\ldots\ldots \times \frac{31}{64} = \frac{1}{2^x}$, then what is the value of x?

4. Production of wheat is $2\frac{1}{4}$ times that of rice, but the cost of rice is $1\frac{1}{4}$ times that of wheat. If a farmer produces wheat in place of rice, then what is his income in terms of the previous income?

5. If a man spends $\frac{5}{6}$th part of money and then earns $\frac{1}{2}$ part of the remaining money, what part of the money is with him now?

6. Eight people are planning to share equally the cost of a rental car. If one person withdraws from the arrangement and the others share equally the entire cost of the car, then by how much is the share of each of the remaining persons is increased in terms of original share?

7. Which of the following fractions is the largest?

(A) $\frac{13}{16}$ (B) $\frac{7}{8}$

(C) $\frac{31}{40}$ (D) $\frac{63}{80}$

8. Which of the following fractions is less than $\frac{7}{8}$ and greater than $\frac{1}{3}$?

(A) $\frac{1}{4}$ (B) $\frac{23}{24}$

(C) $\frac{11}{12}$ (D) $\frac{17}{24}$

9. Madan picks up three different digits from the set {1, 2, 3, 4, 5} and forms a mixed number by placing the digits in the spaces $\square\frac{\square}{\square}$. The fractional part of the mixed number should be less than 1 (for example $4\frac{2}{3}$). What is the difference between the largest and smallest possible mixed number that can be formed?

(A) $4\frac{7}{20}$ (B) $4\frac{3}{10}$

(C) $4\frac{9}{20}$ (D) $4\frac{3}{5}$

10. What fraction of $\frac{4}{7}$ must be added to itself to make the sum $1\frac{1}{14}$?

(A) $\frac{1}{2}$ (B) $\frac{4}{7}$

(C) $\frac{7}{8}$ (D) $\frac{15}{14}$

11. If $a=\left(\frac{1}{10}\right)^2$, $b = \frac{1}{5}$ and $c = \sqrt{\frac{1}{100}}$, then which of the following statements is correct?

(A) $a < b < c$ (B) $a < c < b$

(C) $b < c < a$ (D) $c < a < b$

12. Mohan ate half a pizza on Monday. He ate half of what was left on Tuesday and so on. He followed this pattern for one week. How much of the pizza would he have eaten during the week?

(A) 99.22% (B) 95%

(C) 98.22% (D) 100%

13. Write five equivalent fractions of $\frac{3}{5}$.

14. Ramesh solved $\frac{2}{7}$ part of an exercise while Seema solved $\frac{4}{5}$ of it. Who solved lesser part?

15. Sameera purchased $3\frac{1}{2}$ kg apples and $4\frac{3}{4}$ kg oranges. What is the total weight of fruits purchased by her?

16. Suman studies for $5\frac{2}{3}$ hours daily. She devotes $2\frac{4}{5}$ hours of her time for Science and Mathematics. How much time does she devote for other subjects?

17. In a class of 40 students $\frac{1}{5}$ of the total number of studetns like to study English $\frac{2}{5}$, of the total number like to study mathematics and the remaining students like to study Science.

(i) How many students like to study English?

(ii) How many students like to study Mathematics?

(iii) What fraction of the total number of students like to study Science?

18. Sushant reads $\frac{1}{3}$ part of a book in 1 hour. How much part of the book will he read in $2\frac{1}{5}$ hours?

19. The least fraction that must be added to $1\frac{1}{3} \div 1\frac{1}{2} \div 1\frac{1}{9}$ to make the result an integer is

(A) $\frac{4}{5}$ (B) $\frac{3}{5}$

(C) $\frac{2}{5}$ (D) $\frac{1}{5}$

20. Find the value of x in the following:

$$1\frac{2}{3} \div \frac{2}{7} \times \frac{x}{7} = 1\frac{1}{4} \times \frac{2}{3} \div \frac{1}{6}$$

(A) 0.006 (B) $\frac{1}{6}$

(C) 0.6 (D) 6

1. Given exp. $= \frac{2}{1+\frac{1}{\frac{1}{2}}} \times \frac{3}{\frac{15}{12} \div \frac{5}{4}}$

$= \frac{2}{1+2} \times \frac{3}{\frac{15}{12} \times \frac{4}{5}} = \frac{2}{3} \times \frac{3}{1} = 2.$

2. Given exp. $= \frac{1}{1+\frac{1}{3-\frac{4}{2+\frac{1}{\frac{5}{2}}}}} + \frac{3}{3-\frac{4}{3+\frac{1}{\frac{1}{2}}}}$

$= \frac{1}{1+\frac{1}{3-\frac{4}{2+\frac{2}{5}}}} + \frac{3}{3-\frac{4}{3+2}}$

$= \frac{1}{1+\frac{1}{3-\frac{4}{12/5}}} + \frac{3}{3-\frac{4}{5}} = \frac{1}{1+\frac{1}{3-\frac{20}{12}}} + \frac{3}{\frac{11}{5}}$

$= \frac{1}{1+\frac{1}{16/12}} + \frac{15}{11}$

$= \frac{1}{1+\frac{12}{16}} + \frac{15}{11} = \frac{1}{\frac{7}{4}} + \frac{15}{11} = \frac{4}{7} + \frac{15}{11}$

$= \frac{44+105}{77} = \frac{149}{77}$

3. $\frac{1}{2^x} = \frac{1}{4} \times \frac{2}{6} \times \frac{3}{8} \times \frac{4}{10} \times \frac{5}{12} \times \ldots\ldots \times \frac{30}{62} \times \frac{31}{64}$

$= \frac{1}{(2 \times 2 \times 2 \times \ldots 30\,\text{times}) \times 64}$

$\Rightarrow \frac{1}{2^x} = \frac{1}{2^{30} \times 64} = \frac{1}{2^{30} \times 2^6} = \frac{1}{2^{36}} \Rightarrow x = 36$

4. Let production of rice = x quintals and cost of 1 quintal rice = ` y

Then, original income of the farmer = ` (x + y) = ` xy

Now as per question, production of wheat $= 2\frac{1}{4} \times$ production of rice $= \frac{9}{4}$ x quintals

Cost of wheat $\times 1\frac{1}{4}$ = Cost of rice = ` y

$\Rightarrow$ Cost of wheat = ` $\frac{4}{5}$ y

$\therefore$ Present income of the farmer $= \left(\frac{9x}{4} \times \frac{4y}{5}\right)$

$= \frac{9}{5}$xy $= 1\frac{4}{5}$ times of xy

$= 1\frac{4}{5}$ time the previous income.

5. Let the money with the man at first be ` 1.

$\therefore$ Money spent $= \frac{5}{6}$ of ` 1 = ` $\frac{5}{6}$

Remaining money = ` $\left(1-\frac{5}{6}\right)$ = ` $\frac{1}{6}$

Money earned = of ` $\frac{1}{6}$ = ` $\frac{1}{2}$

$\therefore$ Total money with the man = ` $\left(\frac{1}{6}+\frac{1}{12}\right)$

= ` $\frac{3}{12}$ = ` $\frac{1}{4}$

$\therefore$ The man now has $\frac{1}{4}$ th part of the money.

6. When there are 8 people, the share of each person is $\frac{1}{8}$ of the total cost.

When there are 7 people, each person's share is $\frac{1}{7}$ of the total cost.

$\therefore$ Increase in the share of each person = $\frac{1}{7}-\frac{1}{8}=\frac{1}{56}$, i.e., $\frac{1}{7}$ of $\frac{1}{8}$, i.e., $\frac{1}{7}$ of the original share of each person.

7. (B)

$\frac{13}{16}=\frac{13\times5}{16\times5}=\frac{65}{80}, \frac{7}{8}=\frac{7\times10}{8\times10}=\frac{70}{80}, \frac{31}{40}$

$=\frac{31\times2}{40\times2}$

$=\frac{62}{80}$ and last fraction is $\frac{63}{80}$

Out of these, the largest fraction

$=\frac{70}{80}=\frac{7}{8}$

8. (D)

$\frac{1}{3}=0.333000,\ \frac{7}{8}=0.875$

$\frac{1}{4}=0.25,\ \frac{23}{24}=0.9583000,\ \frac{11}{12}$

$=0.9166000,$

$\frac{17}{24}=0.7083000$

Since $0.7083000\left(=\frac{17}{24}\right)$ is greater than $0.333000\left(=\frac{1}{3}\right)$ and less than $0.875\left(=\frac{7}{8}\right)$

Therefore, $\frac{17}{24}$ lies between $\frac{1}{3}$ and $\frac{7}{8}$.

9. (A)

The largest number = $5\frac{3}{4}$

The smallest number = $1\frac{2}{5}$

$\therefore$ Required difference = $5\frac{3}{4}-1\frac{2}{5}=\frac{23}{4}-\frac{7}{5}$

$=\frac{115-28}{20}=\frac{87}{20}=4\frac{7}{20}$.

10. (C)

Let the required fraction be x. Then,

$x \text{ of } \frac{4}{7}+\frac{4}{7}=1\frac{1}{14}\Rightarrow\frac{4x}{7}+\frac{4}{7}=\frac{15}{14}$

$\Rightarrow \frac{4x}{7}=\frac{15}{14}-\frac{4}{7}=\frac{7}{14}=\frac{1}{2}$

$\Rightarrow x=\frac{1}{2}\times\frac{7}{4}=\frac{7}{8}$.

11. (B)

$a=\frac{1}{100}, b=\frac{1}{5}=\frac{20}{100}, c=\frac{1}{10}=\frac{10}{100}$

$\therefore a < c < b$

12. (A)

Pizza he ate on Monday = $\frac{1}{2}$

Pizza left = $\frac{1}{2}$

Pizza he ate on Tuesday = $\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$

Pizza left = $\frac{1}{2}-\frac{1}{4}=\frac{1}{4}$

Pizza he ate on Wednesday = $\frac{1}{2}\times\frac{1}{4}=\frac{1}{8}$

Pizza left = $\frac{1}{4}-\frac{1}{8}=\frac{1}{8}$

Continuing this pattern, we see that Pizza he ate on Thursday, Friday, Saturday and Sunday is $\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$ and $\frac{1}{128}$ respectively.

$\therefore$ Quantity of pizza he ate during the week

$= \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}+\frac{1}{128}$

$= \frac{64+32+16+8+4+2+1}{128} = \frac{127}{128}$

$= \frac{127}{128} \times 100\% =$ **99.22%**

13. One of the equivalent fractions of $\frac{3}{5}$ is

. Find the other four.

$\frac{3}{5} = \frac{3\times 2}{5\times 2} = \frac{6}{10}$. Find the other four.

14. In order to find who solved lesser part of the exercise, let us compare $\frac{2}{7}$ and $\frac{4}{5}$.

Converting them to like fractions we have,

$\frac{2}{7} = \frac{10}{35}, \frac{4}{5} = \frac{28}{35}$

Since $10 < 28$, so $\frac{10}{35} < \frac{28}{35}$

Thus, $\frac{2}{7} < \frac{4}{5}$

Ramesh solved lesser part than Seema.

15. The total weight of the fruits $\left(3\frac{1}{2}+4\frac{3}{4}\right)$kg

$= \left(\frac{7}{2}+\frac{19}{4}\right)$kg $= \left(\frac{14}{4}+\frac{19}{4}\right)$kg

$= \frac{33}{4}$kg $= 8\frac{1}{4}$kg

16. Total time of Suman's study = $5\frac{2}{3}$h $= \frac{17}{3}$h

Time devoted by her for Science and Mathematics = $2\frac{4}{5} = \frac{14}{5}$h

Thus, time devoted by her for other subjects = $\left(\frac{17}{3}-\frac{14}{5}\right)$h

$= \left(\frac{17\times 5}{15}-\frac{14\times 3}{15}\right)$h $= \left(\frac{85-42}{15}\right)$h

$= \frac{43}{15}$h $= 2\frac{13}{15}$h

17. Total number of students in the class = 40.

(i) Of these $\frac{1}{5}$ of the total number of students like to study English.

Thus, the number of students who like to study English $= \frac{1}{5}$ of $40 = \frac{1}{5}\times 40 = 8$.

(ii) Try yourself.

(iii) The number of students who like English and Mathematics = 8 + 16 = 24. Thus, the number of students who like Science = 40 – 24 = 16.

Thus, the required fraction is $\frac{16}{40}$.

18. The part of the book read by Sushant in 1 hour = $\frac{1}{3}$.

so, the part of the book read by him in $2\frac{1}{5}$ hours = $2\frac{1}{5}\times\frac{1}{3}$

$= \frac{11}{5}\times\frac{1}{3} = \frac{11\times 1}{5\times 3} = \frac{11}{15}$

Let us now find $\frac{1}{2}\times\frac{5}{3}$. We known that

$\frac{5}{3} = \frac{1}{3}\times 5$

So, $\frac{1}{2}\times\frac{5}{3} = \frac{1}{2}\times\frac{1}{3}\times 5 = \frac{1}{6}\times 5 = \frac{5}{6}$

Also, $\frac{5}{6} = \frac{1\times 5}{2\times 3}$. Thus, $\frac{1}{2}\times\frac{5}{3} = \frac{1\times 5}{2\times 3} = \frac{5}{6}$.

19. **(D)**

$1\frac{1}{3}\div 1\frac{1}{2}\div 1\frac{1}{9} = \frac{4}{3}\div\frac{3}{2}\div\frac{10}{9}$

$= \frac{4}{3} \times \frac{2}{3} \div \frac{10}{9} = \frac{8}{9} \times \frac{9}{10} = \frac{4}{5}$

$\therefore \frac{1}{5}$ should be added to $\frac{4}{5}$ to make it an integer.

20. (D)

$$\frac{5}{3} \div \frac{2}{7} \times \frac{x}{7} = \frac{5}{4} \times \frac{2}{3} \div \frac{1}{6}$$

$$\Rightarrow \quad \frac{5}{3} \times \frac{7}{2} \times \frac{x}{7} = \frac{5}{4} \times \frac{2}{3} \times \frac{6}{1}$$

$$\Rightarrow \quad \frac{5}{6} \times x = 5 \quad \Rightarrow \quad x = \frac{5 \times 6}{5} = \mathbf{6.}$$