OPERATION ON INTEGERS

INTRODUCTION

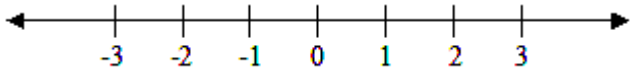
Integers:

Integers are like whole numbers but they also include negative numbers, for example, –4, –3, –2, –1, 0, 1, 2, 3, 4, … .

Positive integers are all the whole numbers greater than zero, ie: 1, 2, 3, 4, 5, … We say that its sign is positive.

Negative integers are the numbers less than zero, ie: –1, –2, –3, –4, –5, … We say that its sign is negative.

Integers extend infinitely in both positive and negative directions. This can be represented on the number line.



Zero is an integer that is neither positive nor negative.

Consecutive Integers:

Consecutive integers are integers that follow in sequence, each number being 1 more than the previous number, for example 22, 23, 24, 25, …

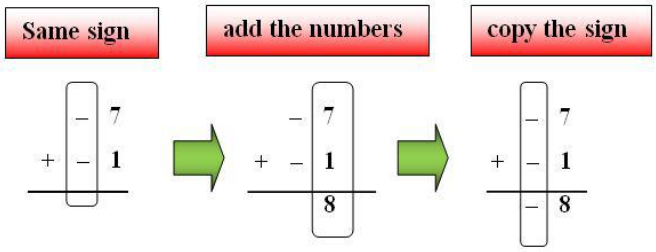
Consecutive integers can be more generally represented by n, n +1, n + 2, n + 3, …, where n is any integer

OPERATIONS ON INTEGERS

(1) Adding Integers:

There are 2 conditions to remember when adding integers:

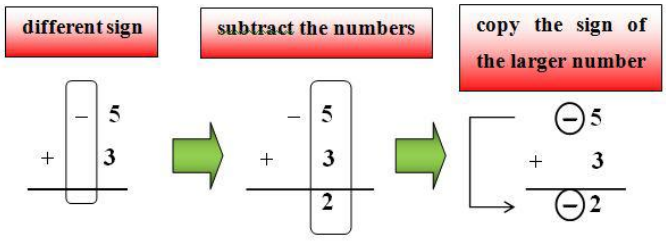
➢ When adding integers having the same sign, add the numbers and then copy the sign.



Ex-1 – 14 + (– 17) = ?

Sol. – 31

➢ When adding two integers having different signs, subtract the numbers and copy the sign of the integer with the larger number.



Ex-2 4 + (– 7) = ?

Sol. – 3

Additive inverse:

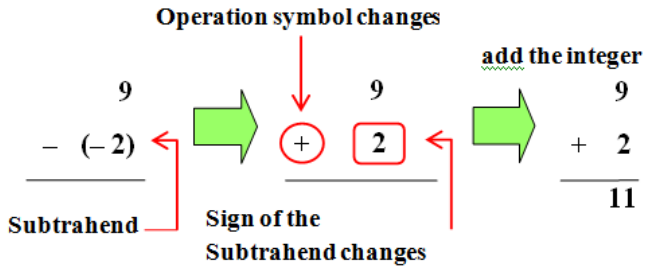
Additive inverse of a number is the number that when added to that number will yield zero. It can also be defined as the opposite of a number.

Ex-3 4 + ? = 0

Sol. 4 + (–4) = 0

(2) Subtracting Integers:

When subtracting two integers, change the sign of the subtrahend and then apply addition of integers.



Parts of a Subtraction Problem:

MINUEND - the first number of the number from which another number is subtracted	**10 → Minuend**
SUBTRAHEND - the second number or the number to be subtracted	**–2 → Subtrahend**
DIFEERENCE - is the resulting number	**8 → Difference**

Ex-4 – 11 – (– 6) =?

Sol. – 11 + 6 = – 5

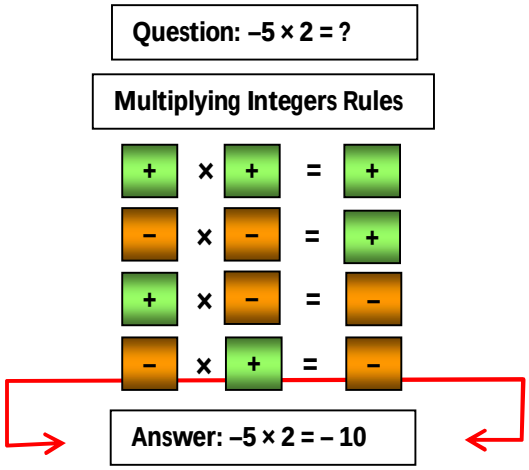
Ex-5 – 3 – (– 10) =?

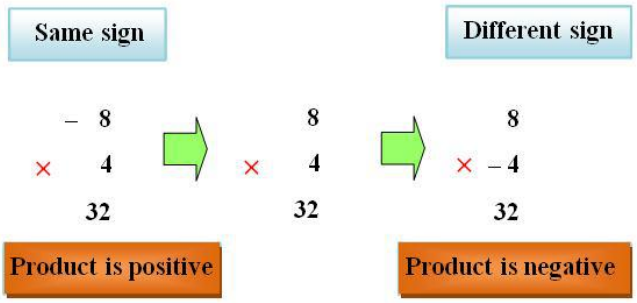
Sol. – 3 + 10 = 7

(3) Multiplying Integers:

Multiplying integers is just multiplying the numbers normally but there are 2 conditions to remember:

- If the two integers have the same sign, then their product is positive.
- If the two integers have different signs, then their product is negative.





Ex-6 6 × – 9 = ?

Sol. – 54

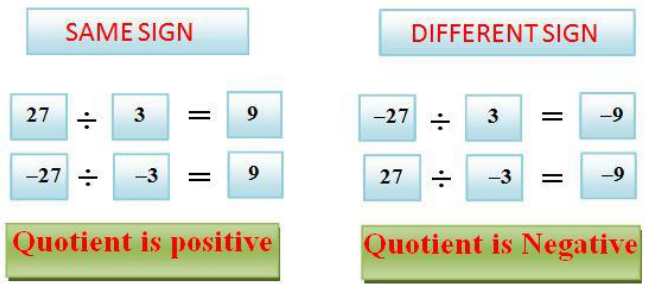
Ex-7 – 8 × – 7 = ?

Sol. 56

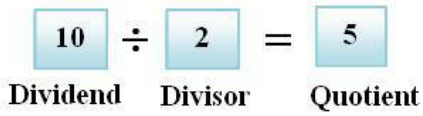
(4) Dividing Integers:

Similar to multiplying integers, dividing integers is just dividing the numbers normally but there are 2 conditions to remember:

- If the two integers have the same sign, then their quotient is positive.
- If the two integers have different signs, then their quotient is negative.



Parts of a Division Problem:



A dividend is divided a divisor to find a quotient

Ex-8 45 ÷ – 3 = ?

Sol. - 15

Ex-9 – 72 ÷ – 8 = ?

Sol. 9

COMPAIRING INTEGERS

There are 3 conditions to remember when comparing integers:

(1) To compare two positive integers, the positive integer with the larger number is greater.

Ex-10 Which is greatest integer between 3 and 7?

Sol. 7 > 3

Ex-11 Is 5 greater than or less than 6?

Sol. 6 > 5

(2) To compare a positive and a negative integer, the positive integer is greater.

Ex-12 Which is the greatest integer between – 3 and 1?

Sol. 1 > – 3

Ex-13 Is 2 greater than or less than – 4?

Sol. 2 > – 4

(3) To compare two negative integers, the negative integer with the smaller number is greater.

Ex-14 Which is the greatest integer between – 5 and – 8?

Sol. – 5 > – 8

Ex-15 Is – 10 greater than or less than – 6 ?

Sol. – 6 > – 10

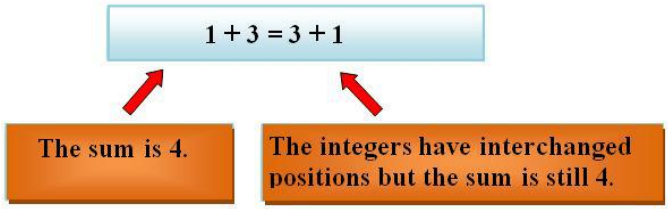
Comparing Integers:

SAME SIGN	For Positive Integers : + (large number) > + (small number) For Negative Integers : – (small number) > – (larger number)
DIFFERENT SIGN	Positive (+) integer > negative (–)integer

PROPERTIES OF INTEGERS

(1) Commutative Property of Addition:

Adding integers will have the same result regardless of the arrangement of the integers.



Ex-16 Is 2 + 6 = 6 + 2 ?

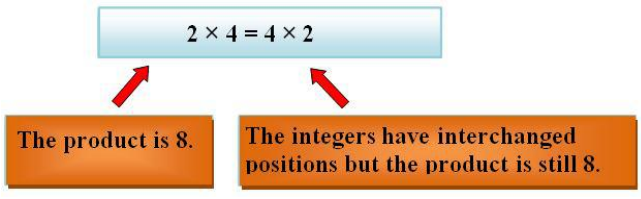
Sol. 8 = 8

Ex-17 By Commutative Property of Addition, 3+4 = ?

Sol. 3 + 4 = 4 + 3

(2) Commutative Property of Multiplication:

Multiplying integers will have the same result regardless of the arrangement of the integers.



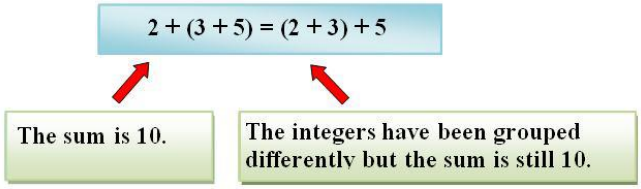
The sum will not change even if the integers are grouped differently.

Ex-18 Is $3 \times 5 = 5 \times 3$?
Sol. $15 = 15$

Ex-19 By Commutative Property of Multiplication, $4 \times 6 = ?$
Sol. $4 \times 6 = 6 \times 4$

(3) Associative Property of Addition:

Adding integers will have the same result regardless of the grouping.



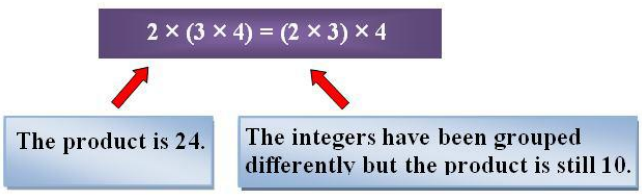
The sum will not change even if the integers are grouped differently.

Ex-20 Is $1 + (4 + 6) = (1 + 4) + 6$?
Sol. $1 + 10 = 5 + 6$
$11 = 11$

Ex-21 By Associative Property of Addition, $3 + (5 + 4)$?
Sol. $3 + (5 + 4) = (3 + 5) + 4$

(4) Associative Property of Multiplication:

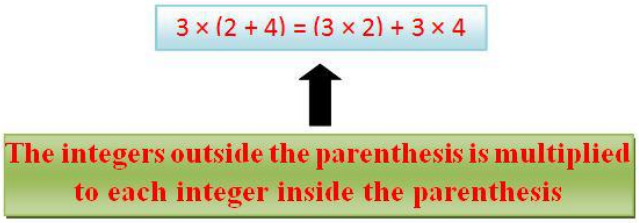
Multiplying integers will have the same result regardless of the grouping.



The product will not change even if the integers are grouped differently.

(5) Distributive Property of Addition and Multiplication:

Distributive property involves the addition of integers being multiplied by another integer.



Each integer inside the parenthesis is multiplied by the integer outside the parenthesis, then the resulting products are added together.

Ex-22 Is $2 \times (1 + 3) = (2 \times 1) + (2 \times 3)$?
Sol. $2 \times 4 = 2 + 6$
$8 = 8$

Ex-23 By Distributive Property of Addition and Multiplication, $4 \times (2 + 3)$?
Sol. $4 \times (2 + 3) = 4 \times 2 + 4 \times 3$

WORKSHEET

1. The difference of an integer a and (-6) is 4. Find the value of a.

2. The sum of two integers is 65. If one of them is -31, find the other.

3. If a = –9 and b = –6, show that (a–b) ≠ (b – a).

4. Find 36 – (–64) and (–64) – 36. Are they equal?

5. If a = –8, b = –7, c = 6, verify that (a + b) + c = a + (b + c).

6. Simplify : {–13–(–27)} + {–25–(–40)}

7. Verify a - (-b) = a + b for the following values of a and b.

(i) a = 21, b = 18

(ii) a = 118, b = 125

(iii) a = 75, b = 84

(iv) a = 28, b = 11

8. Use the sign of >, < or = in the box to make the statements true.

(a) (-8) + (-4) ☐ (-8) - (-4)

(b) (-3) + 7 - (19) ☐ 15 - 8 + (-9)

(c) 23 - 41 + 11 ☐ 23 - 41 - 11

(d) 39 + (-24) - (15) ☐ 36 + (-52) - (-36)

(e) -231 + 79 + 51 ☐ -399 + 159 + 81

9. Write down a pair of integers whose

(a) sum is –3 (b) difference is –5

(c) difference is 2 (d) sum is 0

10. Find each of the following products:

(i) (–18) × (–10) × 9

(ii) (–20) × (–2) × (–5) × 7

(iii) (–1) × (–5) × (– 4) × (– 6)

11. Verify (–30) × [13 + (–3)] = [(–30) × 13] + [(–30) × (–3)]

12. In a class test containing 15 questions, 4 marks are given for every correct answer and (–2) marks are given for every incorrect answer.

(i) Gurpreet attempts all questions but only 9 of her answers are correct. What is her total score?

(ii) One of her friends gets only 5 answers correct. What will be her score?

13. Suppose we represent the distance above the ground by a positive integer and that below the ground by a negative integer, then answer the following:

(i) An elevator descends into a mine shaft at the rate of 5 metre per minute. What will be its position after one hour?

(ii) If it begins to descend from 15 m above the ground, what will be its position after 45 minutes?

14. In a test (+5) marks are given for every correct answer and (–2) marks are given for every incorrect answer.

(i) Radhika answered all the questions and scored 30 marks though she got 10 correct answers.

(ii) Jay also answered all the questions and scored (–12) marks though he got 4 correct answers. How many incorrect answers had they attempted?

15. A shopkeeper earns a profit of Re 1 by selling one pen and incurs a loss of 40 paise per pencil while selling pencils of her old stock.

(i) In a particular month she incurs a loss of Rs 5. In this period, she sold 45 pens. How many pencils did she sell in this period?

(ii) In the next month she earns neither profit nor loss. If she sold 70 pens, how many pencils did she sell?

16. Find each of the products:

(i) $40 \times (-7)$ (ii) -3×25

(iii) -5×-8 (iv) -896×0

(v) $(-10) \times (-10) \times (-10)$

(vi) $(-3) \times (-6) \times (-9) \times (-12)$

(vii) $(-8) \times (-43) \times 0$

(viii) $(-5) \times (6) \times (-7) \times (-20)$

17. Compare: $(-2 - 5) \times (-6)$ and $(-2) + (-5) \times (-6)$.

18. The sum of two integers is –16. If one of them is 53, find the other.

19. Answer True or False:

For all non-zero integers a and b, $a \times b$ is always greater than either a or b.

20. Write (T) for true and (F) for false for each of the following statements.

(i) $0 \div (-4) = 0$

(ii) $(-6) \div 0 = 0$

(iii) $(-5) \div (-1) = -5$

(iv) $(-8) \div 1 = -8$

SOLUTION SHEET

Sol.1 We have:

$a - (-6) = 4$

$\Rightarrow \quad a = 4 + (-6) = -2$

$\therefore \quad a = -2$

Sol. 2 Let the other integer be a.

Then, $-31 + a = 65$

$\Rightarrow \quad a = 65 - (-31) = 96$

$\therefore$ The other integer is 96.

Sol. 3 Here, $(a - b) = -9 - (-6) = -3$

Similarly, $(b - a) = -6 - (-9) = 3$

$\therefore (a - b) \neq (b - a)$

Sol. 4 $36 - (-64) = 36 + 64 = 100$

Now, $(-64) - 36 = (-64) + (-36) = -100$

Here, $100 \neq -100$

Thus, they are not equal.

Sol. 5 $(a + b) + c = (-8 + (-7)) + 6 = -15 + 6 = -9$

$a + (b + c) = -8 + (-7 + 6) = -8 + (-1) = -9$

Hence, $(a + b) + c = a + (b + c)$ [i.e. Pr. Associativity]

Sol. 6 $= \{-13 + 27\} + \{-25 + 40\}$

$= 14 + 15$

$= 29$

Sol. 7 (i) $a = 21, b = 18$

$a - (-b) = 21 - (-18) = 21 + 18 = 39$

$a + b = 21 + 18 = 39$

$\therefore \quad a - (-b) = a + b = 39$

(ii) $a = 118, b = 125$

$a - (-b) = 118 - (-125) = 118 + 125$

$= 243$

$a + b = 118 + 125 = 243$

(iii) $a = 75, b = 84$

$a + b = 75 - (-84) = 75 + 84 = 159$

$a + b = 75 + 84 = 159$

$\therefore \quad a - (-b) = 159$

(iv) $a = 28, b = 11$

$a - (-b) = 28 - (-11) = 28 + 11 = 39$

$a + b = 28 + 11 = 39$

$\therefore \quad a - (-b) = a + b = 39$

Sol. 8 (a) $(-8) + (-4) \quad (-8) - (-4)$

$\Rightarrow \quad -8 - 4 \quad -8 + 4$

$\Rightarrow \quad -12 < -4$

(b) $(-3) + 7 - (19) \quad 15 - 8 + (-9)$

$\Rightarrow \quad -3 + 7 - 19 \quad 15 - 8 - 9$

$\Rightarrow \quad -15 < -4$

(c) $23 - 41 + 11 \quad 23 - 41 - 11$

$\Rightarrow \quad -7 > -29$

(d) $39 + (-24) - 15 \quad 36 + (-52) - (-36)$

$\Rightarrow \quad 39 - 24 - 15 \quad 36 - 52 + 36$

$\Rightarrow \quad 0 > 20$

(e) $-231 + 79 + 51 \quad -399 + 159 + 81$

$\Rightarrow \quad -101 > -159$

Sol. 9 (a) $(-1) + (-2) = -3$ or $(-5) + 2 = -3$

(b) $(-9) - (-4) = -5$ or $(-2) - 3 = -5$

(c) $(-7) - (-9) = 2$ or $1 - (-1) = 2$

(d) $(-10) + 10 = 0$ or $5 + (-5) = 0$

Sol. 10 (i) $(-18) \times (-10) \times 9 = [(-18) \times (-10)] \times 9$

$= 180 \times 9 = 1620$

(ii) $(-20) \times (-2) \times (-5) \times 7 = -20 \times (-2 \times -5) \times 7 = [-20 \times 10] \times 7 = -1400$

(iii) $(-1) \times (-5) \times (-4) \times (-6) = [(-1) \times (-5)] \times [(-4) \times (-6)] = 5 \times 24 = 120$

Sol. 11 $(-30) \times [13 + (-3)] = (-30) \times 10 = -300$

$[(-30) \times 13] + [(-30) \times (-3)] = -390 + 90 = -300$

So, $(-30) \times [13 + (-3)] = [(-30) \times 13] + [(-30) \times (-3)]$

Sol. 12

(i) Marks given for one correct answer = 4

So, marks given for 9 correct answers $= 4 \times 9 = 36$

Marks given for one incorrect answer $= -2$

So, marks given for 6 = (15 – 9) incorrect answers $= (-2) \times 6 = -12$

Therefore, Gurpreet's total score $= 36 + (-12) = 24$

(ii) Marks given for one correct answer = 4

So, marks given for 5 correct answers $= 4 \times 5 = 20$

Marks given for one incorrect answer = (–2)

So, marks given for 10 (=15 – 5) incorrect answers $= (-2) \times 10 = -20$

Therefore, her friend's total score $= 20 + (-20) = 0$

Sol. 13

(i) Since the elevator is going down, so the distance covered by it will be represented by a negative integer.

Change in position of the elevator in one minute = – 5 m

Position of the elevator after 60 minutes $= (-5) \times 60 = -300$ m, i.e., 300 m below ground level.

(ii) Change in position of the elevator in 45 minutes $= (-5) \times 45 = -225$ m, i.e., 225 m below ground level.

So, the final position of the elevator = – 225 + 15 = –210 m, i.e., 210 m below ground level.

Sol. 14

(i) Marks given for one correct answer = 5

So, marks given for 10 correct answers $= 5 \times 10 = 50$

Radhika's score = 30

Marks obtained for incorrect answers = 30 – 50 = –20

Marks given for one incorrect answer = (–2)

Therefore, number of incorrect answers = $(-20) \div (-2) = 10$

(ii) Marks given for 4 correct answers = 5 × 4 = 20 Jay's score = –12

Marks obtained for incorrect answers $= -12 - 20 = -32$

Marks given for one incorrect answer = (–2)

Therefore number of incorrect answers = $(-32) \div (-2) = 16$

Sol. 15

(i) Profit earned by selling one pen = Re 1

Profit earned by selling 45 pens = Rs 45, which we denote by + Rs 45

Total loss given = Rs 5, which we denote by – Rs 5

Profit earned + Loss incurred = Total loss

Therefore, Loss incurred = Total Loss – Profit earned

= Rs (– 5 – 45) = Rs (–50) = –5000 paise

Loss incurred by selling one pencil = 40 paise which we write as – 40 paise

So, number of pencils sold = (–5000) ÷ (– 40) = 125 pencils.

(ii) In the next month there is neither profit nor loss.

So, Profit earned + Loss incurred = 0

i.e., Profit earned = – Loss incurred.

Now, profit earned by selling 70 pens = Rs 70

Hence, loss incurred by selling pencils = Rs 70 which we indicate by – Rs 70 or – 7,000 paise.

Total number of pencils sold = (–7000) ÷ (– 40) = 175 pencils.

Sol. 16 (i) $40 \times (-7) = -(40 \times 7) = -280$

(ii) $-3 \times 25 = -(3 \times 25) = -75$

(iii) $-5 \times -8 = 40$, since $(-) \times (-) = +$

(iv) $-896 \times 0 = 0$

(v) $(-10) \times (-10) \times (-10) = (-10 \times -10) \times 10 = (100 \times -10 = -1000$

(vi) $(-3) \times (-6) \times (-9) \times (-12) = [(-3) \times (-6)] \times [(-9) \times (-12)] = 18 \times 108 = 1944$

(vii) $(-8) \times (-43) \times 0 = -8 \times 0 = 0$

(viii) $(-5) \times (6) \times (-7) \times (-20) = -30 \times 140 = -(30 \times 140) = -4200$.

Sol. 17 $(-2 - 5) \times (-6) = -7 \times -6 = 42$

$-2 + (-5) \times (-6) = -2 + 30 = 28$

Since $42 > 28$, So $(-2 - 5) \times (-6)$ is greater.

Sol. 18 Let the other integer be a. then, we have:

$53 + a = -6$

$\Rightarrow a = -16 - 53 = -69$

$\therefore$ The other integer is –69.

Sol. 19 False, because if a and b are both + ve integers, then the product will be greater than both a and b but if either of a and b is a negative integer then the product will be smaller than a and b.

Sol. 20 (i) True (T). Dividing zero by any integer gives zero.

(ii) False (F). Division by zero gives an indefinite number.

(iii) False (F). $\frac{-5}{-1} = 5$

(iv) True (T). $\frac{-8}{1} = -8$