JEE MAIN 2024
Paper with Solution

Maths | 1st February 2024 _ Shift-2

SECTION – A

1. If the domain of the function $f(x)=\frac{\sqrt{x^2-25}}{(4-x^2)}+\log_{10}(x^2+2x-15)$ is $(-\infty,\alpha)\cup[\beta,\infty)$, then $\alpha^2+\beta^3$ is equal to :

(1) 140 (2) 175 (3) 125 (4) 150

Sol. **(4)**

$f(x) = \frac{\sqrt{x^2-25}}{4-x^2}+\log_{10}(x^2+2x-15)$

to check domain :

$x^2 - 25 \geq 0$ ……. (i)

$4 - x^2 \neq 0$ ……..(ii)

$x^2 + 2x - 15 > 0$ ……..(iii)

Solving (i)

$(x-5)(x+5) \geq 0$

$x \in (-\infty, -5] \cup [5, \infty)$ …….(iv)

Solving (ii)

$x \neq 2$ or -2 …….(v)

solving (iii)

$(x+5)(x-3) > 0$

$x \in (-\infty, -5) \cup (3, \infty)$ ……..(vi)

domain $\Rightarrow$ intersection of (iv), (v), (vi)

$\therefore x \in (-\infty, -5) \cup [5, \infty)$

on comparing $\alpha = -5 \quad \beta = 5$

$\therefore \alpha^2 + \beta^3 = 25 + 125 = 150$

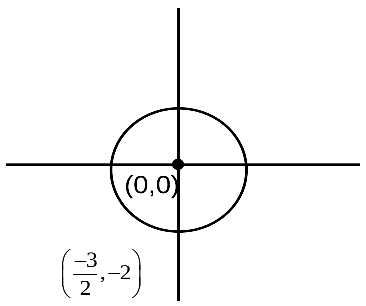
2. If z is a complex number such that $|z| \geq 1$, then the minimum value of $\left|z+\frac{1}{2}(3+4i)\right|$ is :

(1) 2 (2) $\frac{5}{2}$ (3) $\frac{3}{2}$ (4) 3

Sol. **Bonus or Ans. 0**

$|Z| \geq 1$ is a region outside & periphery of circle with centre (0, 0) & radius = one unit

minimum value of $\left|Z-\left(-\frac{3}{2}-2i\right)\right|$ is minimum distance of point in $|Z| \geq 1$ from $\left(-\frac{3}{2}-2\right)$

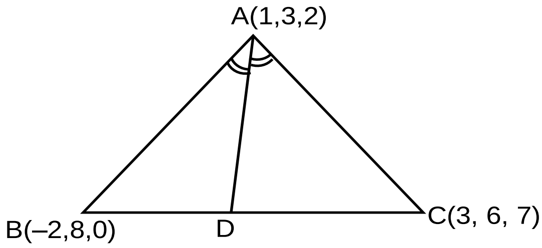


Therefore minimum value of $\left|Z+\frac{1}{2}(3+4i)\right|=0$

3. Consider a $\triangle ABC$ where A(1,3,2), B(-2,8,0) and C(3,6,7). If the angle bisector of $\angle BAC$ meets the line BC at D, then the length of the projection of the vector $\overrightarrow{AD}$ on the vector $\overrightarrow{AC}$ is :

(1) $\frac{37}{2\sqrt{38}}$ (2) $\sqrt{19}$ (3) $\frac{39}{2\sqrt{38}}$ (4) $\frac{\sqrt{38}}{2}$

Sol. **(1)**



D divides BC in the ratio of $|\overrightarrow{AB}|:|\overrightarrow{AC}|$

$\overrightarrow{AB} = -3\hat{i}+5\hat{j}-2\hat{k}$

$\overrightarrow{AC} = 2\hat{i}+3\hat{j}+5\hat{k}$

$|\overrightarrow{AB}| = \sqrt{38}$

$|\overrightarrow{AC}| = \sqrt{38}$

$\therefore \frac{BD}{DC} = \frac{\sqrt{38}}{\sqrt{38}} = \frac{1}{1}$

$\therefore$ D is mid point

coordinates of D $\left(\frac{1}{2}, 7, \frac{7}{2}\right)$

$\overrightarrow{AD} = -\frac{1}{2}\hat{i}+4\hat{j}+\frac{3}{2}\hat{k}$

Projection of $\overrightarrow{AD}$ on $\overrightarrow{AC}$ = $\frac{\overrightarrow{AD}.\overrightarrow{AC}}{|\overrightarrow{AC}|} = \frac{-1+12+\frac{15}{2}}{\sqrt{38}}$

$= \frac{37}{2\sqrt{38}}$

4. Consider the relations R_1 and R_2 defined as

$aR_1b \Leftrightarrow a^2+b^2=1$ for all $a,b \in R$ and $(a,b)R_2(c,d) \Leftrightarrow a+d=b+c$ for all $(a,b),(c,d) \in N\times N$. Then

(1) R_1 and R_2 both are equivalence relations (2) Only R_1 is an equivalence relation

(3) Only R_2 is an equivalence relation (4) Neither R_1 nor R_2 is an equivalence relation

Sol. **(3)**

a, R, b $\Leftrightarrow$ $a^2+b^2=1$ $\forall$ a, b, $\in$ R

checking reflexive

a = b

$a^2 + a^2 = 1$

$a = \pm\frac{1}{\sqrt{2}}$

$\therefore$ only $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ & $\left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ will be present in R_1

So not reflexive, $\therefore$ not equivalence

$(a, b)\ R_2\ (c, d) \Leftrightarrow a + d = b + c\ \forall\ (a, b), (c, d) \in N \times N$

Checking reflexive, put $a = b = c = d$

$a + a = a + a$

Which is satisfied $\forall$ ordered pair $\in N \times N$, $\therefore$ R_2 is reflexive

Checking symmetric

Let suppose $((a, b), (c, d))$ is in R_2

So, $a + d = b + c$ ……(i)

to check $((c, d), (a, b))$ lies in R_2,

(i) can be expressed as,

$c + b = d + a$

$\therefore$ R_2 is symmetric

checking transitive,

Let suppose $((a, b), (c, d))$ & $((c, d) (e, f))$ are in R_2

So, $a + d = b + c$ ……(ii)

$c + f = d + e$ ……(iii)

adding (ii) & (iii)

$a + f = b + e$

$\therefore$ $(a, b), (e, f)$ also in R_2

$\therefore$ R_2 is transitive

$\therefore$ R_2 is equivalence

$\therefore$ only R_2 is equivalence relation

5. Let the system of equations $x + 2y + 3z = 5$, $2x + 3y + z = 9$, $4x + 3y + \lambda z = \mu$ have infinite number of solutions. Then $\lambda + 2\mu$ is equal to :

(1) 22 (2) 17 (3) 15 (4) 28

Sol. **(2)**

$x + 2y + 3z = 5$

$2x + 3y + z = 9$

$4x + 3y + \lambda z = \mu$

for infinite solution $D = D_1 = D_2 = D_3 = 0$

$$D = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 4 & 3 & \lambda \end{vmatrix} = 0$$

$(3\lambda - 3) - 2(2\lambda - 4) + 3(6 - 12) = 0$

$\lambda = -13$

$$D_3 = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 9 \\ 4 & 3 & \mu \end{vmatrix} = 0$$

$3\mu - 27 - 2(2\mu - 36) + 5(6 - 12) = 0$

$\mu = 15$

$\lambda + 2\mu = -13 + 30 = 17$

6. If $\int_0^{\frac{\pi}{3}} \cos^4 x\,dx = a\pi + b\sqrt{3}$, where a and b are rational numbers, then 9a + 8b is equal to

(1) 2 (2) 1 (3) 3 (4) $\frac{3}{2}$

Sol. **(1)**

$$\int_0^{\frac{\pi}{3}} \cos^4 x dx = \int_0^{\frac{\pi}{3}} \cos^2 x.\cos^2 x dx$$

$$= \int_0^{\frac{\pi}{3}} \cos^2 x(1-\sin^2 x)dx$$

$$= \int_0^{\frac{\pi}{3}} \cos^2 x dx - \int_0^{\frac{\pi}{3}} 4\frac{\sin^2 x \cos^2 x}{4}dx$$

$$\int_0^{\frac{\pi}{3}} \left(\frac{1+\cos 2x}{2}\right)dx - \frac{1}{4}\int_0^{\frac{\pi}{3}} (\sin 2x)^2 dx$$

$$= \frac{1}{2}\left(\frac{1+\cos 2x}{2}\right)_0^{\frac{\pi}{3}} - \frac{1}{4}\int_0^{\frac{\pi}{3}} \left(\frac{1-\cos 4x}{2}\right)dx$$

$$= \frac{1}{2}\left[\frac{\pi}{3} + \frac{\sqrt{3}}{4}\right] - \frac{1}{8}\left[x - \frac{\sin 4x}{4}\right]_0^{\pi/3}$$

$$= \frac{\pi}{6} + \frac{\sqrt{3}}{8} - \frac{\pi}{24} - \frac{\sqrt{3}}{64}$$

$$= \frac{\pi}{8} + \frac{7\sqrt{3}}{64}$$

$a = \frac{1}{8}$ $b = \frac{7}{64}$

$9a + 8b = \frac{9}{8} + \frac{7}{8} = 2$

7. Let α and β be the roots of the equation $px^2 + qx - r = 0$, where $p \neq 0$. If p, q and r be the consecutive terms of a non constant G.P. and $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{3}{4}$, then the value of $(\alpha - \beta)^2$ is :

(1) 8 (2) 9 (3) $\frac{20}{3}$ (4) $\frac{80}{9}$

Sol. **(4)**

p, q, r, in G.P. , $\therefore$ $q^2 = pr$

$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{3}{4}$

$\frac{\alpha+\beta}{\alpha\beta}=\frac{3}{4}$

$\frac{-\frac{q}{p}}{-\frac{r}{p}}=\frac{3}{4}$

$|\alpha-\beta| = \frac{\sqrt{q^2+4pr}}{|P|}$

$(\alpha-\beta)^2 = \frac{q^2+4pr}{p^2}$

$= \frac{5q^2}{p^2}=\frac{5q^2r^2}{q^4}=5\frac{r^2}{q^2}$

$= 5 \times \frac{16}{9}=\frac{80}{9}$

8. Let Ajay will not appear in JEE exam with probability $p=\frac{2}{7}$, while both Ajay and Vijay will appear in the exam with probability $q=\frac{1}{5}$. Then the probability, that Ajay will appear in the exam and Vijay will not appear is :

(1) $\frac{9}{35}$ (2) $\frac{3}{35}$ (3) $\frac{24}{35}$ (4) $\frac{18}{35}$

Sol. **(4)**

$P(A) = \frac{5}{7}$

$P(A\cap V)=\frac{1}{5}=P(A).P(V)$ $\therefore$ $P(V)=\frac{7}{25}$

$P(A\cap \bar{V})=P(A).P(\bar{V})$

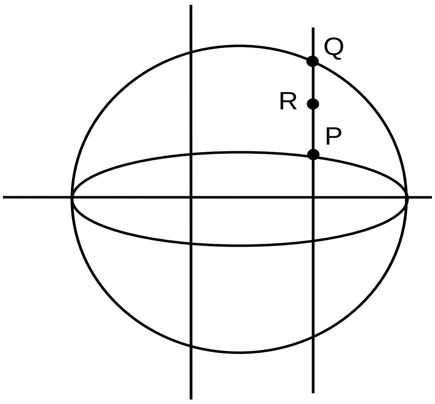
$= \frac{5}{7}\times\left(1-\frac{7}{25}\right)$

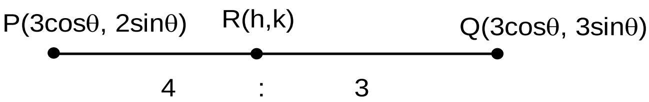
$= \frac{18}{35}$

9. Let P be a point on the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$. Let the line passing through P and parallel to y-axis meet the circle $x^2 + y^2 = 9$ at point Q such that P and Q are on the same side of the x-axis. Then, the eccentricity of the locus of the point R on PQ such that PR : RQ = 4 : 3 as P moves on the ellipse, is :

(1) $\frac{13}{21}$ (2) $\frac{\sqrt{139}}{23}$ (3) $\frac{\sqrt{13}}{7}$ (4) $\frac{11}{19}$

Sol. **(3)**





$h = \dfrac{12\cos\theta + 9\cos\theta}{7}$ $\qquad k = \dfrac{6\sin\theta + 12\sin\theta}{7}$

$h = 3\cos\theta$ $\qquad k = \dfrac{18\sin\theta}{7}$

$$\frac{h^2}{3^2} + \frac{k^2}{\left(\frac{18}{7}\right)^2} = 1$$

$$\left(\frac{18}{7}\right)^2 = 3^2(1 - e^2)$$

$$e^2 = 1 - \frac{36}{49}$$

$$e = \frac{\sqrt{13}}{7}$$

10. Consider 10 observations $x_1, x_2, \ldots, x_{10}$ such that $\sum_{i=1}^{10}(x_i - \alpha) = 2$ and $\sum_{i=1}^{10}(x_i - \beta)^2 = 40$, where α, β are positive integers. Let the mean and the variance of the observations be $\frac{6}{5}$ and $\frac{84}{25}$ respectively. Then $\frac{\beta}{\alpha}$ is equal to :

(1) 2 (2) 1 (3) $\frac{5}{2}$ (4) $\frac{3}{2}$

Sol. **(1)**

$\bar{x} = \dfrac{\sum_{i=1}^{10} x_i}{10} = \dfrac{6}{5}$, $\qquad \dfrac{\sum x_i^2}{10} - (\bar{x})^2 = \dfrac{84}{25}$

$\sum_{i=1}^{10}(x_i - \alpha) = 2$ $\qquad \dfrac{\sum x_i^2}{10} - \dfrac{36}{25} = \dfrac{84}{25}$

$\sum_{i=1}^{10} x_i - 10\alpha = 2$ $\qquad \dfrac{\sum x_i^2}{10} = \dfrac{120}{25}$

$10\alpha = 10$ $\quad\quad\quad\quad\quad\quad \sum x_i^2 = 48$

$\alpha = 1$

$$\sum_{i=1}^{10}(x_i - \beta)^2 = 40$$

$$\sum_{i=1}^{10} x_i^2 + 10\beta^2 - 2\beta\sum_{i=1}^{10} x_i = 40$$

$48 + 10\beta^2 - 24\beta = 40$

$10\beta^2 - 24\beta + 8 = 0$

$5\beta^2 - 12\beta + 4 = 0$

$\beta = 2$ or $\beta = \frac{2}{5}$ (reject)

$$\frac{\beta}{\alpha} = 2$$

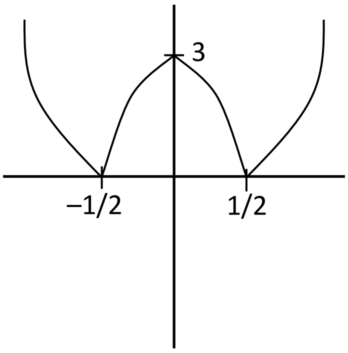
11. Let $f(x) = |2x^2 + 5|\,x\,| - 3\,|$, $x \in R$. If m and n denote the number of points where f is not continuous and not differentiable respectively, then m + n is equal to:

(1) 5 $\quad$ (2) 3 $\quad$ (3) 2 $\quad$ (4) 0

Sol. **(2)**

$$y = \left|2x^2 + 5|x| - 3\right|$$

By graphical transformation



Clearly f is continuous everywhere and not differentiable at three points.

$\therefore$ m = 0, n = 3

m + n = 3

12. The number of solutions of the equation $4\sin^2 x - 4\cos^3 x + 9 - 4\cos x = 0$; $x \in [-2\pi, 2\pi]$ is :

(1) 0 $\quad$ (2) 3 $\quad$ (3) 1 $\quad$ (4) 2

Sol. **(1)**

$$4\sin^2 x - 4\cos^3 x + 9 - 4\cos x = 0$$

$$4(1 - \cos^2 x) - 4\cos^3 x + 9 - 4\cos x = 0$$

$$4\cos^3 x + 4\cos^2 x + 4\cos x - 13 = 0$$

Max value of cos x = 1

So $4\cos^3 x + 4\cos^2 x + 4\cos x$ can never be greater than 12 therefore no solution.

13. Let the locus of the mid points of the chords of the circle $x^2 + (y-1)^2 = 1$ drawn from the origin intersect the line x + y = 1 at P and Q. Then, the length of PQ is :

(1) $\frac{1}{2}$ (2) 1 (3) $\frac{1}{\sqrt{2}}$ (4) $\sqrt{2}$

Sol. **(3)**

Eq. of chord whose mid point is (h, k)

$T = S_1$

$xh + yk - (y+k) = h^2 + k^2 - 2k$

$xh + yk - y = h^2 + k^2 - k$

It passes through (0, 0)

$\therefore\ h^2 + k^2 - k = 0$

Locus : $x^2 + y^2 - y = 0$

Locus intersect x + y = 1 at P and Q

$\therefore\ y^2 - y + (1-y)^2 = 0$

$2y^2 - 3y + 1 = 0$

$(y-1)(2y-1) = 0$

y = 1 or y = $\frac{1}{2}$

x = 0 $\quad x = \frac{1}{2}$

P(0, 1) and $Q\left(\frac{1}{2}, \frac{1}{2}\right)$

$PQ = \sqrt{\frac{1}{4} + \frac{1}{4}} = \frac{1}{\sqrt{2}}$

14. Let α be a non-zero real number. Suppose f : R→R is a differentiable function such that f(0) = 2 and $\lim_{x\to-\infty} f(x) = 1$. If f '(x) = αf(x) + 3, for all x ∈ R, then f(– $\log_e 2$) is equal to

(1) 7 (2) 9 (3) 3 (4) 5

Sol. **(2) or Bonus**

y = f(x)

$\frac{dy}{dx} - \alpha y = 3$

I.F. = $e^{\int -\alpha dx} = e^{-\alpha x}$

Sol. of differential equation

Y(I.F.) = $\int (I.F.)(3)dx + C$

$y.e^{-\alpha x} = 3\int e^{-\alpha x}dx + C_1$

$e^{-\alpha x}.y = \frac{-3e^{-\alpha x}}{\alpha} + C$

$\lim_{x\to-\infty} f(x)=1$

$\lim_{x\to-\infty} f(x)=\frac{-3}{\alpha}+C\lim_{x\to-\infty} e^{\alpha x}$

$1=\frac{-3}{\alpha}$

$\boxed{\alpha=-3}$ (α should be positive for finite value of C)

$f(x)=1+Ce^{-3x}$

Put x = 0

2 = 1 + C

$\boxed{c=1}$

$\therefore\ f(x)=1+e^{-3x},\ f(-\ln 2)=1+e^{3\ln 2}=9$

15. Let P and Q be the points on the line $\frac{x+3}{8}=\frac{y-4}{2}=\frac{z+1}{2}$ which are at distance of 6 units from the point R(1,2,3). If the centroid of the triangle PQR is (α, β, γ), then $\alpha^2+\beta^2+\gamma^2$ is :

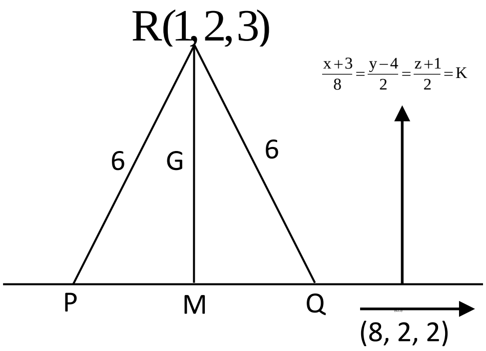
(1) 18 (2) 24 (3) 26 (4) 36

Sol. **(1)**

M is mid point of PQ and $\perp^r$ to line

G is centroid of ΔPQR such that RG : GM ≡ 2 : 1

Coordinates of $M(8k-3, 2k+4, 2k-1)$



$\overrightarrow{RM}.(8\hat{i}+2\hat{j}+2k)=0$

$8(8K-4)+(2K+2)2+(2K-4)2=0$

72 K – 36 = 0

$K=\frac{1}{2}$

$M(1,5,0)$

$\therefore\ G\left(\frac{2\times1+1\times1}{3},\frac{2\times5+1\times2}{3},\frac{2\times0+1\times3}{3}\right)$

G(1, 4, 1)

$\alpha^2+\beta^2+\gamma^2=18$

16. The value of $\int_0^1 \left(2x^3-3x^2-x+1\right)^{\frac{1}{3}} dx$ is equal to :

(1) –1 (2) 2 (3) 0 (4) 1

Sol. (3)

$$I = \int_0^1 (2x^3 - 3x^2 - x + 1)^{1/3}.dx$$

Applying king replace $x \to (1 - x)$

$$I = \int_0^1 [2(1-x)^3 - 3(1-x)^2 - (1-x) + 1]^{1/3}.dx$$

$$I = \int_0^1 [2 - 2x^3 - 6x + 6x^2 - 3 - 3x^2 + 6x - 1 + x + 1]^{1/3}.dx$$

$$I = \int_0^1 [-2x^3 + 3x^2 + x - 1]^{1/3}.dx$$

$$I = -\int_0^1 [2x^3 - 3x^2 - x + 1]^{1/3}.dx$$

$I = -I$

$2I = 0 \Rightarrow I = 0$

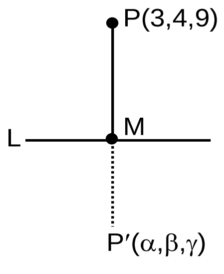
Ans. (3)

17. If the mirror image of the point P(3,4,9) in the line $\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-2}{1}$ is (α, β, γ), then $14(\alpha + \beta + \gamma)$ is :

(1) 102 (2) 138 (3) 132 (4) 108

Sol. (4)

$$L : \frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{1} = \lambda, \quad p(3,4,9)$$



$M(3\lambda + 1, 2\lambda - 1, \lambda + 2)$

$\overline{PM} = (3\lambda - 2, 2\lambda - 5, \lambda - 7)$

DR's of L : (3,2,1)

$\overline{PM} \perp$ line L,

So, $3(3\lambda - 2) + 2(2\lambda - 5) + 1(\lambda - 7) = 0$

$9\lambda - 6 + 4\lambda - 10 + \lambda - 7 = 0$

$14\lambda = 23 \Rightarrow \lambda = \frac{23}{14}$

$M\left(\frac{83}{14}, \frac{32}{14}, \frac{51}{14}\right)$

Now, As M is mid-point of pp′

$\therefore$ co-ordinates of p′ are

$\frac{\alpha+3}{2} = \frac{83}{14} \Rightarrow \alpha = \frac{62}{7}$

$\frac{\beta+4}{2} = \frac{32}{14} \Rightarrow \beta = \frac{4}{7}$

$\frac{\gamma+9}{2} = \frac{51}{14} \Rightarrow \gamma = \frac{-12}{7}$

So, $14(\alpha + \beta + \gamma) = 14 \times \frac{54}{7} \Rightarrow 108$

Ans. (4)

18. Let S_n denote the sum of the first n terms of an arithmetic progression. If $S_{10} = 390$ and the ratio of the tenth and the fifth terms is 15 : 7, then $S_{15} - S_5$ is equal to :

(1) 800 (2) 890 (3) 790 (4) 690

Sol. **(3)**

Given :

$S_{10} = 390$, $\frac{T_{10}}{T_7} = \frac{15}{7}$

$S_{10} = \frac{10}{2}[2a + 9d] = 390$

$2a + 9d = 78$ ....(1)

$\frac{T_{10}}{T_5} = \frac{a+9d}{a+4d} = \frac{15}{7}$

Ta + 63d = 15a + 60d

8a = 3d ....(2)

From (1) & (2)

a = 3, d = 8

Now, $S_{15} - S_5 = \frac{15}{2}[2a+14d] - \frac{5}{2}[2a+4d]$

$= \frac{5}{2}[6a + 42d - 2a - 4d]$

$= \frac{5}{2}[4a + 38d] = 5[2a + 19d]$

$= 5[6 + 19 \times 8]$

$= 790$

Ans. (3)

19. Let m and n be the coefficients of seventh and thirteenth terms respectively in the expansion of $\left(\frac{1}{3}x^{\frac{1}{3}}+\frac{1}{2x^{\frac{2}{3}}}\right)^{18}$. Then $\left(\frac{n}{m}\right)^{\frac{1}{3}}$ is :

(1) $\frac{1}{9}$ (2) $\frac{1}{4}$ (3) $\frac{4}{9}$ (4) $\frac{9}{4}$

Sol. **(3)**

$\left(\frac{1}{3}x^{1/3}+\frac{1}{2.x^{2/3}}\right)^{18}$ coeff. of $T_7 \to m$

coeff. of $T_{13} \to n$

$$\frac{m}{n}=\frac{18c_6\left(\frac{1}{3}x^{1/3}\right)^{12}\cdot\left(\frac{1}{2x^{2/3}}\right)^{6}}{18c_{12}\left(\frac{1}{3}x^{1/3}\right)^{6}\cdot\left(\frac{1}{2x^{2/3}}\right)^{12}} \qquad (\because nc_r = nc_{n-r})$$

$$\left(\frac{m}{n}\right)^{1/3}=\left(\frac{1}{3^{12}}\times 3^6\times\frac{1}{2^6}\times 2^{12}\right)^{1/3}\Rightarrow\left(\frac{2^6}{3^6}\right)^{1/3}\Rightarrow\frac{4}{9}$$

20. Let $f(x)=\begin{cases}x-1, & x \text{ is even,}\\ 2x, & x \text{ is odd,}\end{cases}$ $x\in N$. If for some $a\in N$, $f(f(f(a)))=21$, then $\lim\limits_{x\to a^-}\left\{\frac{|x|^3}{a}-\left[\frac{x}{a}\right]\right\}$, where [t] denotes the greatest integer less than or equal to t, is equal to :

(1) 169 (2) 121 (3) 225 (4) 144

Sol. **(4)**

$f(x) = \begin{cases}x-1, & x \text{ is even}\\ 2x, & x \text{ is odd}\end{cases}$

$f\left(\underbrace{f(f(a))}_{\text{even}}\right)=21$

i.e. $f\underbrace{(f(a))}_{\text{odd}}=22$

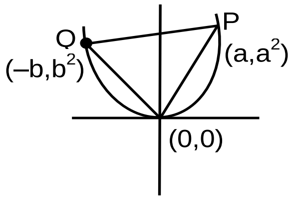
So, a = 12

Now, $\lim\limits_{x\to 12^-}\left\{\frac{12^3}{12}-\left[\frac{12}{12^-}\right]\right\}\Rightarrow\left\{(12)^2-0\right\}\Rightarrow 144$

SECTION – B

21. Three points O(0, 0), P(a, a^2), Q(–b, b^2), a > 0, b > 0, are on the parabola $y = x^2$. Let S_1 be the area of the region bounded by the line PQ and the parabola, and S_2 be the area of the triangle OPQ. If the minimum value of $\frac{S_1}{S_2}$ is $\frac{m}{n}, \gcd(m,n) = 1$, then m + n is equal to_____.

Sol. 7



$y = x^2$ $\quad$ $PQ; (y - b^2) = \frac{a^2 - b^2}{a+b}(x+b)$

$PQ; (a-b)x - y + ab = 0$

Now, $S_1 = \int_{-b}^{a} [(a-b)x + ab] - [x^2].dx$

$$S_1 = \left[(a-b)\frac{x^2}{2} + abx - \frac{x^3}{3}\right]_{-b}^{a}$$

$$S_1 = (a-b)\frac{(a^2-b^2)}{2} + ab(a+b) - \frac{1}{3}[a^3+b^3]$$

$$S_1 = \frac{(a+b)}{6}\left[a^2+b^2+2ab\right] \Rightarrow \frac{(a+b)^3}{6}$$

Area of Δ OPQ(S_2) $= \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ a & a^2 & 1 \\ -b & b^2 & 1 \end{vmatrix} = \frac{1}{2}[ab(a+b)]$

$$\frac{S_1}{S_2} = \frac{\frac{(a+b)^3}{6}}{\frac{ab}{2}(a+b)} \Rightarrow \frac{1}{3ab}(a+b)^2 \Rightarrow \frac{1}{3}\left[\frac{a}{b} + \frac{b}{a} + 2\right]$$

as , a, b > 0

So, using A.M., G.M.

$$\frac{\frac{a}{b} + \frac{b}{a}}{2} \ge \left(\frac{a}{b}.\frac{b}{a}\right)^{1/2}$$

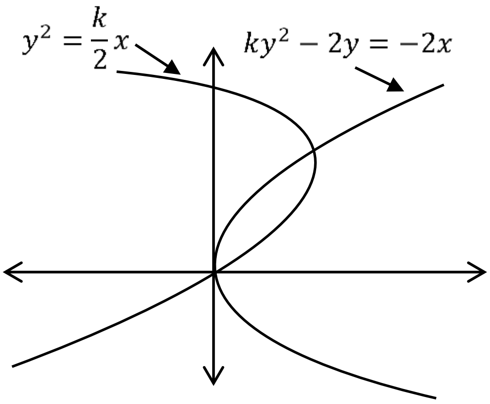
$$\frac{a}{b} + \frac{b}{a} \ge 2$$

$$\min\left(\frac{S_1}{S_2}\right) = \frac{4}{3}$$

$\therefore$ m + n = 7

22. The sum of squares of all possible values of k, for which area of the region bounded by the parabolas $2y^2 = kx$ and $ky^2 = 2(y - x)$ is maximum, is equal to______.

Sol. **8**



$$\text{Area} = A = \int_0^{\frac{2k}{k^2+4}} \left[\left(\frac{ky^2 - 2y}{-2}\right) - \left(\frac{2y^2}{k}\right)\right] dy$$

$$= \left[\left(\frac{-k}{2} - \frac{2}{k}\right)\frac{y^3}{3} + \frac{y^2}{2}\right]_0^{\frac{2k}{k^2+4}}$$

$$= \frac{-(k^2+4)}{2k} \times \left(\frac{2k}{k^2+4}\right)^3 \times \frac{1}{3} + \left(\frac{2k}{k^2+4}\right)^2 \times \frac{1}{2}$$

$$= -\left(\frac{2k}{k^2+4}\right)^2 \times \frac{1}{3} + \left(\frac{2k}{k^2+4}\right)^2 \times \frac{1}{2}$$

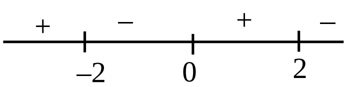
$$= \frac{1}{6} \times \frac{4k^2}{(k^2+4)^2}$$

$$A(k) = \frac{2}{3}\frac{k^2}{(k^2+4)^2}$$

$$A'(K) = \frac{2}{3}\frac{\left[2K(K^2+4)^2 - 2K^2(K^2+4)(2K)\right]}{(K^2+4)^4} = 0$$

$$\Rightarrow \frac{2}{3}(2K)(K^2+4)\left[K^2 + 4 - 2K^2\right] = 0$$

$K = 0$ or $K = 2$ or $K = -2$



$\therefore$ $K = 0$ is local minima

$K = -2$ & 2 are local maxima

$\therefore$ $(-2)^2 + 2^2 = 8$

23. If $y=\dfrac{(\sqrt{x}+1)\left(x^2-\sqrt{x}\right)}{x\sqrt{x}+x+\sqrt{x}}+\dfrac{1}{15}\left(3\cos^2 x-5\right)\cos^3 x$, then $96y'\left(\dfrac{\pi}{6}\right)$ is equal to_____.

Sol. **105**

$$y=\frac{(\sqrt{x}+1)(x^2-\sqrt{x})}{x\sqrt{x}+x+\sqrt{x}}+\frac{1}{15}(3\cos^2 x-5)\cos^3 x$$

$$y=\frac{(\sqrt{x}+1).\sqrt{x}(x\sqrt{x}-1)}{x\sqrt{x}+x+\sqrt{x}}+\frac{1}{15}(3\cos^2 .x-5)\cos^3 x$$

$$y=\frac{(\sqrt{x}+1).\sqrt{x}((\sqrt{x})^3-1^3)}{x\sqrt{x}+x+\sqrt{x}}+\frac{1}{15}[3\cos^5 x-5\cos^3 x]$$

$$y=\frac{(\sqrt{x}+1).\sqrt{x}(\sqrt{x}-1)(x+\sqrt{x}+1)}{\sqrt{x}(x+\sqrt{x}+1)}+\frac{1}{15}[3\cos^5 x-5\cos^3 x]$$

$$y = (x-1) + \frac{1}{15}[3\cos^5 x-5\cos^3 x]$$

Now, diff w.r.t x

$$y' = 1-0+\frac{1}{15}[15\cos^4 x.(-\sin x)-15\cos^2 x(-\sin x)]$$

$$y' = 1-\sin x.[\cos^4 x-\cos^2 x]$$

$$y'(\pi/6) = 1-\sin\frac{\pi}{6}\left[\cos^4\frac{\pi}{6}-\cos^2\frac{\pi}{6}\right]$$

$$= 1-\frac{1}{2}\left[\frac{9}{16}-\frac{3}{4}\right]$$

$$= 1-\frac{1}{2}\left[\frac{9-12}{16}\right] \Rightarrow 1+\frac{3}{32}\Rightarrow\frac{35}{32}$$

$$96.\ y'(\pi/6) = 96\times\frac{35}{32} \Rightarrow 105$$

24. If $\dfrac{dx}{dy}=\dfrac{1+x-y^2}{y}, x(1)=1,$ then $5x(2)$ is equal to _____.

Sol. **5**

$$\frac{dx}{dy}=\frac{1+x-y^2}{y}$$

$ydx = dy + x.dy - y^2dy$

$ydx - x\,dy = dy - y^2.\,dy$

divide by y^2 both sides

$$\frac{y\,dx-xdy}{y^2}=\frac{1}{y^2}.dy-dy$$

$$\int d(x/y)=\int\frac{1}{y^2}.dy-\int dy$$

$\frac{x}{y}=\frac{-1}{y}-y+c$

1 = –1 –1 + C

1 = –1 –1 + C ⇒ [C = 3]

$\frac{x}{y}=\frac{-1}{y}-y+3$

$x = 3y - y^2 - 1$

$5\,x(2) \Rightarrow 5\,(3 \times 2 - (2)^2 - 1) \Rightarrow 5(1) = 5$

25. Let $f:(0,\infty)\to R$ and $F(x)=\int_0^x tf(t)dt$. If $F(x^2)=x^4+x^5$, then $\sum_{r=1}^{12} f(r^2)$ is equal to _____.

Sol. **219**

$F(x)=\int_0^x t\cdot f(t)dt$

$F'(x)=xf(x)$

Given $F(x^2)=x^4+x^5$ Let^2 = t

$F(t)=t^2+t^{5/2}$

$F'(t)=2t+5/2t^{3/2}$

$t\cdot f(t)=2t+5/2t^{3/2}$

$f(t)=2+5/2r^{1/2}$

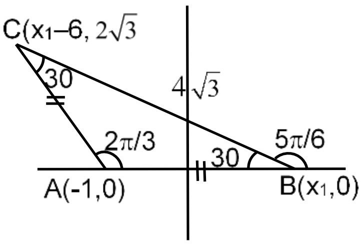
$\sum_{r=1}^{12} f(r^2)=\sum_{r=1}^{12} 2+\frac{5}{2}r$

$=24+5/2\left[\frac{12(13)}{2}\right]$

= 219

26. Let ABC be an isosceles triangle in which A is at $(-1,0), \angle A=\frac{2\pi}{3}, AB=AC$ and B is on the positve x-axis. If $BC=4\sqrt{3}$ and the line BC intersects the line y = x + 3 at (α, β), then $\frac{\beta^4}{\alpha^2}$ is_____.

Sol. **36**



co-ordinates of 'C' are

$$\frac{x-x_1}{\cos\frac{5\pi}{6}}=\frac{y-0}{\sin\frac{5\pi}{6}}=4\sqrt{3}$$

$$x = 4\sqrt{3}\cos\frac{5\pi}{6}+x_1 \Rightarrow 4\sqrt{3}\times\left(-\frac{\sqrt{3}}{2}\right)+x_1 \Rightarrow (x_1-6)$$

$$y=4\sqrt{3}\sin\frac{5\pi}{6}\Rightarrow 4\sqrt{3}\times\frac{1}{2}\Rightarrow 2\sqrt{3}$$

$$C\left(x_1-6, 2\sqrt{3}\right)$$

$$m_{AC} = -\sqrt{3}$$

$$\frac{0-2\sqrt{3}}{-1-x_1+6}=-\sqrt{3}\Rightarrow x_1=3$$

BC : $x + \sqrt{3}y=3$, $y = x + 3$

$y - 3 + \sqrt{3}y=3$

$$y = \frac{6}{\sqrt{3}+1},\ x = \frac{6}{\sqrt{3}+1}-3 \Rightarrow \frac{6-3\sqrt{3}-3}{\sqrt{3}+1}$$

$$x = \frac{3-3\sqrt{3}}{\sqrt{3}+1}$$

$$\frac{-6}{\left(1+\sqrt{3}\right)^2}$$

So, $\frac{\beta^4}{\alpha^2}=36$

27. Let A = $I_2 - 2MM^T$, where M is a real matrix of order 2×1 such that the relation M^I M = I_1 holds. If λ is a real number such that the relation AX = λX holds for some non-zero real matrix X of order 2×1, then the sum of squares of all possible values of λ is equal to_____.

Sol. **2**

$$A=I_2-2MM^T$$

$$A^2=\left(I_2-2MM^T\right)\left(I_2-2MM^T\right)$$

$$=I_2-2MM^T-2MM^T+4MM^TMM^T$$

$$=I_2-4MM^T+4MM^T$$

$$=I_2$$

$$AX=\lambda X$$

$$A^2X=\lambda AX$$

$$X=\lambda(\lambda X)$$

$$X=\lambda^2 X$$

$$X\left(\lambda^2-1\right)=0$$

$$\lambda^2=1$$

$$\lambda=\pm 1$$

Sum of square of all possible values = 2

28. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = -\hat{i} - 8\hat{j} + 2\hat{k}$ and $\vec{c} = 4\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three vectors such that $\vec{b} \times \vec{a} = \vec{c} \times \vec{a}$. If the angle between the vector $\vec{c}$ and the vector $3\hat{i} + 4\hat{j} + \hat{k}$ is θ, then the greatest integer less than or equal to $\tan^2\theta$ is_____.

Sol. **38**

$\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = -\hat{i} - 8\hat{j} + 2\hat{k}$, $\vec{c} = 4\hat{i} + c_2\hat{j} + c_3\hat{k}$

Given, $\vec{b} \times \vec{a} = \vec{c} \times \vec{a}$

$-10\hat{i} + 3\hat{j} + 7\hat{k} = (c_2 - c_3)\,\hat{i} + (c_3 - 4)\,\hat{j} + (4 - c_2)\,\hat{k}$

$c_2 - c_3 = -10$; $c_3 - 4 = 3$; $4 - c_2 = 7$

$c_3 = 7 \quad c_2 = -3$

So, $\vec{c} = 4\hat{i} - 3\hat{j} + 7\hat{k}$

angle b/w $\vec{c}$ & $(3\hat{i} + 4\hat{j} + \hat{k})$ is θ

$$\therefore \cos\theta = \frac{12 - 12 + 7}{\sqrt{74}.\sqrt{26}} \Rightarrow \frac{7}{\sqrt{74}.\sqrt{26}}$$

$$\tan^2\theta = \sec^2\theta - 1 \Rightarrow \frac{74 \times 26}{49} - 1$$

$$\tan^2\theta = \frac{1924 - 49}{49} = \frac{1875}{49} = 38.26$$

$[\tan^2\theta] = 38$

29. If three successive terms of a G.P. with common ratio $r(r > 1)$ are the lengths of the sides of a triangle and $[r]$ denotes the greatest integer less than or equal to r, then $3[r] + [-r]$ is equal to_____.

Sol. **1**

Let side of triangle be a, ar, ar^2

Now, $a + ar > ar^2$

$1 + r > r^2$

or $r^2 - r - 1 < 0$

$$\left(r - \frac{1+\sqrt{5}}{2}\right)\left(r - \frac{-1+\sqrt{5}}{2}\right) < 0$$

$$r \in \left(\frac{-1+\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right)$$

but, $r > 1$ (given)

$$\therefore r \in \left(1, \frac{1+\sqrt{5}}{2}\right)$$

$[r] = 1$

$[-r] = -2$

So, $3[r] + [-r] = 3 - 2 \Rightarrow 1$

30. The lines $L_1, L_2, \ldots, L_{20}$ are distinct. For n =1, 2, 3,…, 10 all the lines L_{2n-1} are parallel to each other and all the lines L_{2n} pass through a given point P. The maximum number of points of intersection of pairs of lines from the set $\{L_1, L_2, \ldots, L_{20}\}$ is equal to_____.

Sol. **101**

Given: $L_1, L_3, L_5 \ldots\ldots L_{19}$ are parallel

$L_2, L_4, L_6 \ldots\ldots L_{20}$(concurrent)

Total At. of intersection

$$= {}^{20}c_2 - \underbrace{{}^{10}c_2}_{\text{(due to parallel lines)}} - \underbrace{{}^{10}c_2 + 1}_{\text{(due to concurrent lines)}}$$

= 190 – 45 – 45 + 1

= 101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