JEE MAIN 2024
Paper with Solution

Maths | 31st January 2024 _ Shift-2

SECTION – A

1. Let 2nd, 8th and 44th terms of a non-constant A.P. be respectively the 1st, 2nd and 3rd terms of a G.P. If the first term of A.P. is 1, then the sum of its first 20 terms is equal to-

(1) 970 (2) 980 (3) 960 (4) 990

Sol. **(1)**

$$\begin{array}{l} \quad a_2, \quad a_8, \quad a_{44} \rightarrow A.P \\ \text{If} \downarrow \quad \downarrow \quad \downarrow \\ \quad A_1 \quad A_2 \quad A_3 \rightarrow G.P. \end{array}$$

$a_2 = a + d$

$a_8 = a + 7d$ where $\{a = 1\}$

$a_{44} = a + 43d$

and $A_1, A_2, A_3 \rightarrow$ G.P

$$\frac{A_2}{A_1} = \frac{A_3}{A_2}$$

$$\frac{a+7d}{a+d} = \frac{a+43d}{a+7d}\{a = 1\}$$

$(1+7d)^2 = (1+43d)(1+d)$

$1 + 49d^2 + 14d = 1 + 44d + 43d^2$

$6d^2 - 30d = 0$

$6d(d-5) = 0$

$d \underset{\times}{=} 0, d = 5$

Sum of first 20 terms

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{20} = \frac{20}{2}(2 \times 1 + 19 \times 5)$$

$= 10(97) = 970$

2. A coin is biased so that a head is twice a likely to occur as tail. If the coin is tossed 3 times, then the probability of getting two tails and one head is

(1) $\frac{1}{27}$ (2) $\frac{1}{9}$ (3) $\frac{2}{27}$ (4) $\frac{2}{9}$

Sol. **(4)**

$p(H) = 2p(T)$

Let $p(T) = x$

$p(H) = 2x$

$x + 2x = 1$

$x=\frac{1}{3}$

$p(T)=\frac{1}{3}$ $\quad p(H)=\frac{2}{3}$

Probability of 2T & 1H is

$^{3}C_{2}\left(\frac{1}{3}\right)^{2}\left(\frac{2}{3}\right)$

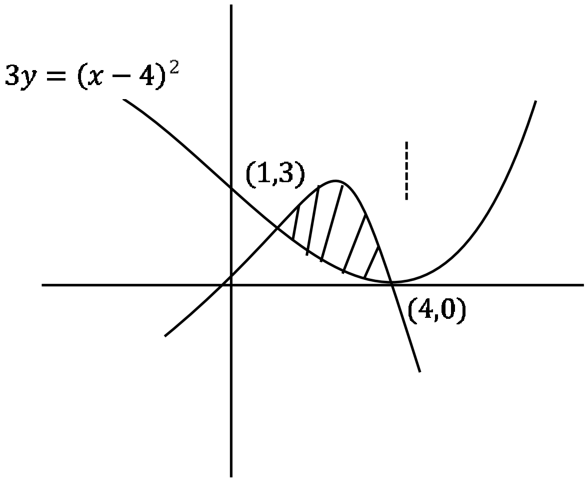
$3\left(\frac{1}{9}\right)\left(\frac{2}{3}\right)=\frac{2}{9}$

3. The area of the region enclosed by the parabolas $y=4x-x^2$ and $3y=(x-4)^2$ is equal to-

(1) 4 (2) 6 (3) $\frac{32}{9}$ (4) $\frac{14}{3}$

Sol. (2)

The area region enclosed by the parabolas $y=4x-x^2$ and $3y=(x-4)^2$



$3(4x-x^2)=(x-4)^2$

$12x-3x^2=x^2-8x+16$

$4x^2-20x+16=0$

$x^2-5x+4=0$

$(x-1)(x-4)=0$

$x=1,4$

If $x=1$

$y=4x-x^2=4-1=3$

If $x=4$, $y=0$

Area is

$$A=\left|\int_1^4\left(\left(4x-x^2\right)-\frac{(x-4)^2}{3}\right)dx\right|$$

$$A=\left|\left(\frac{4x^2}{2}-\frac{x^3}{3}\right)-\frac{(x-4)^3}{9}\right|_1^4$$

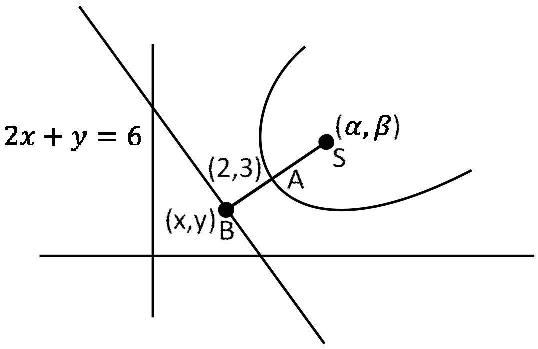
$$A=\left|\left(32-\frac{64}{3}-0\right)-\left(2-\frac{1}{3}+3\right)\right|$$

$$A=\left|\frac{32}{3}-\left(5-\frac{1}{3}\right)\right|=6$$

4. Let P be a parabola with vertex (2, 3) and directrix $2x+y=6$. Let an ellipse $E:\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$, of eccentricity $\frac{1}{\sqrt{2}}$ pass through the focus of the parabola P. Then the square of the length of the latus rectum of E, is

(1) $\frac{512}{25}$ (2) $\frac{385}{8}$ (3) $\frac{656}{25}$ (4) $\frac{347}{8}$

Sol. (3)



$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=-\left(\frac{ax_1+by_1+c}{a^2+b^2}\right)$$

$$\frac{x-2}{2}=\frac{y-3}{1}=-\left(\frac{2\times2+3-6}{4+1}\right)$$

$$\frac{x-2}{2}=\frac{y-3}{1}=-\frac{1}{5}$$

$$x=\frac{-2}{5}+2=\frac{8}{5}$$

$$y=\frac{-1}{5}+3=\frac{14}{5}$$

So, $\frac{\alpha+x}{2}=2$

$$\Rightarrow \alpha+\frac{8}{5}=4$$

$\Rightarrow \alpha = 4 - \frac{8}{5} = \frac{12}{5}$

$\frac{\beta + y}{2} = 3$

$\beta = 6 - \frac{14}{5} = \frac{16}{5}$

$(\alpha, \beta) = \left(\frac{12}{5}, \frac{16}{5}\right)$ lies on the ellipse

Now ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{\left(\frac{12}{5}\right)^2}{a^2} + \frac{\left(\frac{16}{5}\right)^2}{b^2} = 1$

and $\frac{144}{a^2} + \frac{256}{b^2} = 25$ …(1)

eccentricity $\Rightarrow \sqrt{1 - \frac{b^2}{a^2}} = \frac{1}{\sqrt{2}}$

$1 - \frac{b^2}{a^2} = \frac{1}{2}$

$\frac{b^2}{a^2} = \frac{1}{2}$

$a^2 = 2b^2$ …(2)

Using eq. (2) in equation (1)

$\frac{144}{2b^2} + \frac{256}{b^2} = 25$

$72 + 256 = 25b^2$

$\frac{328}{25} = b^2$

Length of L.R. $\Rightarrow \frac{2b^2}{a}$

Sq. of Length of L.R. $= \left(\frac{2b^2}{a}\right)^2 = \frac{4b^4}{a^2}$

$\Rightarrow \frac{4b^4}{2b^2}$

$2b^2 \Rightarrow 2 \times \frac{328}{25}$

$\Rightarrow \frac{656}{25}$

5. The shortest distance, between lines L_1 and L_2, where $L_1 : \frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+4}{2}$ and L_2 is the line, passing through the points A(–4, 4, 3), B(–1, 6, 3) and perpendicular to the line $\frac{x-3}{-2} = \frac{y}{3} = \frac{z-1}{1}$, is

(1) $\frac{42}{\sqrt{117}}$ (2) $\frac{24}{\sqrt{117}}$ (3) $\frac{121}{\sqrt{221}}$ (4) $\frac{141}{\sqrt{221}}$

Sol. **(4)**

$L_2 : \frac{x+4}{3} = \frac{y-4}{2} = \frac{z-3}{0}$

$L_1 : \frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+4}{2}$

$$S \cdot D = \left| \frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$\vec{a}_1 = \hat{i} - \hat{j} - 4\hat{k}$ $\vec{b}_2 = 3\hat{i} + 2\hat{j} + 0\hat{k}$

$\vec{a}_2 = -4\hat{i} + 4\hat{j} + 3\hat{k}$ $\vec{b}_1 = 2\hat{i} - 3\hat{j} + 2\hat{k}$

$\vec{b}_1 \times \vec{b}_2 = -4\hat{i} + 6\hat{j} + 13\hat{k}$

$$S \cdot D = \left| \frac{(-5\hat{i} + 5\hat{j} + 7\hat{k})\cdot(-4\hat{i} + 6\hat{j} + 13\hat{k})}{\sqrt{221}} \right|$$

$$= \left| \frac{141}{\sqrt{221}} \right| = \frac{141}{\sqrt{221}}$$

6. If for some $m, n;\ {}^6C_m + 2({}^6C_{m+1}) + {}^6C_{m+2} > {}^8C_3$ and ${}^{n-1}P_3 : {}^nP_4 = 1 : 8$, then ${}^nP_{m+1} + {}^{n+1}C_m$ is equal to

(1) 384 (2) 376 (3) 372 (4) 380

Sol. **(3)**

$6_{C_m} + 2(6_{C_{m+1}}) + 6_{C_{m+2}} > 8_{C_3}$

$6_{C_m} + 6_{C_{m+1}} + 6_{C_{m+1}} + 6_{C_{m+2}} > 8_{C_3}$

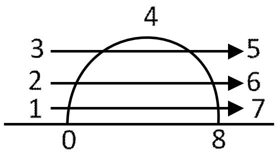
$\{{}^nC_r + {}^nC_{r-1} \Rightarrow {}^{n+1}C_r\}$

${}^7C_{m+1} + {}^7C_{m+2} > {}^8C_3$

${}^8C_{m+2} > {}^8C_3$

${}^8C_{m+2} > {}^8C_3$

$\Rightarrow m = 2$



$$\frac{^{n-1}P_3}{^{n}P_4}=\frac{1}{8}$$

$$\frac{(n-1)(n-2)(n-3)}{n(n-1)(n-2)(n-3)}=\frac{1}{8}$$

n = 8

Now

$$^{8}P_{2+1}+{}^{8+1}C_2$$

$$\Rightarrow 8\times7\times6+\frac{9\times8}{2}$$

$$\Rightarrow 336+36=372$$

7. Let A be a 3×3 real matrix such that $A\begin{pmatrix}1\\0\\1\end{pmatrix}=2\begin{pmatrix}1\\0\\1\end{pmatrix}, A\begin{pmatrix}-1\\0\\1\end{pmatrix}=4\begin{pmatrix}-1\\0\\1\end{pmatrix}, A\begin{pmatrix}0\\1\\0\end{pmatrix}=2\begin{pmatrix}0\\1\\0\end{pmatrix}$. Then, the system

$(A-3I)\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\2\\3\end{pmatrix}$ has

(1) exactly two solutions

(2) unique solution

(3) no solution

(4) infinitely many solutions

Sol. **(2)**

$A\rightarrow 3\times3$ matrix

$$A\begin{pmatrix}1\\0\\1\end{pmatrix}=2\begin{pmatrix}1\\0\\1\end{pmatrix}$$

$$A\begin{pmatrix}-1\\0\\1\end{pmatrix}=4\begin{pmatrix}-1\\0\\1\end{pmatrix}$$

$$A\begin{pmatrix}0\\1\\0\end{pmatrix}=2\begin{pmatrix}0\\1\\0\end{pmatrix}$$

Then $(A-3I)\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\2\\3\end{pmatrix}$

Let

$$A=\begin{bmatrix}x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\\ x_3 & y_3 & z_3\end{bmatrix}$$

Now using given condition

(i) $\begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix}_{3\times3} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}_{3\times1} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$

$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} \Rightarrow \begin{matrix} y_1 = 0 \\ y_2 = 2 \\ y_3 = 0 \end{matrix}$

(ii) $\begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix}_{3\times3} \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \\ 4 \end{pmatrix}$

$-x_1 + z_1 = -4$

$-x_2 + z_2 = 0$

$-x_3 + z_3 = 4$

(iii) $\begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix}_{3\times3} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$

$x_1 + z_1 = 2$

$x_2 + z_2 = 0$

$x_3 + z_3 = 2$

$-x_1 + z_1 = -4$

$x_1 + z_1 = 2$

$\Rightarrow z_1 = -1$

$x_1 = 3$

$\begin{matrix} x_2 + z_2 = 0 \\ -x_2 + z_2 = 0 \end{matrix} \Rightarrow z_2 = 0 \Rightarrow x_2 = 0$

$x_3 + z_3 = 2$

$-x_3 + z_3 = 4$

$z_3 = 3$

$x_3 = -1$

So now

$A = \begin{bmatrix} 3 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 3 \end{bmatrix}$

$A - 3I = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

$(A - 3I)\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

$$\begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$-z = 3 \Rightarrow z = -3$

$-y = 2 \Rightarrow y = -2$ $\qquad \Rightarrow$ unique Solution

$-x = 1 \Rightarrow x = -1$

8. Let a variable line passing through the centre of the circle $x^2 + y^2 - 16x - 4y = 0$, meet the positive co-ordinate axes at the points A and B. Then the minimum value of $OA + OB$, where O is the origin is equal to

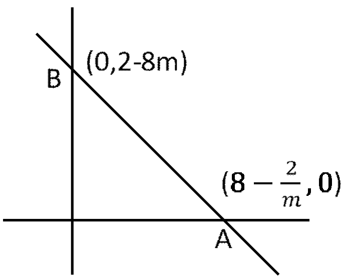
(1) 18 (2) 24 (3) 20 (4) 12

Sol. **(1)**

certre $(8,2)$

equation of line

$y - 2 = m(x - 8)$



$$f(m) = OA + OB = 8 - \frac{2}{m} + 2 - 8m$$

$$f(m) = 10 - \frac{2}{m} - 8m$$

$$f'(m) = \frac{2}{m^2} - 8 = 0$$

$$\frac{2}{m^2} = 8$$

$$m^2 = \frac{1}{4}$$

$$m = \pm\frac{1}{2}$$

$$m = \frac{1}{2} \quad \times$$

$$m = -\frac{1}{2}$$

$$f\left(-\frac{1}{2}\right) = 10 - \frac{2}{\left(-\frac{1}{2}\right)} - 8\left(-\frac{1}{2}\right)$$

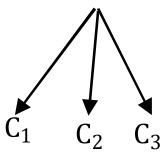
$$= 10 + 4 + 4 = 18$$

9. The number of ways in which 21 identical apples can be distributed among three children such that each child gets at least 2 apples, is

(1) 406 (2) 142 (3) 136 (4) 130

Sol. **(3)**

21 identical apples



2 2 2 (-atlest)

$21-6=15$

$\downarrow$

in 3 children

So $15+3-1_{C_{3-1}} = {}^{17}C_2$

$\Rightarrow \frac{17\times 16}{2}=136$

10. Let $f,g:(0,\infty)\to\mathbb{R}$ be two functions defined by $f(x)=\int_{-x}^{x}\left(|t|-t^2\right)e^{-t^2}dt$ and $g(x)=\int_0^{x^2} t^{1/2}e^{-t}dt$. Then, the value of $9\left(\left(f\sqrt{\log_e 9}\right)+g\left(\sqrt{\log_e 9}\right)\right)$ is equal to

(1) 10 (2) 9 (3) 8 (4) 6

Sol. **(3)**

$f(x)=\int_{-x}^{x}|t|-t^2e^{-t^2}dt$

$g(x)=\int_0^{x^2} t^{\frac{1}{2}}e^{-t}dt$

$f(x)=2\int_0^{x}\left(t-t^2\right)e^{-t^2}dt$

$f(x)=2\left[\int_0^{x} te^{-t^2}dt-\int_0^{x} t^2e^{-t^2}dt\right]$

$g(x)=\int_0^{x^2}\sqrt{t}\,e^{-t}dt$ $\qquad \sqrt{t}=y \Rightarrow \frac{dt}{2\sqrt{t}}=dy$

$g(x)=2\int_0^{x} y^2\cdot e^{-y^2}dy$

$f(x)+g(x)=2\left(\frac{1-e^{-x^2}}{2}\right)$

$=1-e^{-x^2}$

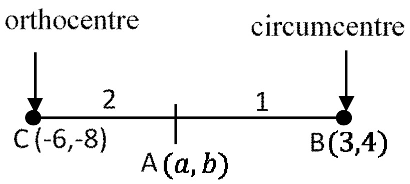
$\Rightarrow 9\left(f\left(\sqrt{\log_e 9}\right)+g\left(\sqrt{\log_e 9}\right)\right)=9\times\left(1-\frac{1}{9}\right)$

$=9\times\frac{8}{9}=8$

11. Let $A(a,b), B(3,4)$ and $C(-6,-8)$ respectively denote the centroid, circumcenter and orthocenter of a triangle. Then, the distance of the point $P(2a+3, 7b+5)$ from the line $2x+3y-4=0$ measured parallel to the line $x-2y-1=0$ is

(1) $\frac{\sqrt{5}}{17}$ (2) $\frac{15\sqrt{5}}{7}$ (3) $\frac{17\sqrt{5}}{7}$ (4) $\frac{17\sqrt{5}}{6}$

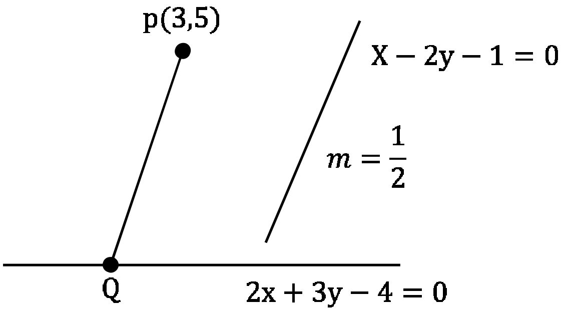
Sol. **(3)**



$$a = \frac{2\times 3-6}{3} = 0$$

$$b = \frac{2\times 4-8}{3} = 0$$

So point $P(3,5)$



Eq. of PQ

$$y-5 = \frac{1}{2}(x-3)$$

$$2y-10 = x-3$$

$$2y-x-7=0$$

Intersection point

$$2(2y-x-7=0)$$

$$3y+2x-4=0$$

$$7y=18$$

$$y=\frac{18}{7}$$

$$2\left(\frac{18}{7}\right)-x-7=0$$

$$\frac{36-49}{7}=x$$

$x = \frac{-13}{7}$

Point $Q\left(-\frac{13}{7}, \frac{18}{7}\right)$

$P(3,5), \quad Q\left(\frac{-13}{7}, \frac{18}{7}\right)$

$PQ = \sqrt{\left(\frac{-13}{7} - 3\right)^2 + \left(\frac{18}{7} - 5\right)^2}$

$PQ = \sqrt{\left(\frac{34}{7}\right)^2 + \left(\frac{17}{7}\right)^2}$

$PQ = \frac{\sqrt{1156 + 289}}{7} = \frac{17\sqrt{5}}{7}$

12. Let z_1 and z_2 be two complex numbers such that $z_1 + z_2 = 5$ and $z_1^3 + z_2^3 = 20 + 15i$. Then, $\left|z_1^4 + z_2^4\right|$ equals-

(1) $25\sqrt{3}$ (2) $30\sqrt{3}$ (3) $15\sqrt{15}$ (4) 75

Sol. **(4)**

$z_1 + z_2 = 5$

$z_1^3 + z_2^3 = 20 + 15i$

$\left|z_1^4 + z_2^4\right| = ?$

$z_1^3 + z_2^3 = (z_1 + z_2)^3 - 3z_1z_2(z_1 + z_2)$

$z_1^3 + z_2^3 = 125 - 15z_1z_2$

$20 + 15i = 125 - 15z_1z_2$

$z_1z_2 = 7 - i$

Now

$(z_1 + z_2)^2 = 5^2$

$z_1^2 + z_2^2 + 2z_1z_2 = 25$

$z_1^2 + z_2^2 = 25 - 2(7 - i)$

$= 11 + 2i$

$\left(z_1^2 + z_2^2\right)^2 = (11 + 2i)^2$

$z_1^4 + z_2^4 + 2(7 - i)^2 = 117 + 44i$

$z_1^4 + z_2^4 = 21 + 72i$

$\left|z_1^4 + z_2^4\right| = \sqrt{(21)^2 + (72)^2}$

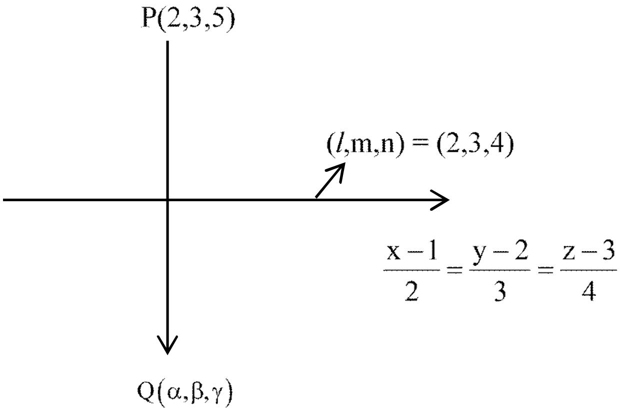
$= \sqrt{441 + 5184} = \sqrt{5625}$

$= 75$

13. Let (α,β,γ) be the mirror image of the point (2, 3, 5) in the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$. Then $2\alpha+3\beta+4\gamma$ is equal to

(1) 31 (2) 34 (3) 32 (4) 33

Sol. **(4)**



Direction Ratios of PQ

$\alpha-2,\beta-3,\gamma-5$

PQ ⊥ line

$2(\alpha-2)+3(\beta-3)+(\gamma-5)4=0$

$2\alpha-4+3\beta-9+4\gamma-20=0$

$2\alpha+3\beta+4\gamma=20+9+4=33$

14. The number of solutions, of the equation $e^{\sin x}-2e^{-\sin x}=2$, is :

(1) 0 (2) 1 (3) more than 2 (4) 2

Sol. **(1)**

$e^{\sin x}-2e^{-\sin x}=2$

$e^{\sin x}=t$

$t-\frac{2}{t}=2$

$t^2-2t-2=0$

$t=\frac{2\pm\sqrt{4+8}}{2}$

$t=\frac{2\pm2\sqrt{3}}{2}\Rightarrow 1\pm\sqrt{3}$

$t=1+\sqrt{3},\quad t=1-\sqrt{3}<0$ (Rejected)

$t=1+\sqrt{3}$

$e^{\sin x}=2.732$ $(\because e^{\sin x}\le 2.71$ as maximum value of sin x =1)

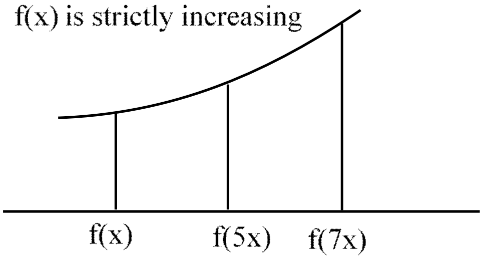
(Rejected)

$\Rightarrow$ no Solution.

15. Let $f: \to R \to (0,\infty)$ be strictly increasing function such that $\lim_{x\to\infty}\frac{f(7x)}{f(x)}=1$. Then, the value of $\lim_{x\to\infty}\left[\frac{f(5x)}{f(x)}-1\right]$ is equal to

(1) 7/5 (2) 4 (3) 1 (4) 0

Sol. **(4)**



$f(x) < f(5x) < f(7x)$

$\frac{f(x)}{f(x)} < \frac{f(5x)}{f(x)} < \frac{f(7x)}{f(x)}$

$\lim_{x\to\infty} 1 < \lim_{x\to\infty}\frac{f(5x)}{f(x)} < \lim_{x\to\infty}\frac{f(7x)}{f(x)}$

$1 < \lim_{x\to\infty}\frac{f(5x)}{f(x)} < 1$

$1-1 < \lim_{x\to\infty}\frac{f(5x)}{f(x)} < 1-1$

$\boxed{\lim_{x\to\infty}\left[\frac{f(5x)}{f(x)}-1\right]=0}$

16. If the function $f:(-\infty,-1)\to(a,b]$ defined by $f(x)=e^{x^3-3x+1}$ is one-one and onto, the distance of the point $P(2b+4, a+2)$ from the line $x+e^{-3}y=4$ is :

(1) $\sqrt{1+e^6}$ (2) $2\sqrt{1+e^6}$ (3) $4\sqrt{1+e^6}$ (4) $3\sqrt{1+e^6}$

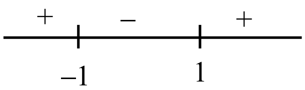
Sol. **(2)**

$f:(-\infty,-1)\to(a,b]$

$f(x)=e^{x^3-3x+1}$

$f'(x)=e^{x^3-3x+1}(3x^2-3)$

$f'(x)=e^{x^3-3x+1}3(x-1)(x+1)$



$(-\infty,-1] \rightarrow \uparrow F^n$

So $x \to -\infty$

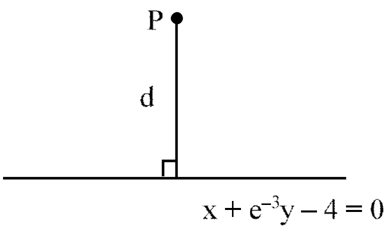
$\Rightarrow f(x) = e^{-\infty} \to 0 = a$

$x \to -1$

$\Rightarrow F(-1) = e^{-1+3+1} = e^3 = b$

$(a, b] \Rightarrow (0, e^3]$

$P(2e^3 + 4, 2)$



$$d = \left| \frac{2e^3 + 4 + e^{-3}(2) - 4}{\sqrt{1 + (e^{-3})^2}} \right|$$

$$d = \frac{2(e^3 + e^{-3})}{\sqrt{\frac{e^6 + 1}{e^6}}}$$

$$v = \frac{\frac{2}{e^3}\left(1 + e^6\right)}{\frac{\sqrt{1 + e^6}}{e^3}}$$

$$\boxed{d = 2\sqrt{1 + e^6}}$$

17. Let the mean and the variance of 6 observations a, b, 68, 44, 48, 60 be 55 and 194, respectively. If $a > b$, then $a + 3b$ is

(1) 210 (2) 180 (3) 190 (4) 200

Sol. **(2)**

$$\text{mean} = \frac{a + b + 68 + 44 + 48 + 60}{6} = 55$$

$a + b + 220 = 55 \times 6 \Rightarrow 330$

$\boxed{a + b = 110}$

$$\text{variance} = \frac{\Sigma x_i^2}{n} - (\bar{x})^2 = 194$$

$$\frac{a^2 + b^2 + (68)^2 + (44)^2 + (48)^2 + (60)^2}{6}$$

$-(55)^2 = 194$

$a^2 + b^2 + 4624 + 1936 + 2304 + 3600 = 6 \times (3205 + 194)$

$a^2 + b^2 = 6850$

Now $(a+b)^2 = (110)^2$

$a^2 + b^2 + 2ab = 12100$

$6850 + 2ab = 12100$

$2ab = 12100 - 6850$

$2ab = 5250$

$\boxed{ab = 2625}$

$\Rightarrow (110-b)b = 2625 \quad (\because a = 110 - b)$

$b^2 - 110b + 2625 = 0$

$b = 35, 75$

but $\boxed{a > b}$

$a + b = 110$

So $b = 35$ and $a = 75$

Now $\Rightarrow a + 3b$

$\Rightarrow 75 + 3(35) \Rightarrow 180$

18. The temperature T(t) of a body at time $t = 0$ is $160°F$ and it decreases continuously as per the differential equation $\frac{dT}{dt} = -K(T-80)$, where K is a positive constant. If $T(15) = 120°F$, then T(45) is equal to

(1) 85°F (2) 95°F (3) 90°F (4) 80°F

Sol. **(3)**

$\frac{dT}{dt} = -KT + 80K$

$\frac{dT}{dt} + KT = 80K$

L.D.E. in T

$T(I.F) = \int (I.F) Q(t)\, dt$

$I.F = e^{\int K dt} = e^{Kt+C}$

$T\left(e^{Kt+C}\right) = \int e^{Kt}.80Ke^{C} dt$

$Te^{Kt+C} = \frac{e^{Kt}}{K} 80Ke^{C} + C$

$Te^{Kt} = 80e^{Kt} + C', \qquad C' = \frac{C}{e^{C}}$

$t = 0 \Rightarrow T = 160$

$160 = 80 + C'$

$\boxed{C' = 80}$

$Te^{Kt} = 80(e^{Kt} + 1)$

$T=\frac{80(e^{Kt}+1)}{e^{Kt}}$

at t = 15, T = 120º F

$120=80\frac{(e^{15K}+1)}{e^{15K}}$

$3e^{15K}=2e^{15K}+2$

$e^{15K}=2$

$\boxed{K=\frac{\ln 2}{15}}$

So now equation

$$T=\frac{80\left(e^{\frac{\ln 2}{15}t}+1\right)}{e^{\frac{\ln 2}{15}t}}$$

if t = 45

$$T=\frac{80\left(e^{\frac{\ln 2}{15}\times 45}+1\right)}{e^{\frac{\ln 2}{15}\times 45}}$$

$T=\frac{80(9)}{8}=90F$

19. If $\alpha=\sin^{-1}(\sin(5))$ and $b=\cos^{-1}(\cos(5))$, then a^2+b^2 is equal to

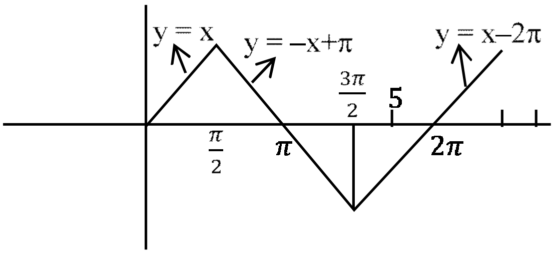
(1) $8\pi^2-40\pi+50$ (2) $4\pi^2-20\pi+50$ (3) $4\pi^2+25$ (4) 25

Sol. **(1)**

$a=\sin^{-1}(\sin(5))$

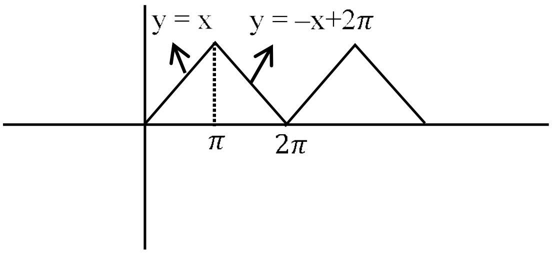
$b=\cos^{-1}(\cos(5))$

$\sin^{-1}(\sin x)=$



$\boxed{a=5-2\pi}$

$\cos^{-1}(\cos x) =$



$\boxed{b = 2\pi - 5}$

Now $a^2 + b^2$

$\Rightarrow (5-2\pi)^2 + (2\pi-5)^2$

$\Rightarrow 25 + 4\pi^2 - 20\pi + 4\pi^2 - 20\pi + 25$

$\Rightarrow 8\pi^2 - 40\pi + 50$

20. Consider the function $f:(0,\infty) \to \mathbb{R}$ defined by $f(x) = e^{-|\log_e x|}$. If m and n be respectively the number of points at which f is not continuous and f is not differentiable, then m + n is

(1) 2 (2) 3 (3) 0 (4) 1

Sol. **(4)**

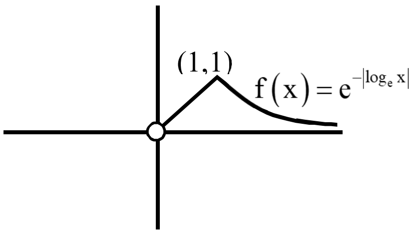
$f(x) = e^{-|\ln x|}$

$f:(0,\infty) \to R$

$f(x) = e^{-|\log_e x|}$

$$f(x) = \frac{1}{e^{|\ln x|}} = \begin{cases} \dfrac{1}{e^{-\ln x}}, & 0 < x < 1 \\ \dfrac{1}{e^{\ln x}}, & x \ge 1 \end{cases}$$

$$= \begin{cases} x, & 0 < x < 1 \\ \dfrac{1}{x}, & x \ge 1 \end{cases}$$



f(x) is continuous in $(0,\infty) \Rightarrow m = 0$

from graph, f(x) is not differentiable at 1 point at x = 1

$\Rightarrow n = 1$

therefore, $m + n = 1$

SECTION – B

21. Let a, b, c be the lengths of three sides of a triangle satisfying the condition $(a^2+b^2)x^2-2b(a+c)x+(b^2+c^2)=0$. If the set of all possible values of x is the interval (α,β), then $12(\alpha^2+\beta^2)$ is equal to ______.

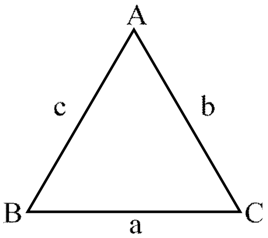
Sol. **36**

$(a^2+b^2)x^2-2b(a+c)x+b^2+c^2=0$

$(ax-b)^2+(bx-c)^2=0$

$\boxed{x=\frac{b}{a}}$ $\boxed{x=\frac{c}{b}}$

Now



$|a-b|<c<(a+b)$

$\boxed{b^2=ac}$

$|a-c|<b<a+c$ $\qquad b^2=ac\times\frac{a}{a}$

$\left|1-\frac{c}{a}\right|<\frac{b}{a}<1+\frac{c}{a}$ $\qquad \frac{b^2}{a^2}=\frac{c}{a}$

$\left|1-\frac{c}{a}\right|<x<1+\frac{c}{a}$ $\qquad x^2=\frac{c}{a}$

$|1-x^2|<x<1+x^2$

Case I: $x<1+x^2$

always +ve

Case II: $|1-x^2|<x$

$-x<1-x^2<x$

Now $1-x^2<x$

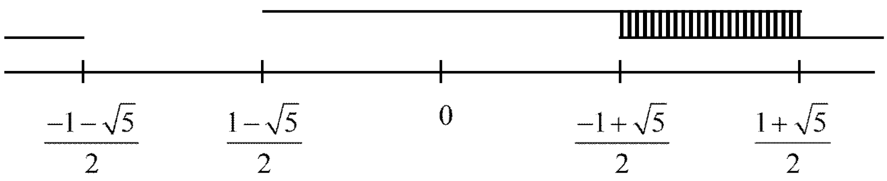
$x^2+x-1>0$

$x=\frac{-1\pm\sqrt{5}}{2}$

Now $-x<1-x^2$

$x^2-x-1<0$

$x=\frac{1\pm\sqrt{5}}{2}$



$\alpha = \frac{-1+\sqrt{5}}{2}$, $\beta = \frac{1+\sqrt{5}}{2}$

Now $12\left(\left(\frac{-1+\sqrt{5}}{2}\right)^2 + \left(\frac{1+\sqrt{5}}{2}\right)^2\right)$

$= 36$

22. Let A = {1, 2, 3.........100}. Let R be a relation on A defined by $(x, y) \in R$ if and only if $2x = 3y$. Let R_1 be a symmetric relation on A such that $R \subset R_1$ and the number of elements in R_1 is n. Then, the minimum value of n is _______.

Sol. **66**

$A = \{1, 2, 3 \ldots 100\}$

$R \Rightarrow 2x = 3y \Rightarrow y = \frac{2x}{3}$

$R = \{(3,2), (6,4), (9,6) \ldots\ldots (99,66)\}$

$n(R) = 33$

$R \subset R_1$

Now

$R_1 = \{(3,2), (6,4), (9,6) \ldots\ldots (99,66),$
$(2,3), (4,6), (6,9) \ldots\ldots (66,99)\}$

$\Rightarrow$ minimum number of elements in $R_1 = 66$

23. Let $\vec{a} = 3\hat{i} + 2\hat{j} + \hat{k}, \vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c}$ be a vector such that $(\vec{a} + \vec{b}) \times \vec{c} = 2(\vec{a} \times \vec{b}) + 24\hat{j} - 6\hat{k}$ and $(\vec{a} - \vec{b} + \hat{i}).\vec{c} = -3$. Then $|\vec{c}|^2$ is equal to _____.

Sol. **38**

$\vec{a} = 3\hat{i} + 2\hat{j} + \hat{k}$

$\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$

Now

$(\vec{a} + \vec{b}) \times \vec{c} = 2(\vec{a} \times \vec{b}) + 24\hat{j} - 6\hat{k}$ ...(1)

$(\vec{a} - \vec{b} + \hat{i}) \cdot \vec{c} = -3$ ...(2)

$\vec{a} + \vec{b} = 5\hat{i} + \hat{j} + 4\hat{k}$

$\vec{a} - \vec{b} = \hat{i} + 3\hat{j} - 2\hat{k}$

let $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\left(5\hat{i} + \hat{j} + 4\hat{k}\right) \times \left(x\hat{i} + y\hat{j} + z\hat{k}\right) = 2(\vec{a} \times \vec{b}) + 24\hat{j} - 6\hat{k}$$

$$(z - 4y)\hat{i} + (4x - 5z)\hat{j} + (5y - x)\hat{k} = 14\hat{i} + 10\hat{j} - 20\hat{k}$$

compare $z - 4y = 14$ ...(1)

$4x - 5z = 10$ ...(2)

$5y - x = -20$ ...(3)

Now by using ...(2)

$$\left(\hat{i} + 3\hat{j} - 2\hat{k} + \hat{i}\right) \cdot \left(x\hat{i} + y\hat{j} + z\hat{k}\right) = 3$$

$2x + 3y - 2z = -3$ ...(4)

Using (1), (2), (3), (4)

$y = -3$, $x = 5$, $z = 2$

$|c|^2 = 9 + 25 + 4 = 38$

24. Let A be a 3×3 matrix and $\det(A) = 2$. If $n = \det\left(\underbrace{\text{adj}\left(\text{adj}\left(.....\left(\text{adj}A\right)\right)\right)}_{2024-\text{times}}\right)$ then the remainder when n is divided by 9 is equal to ______.

Sol. **7**

$|A| = 2$

$$n = \left|\underbrace{\text{adj}\left(\text{adj}\left(.....\left(\text{adj}A\right)\right)\right)}_{2024-\text{times}}\right|$$

$= |A|^{(x-1)^{2024}}$

$\Rightarrow |A|^{2^{2024}} \Rightarrow 2^{2^{2024}}$

Now $2^{2024} \Rightarrow 2^2(2)^{2022}$

$\Rightarrow 2^2(2^3)^{674}$

$\Rightarrow 2^2(8)^{674}$

$\Rightarrow 4(9-1)^{674}$

$\Rightarrow 4(9-1)^{674} \rightarrow$ divided by 9. Thus remainder is 4

So, $2^{2024} \rightarrow 9K + 4$ $(K = \text{even})$

Now $\rightarrow 2^{9K+4} \Rightarrow 16.2^{9K}$

$\Rightarrow 16(2^3)^{3K}$

$\Rightarrow 16(8)^{3K}$

$\Rightarrow 16(9-1)^{3K} \rightarrow$ even

$\Rightarrow 9\alpha + 16\{\alpha \rightarrow \text{integer}\}$

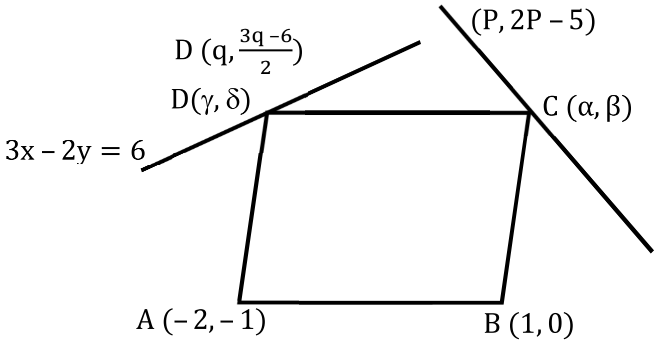
$\Rightarrow 9\alpha + 9 + 7$

$\Rightarrow 9(\alpha + 1) + 7$

$\boxed{\text{remainder} \rightarrow 7}$

25. Let $A(-2,-1), B(1,0), C(\alpha,\beta)$ and D (γ,δ) be the vertices of a parallelogram ABCD. If the point C lies on $2x-y=5$ and the point D lies on $3x-2y=6$, then the value of $|\alpha+\beta+\gamma+\delta|$ is equal to _____.

Sol. **32**



mid point of diagonals

$\frac{-2+p}{2}=\frac{q+1}{2}$

$p-q-3=0$

$\frac{-1+2p-5}{2}=\frac{0+3q-6}{4}$

$2p-6=\frac{3q-6}{2}$

$4p-3q-6=0$

$3(p-q-3=0)$

$p+3=0$

$p=-3$

$q=-6$

$c(p,2p-5)$

$c(-3,-11)=(\alpha,\beta)$

$D(-6,-12)=(\gamma,\delta)$

$|\alpha+\beta+\gamma+\delta|$

$\Rightarrow|-3-11-6-12|\Rightarrow 32$

26. Let the coefficient of x^r in the expansion of $(x+3)^{n-1}+(x+3)^{n-2}(x+2)+(x+3)^{n-3}(x+2)^2+.....+(x+2)^{n-1}$ be α_r. If $\sum_{r=0}^{n}\alpha_r=\beta^n-\gamma^n, \beta,\gamma\in\mathbb{N}$, then the value of $\beta^2+\gamma^2$ equals ______.

Sol. **25**

$(x+3)^{n-1}+(x+3)^{n-2}(x+2)\ldots+(x+2)^{n-1}$

$\Rightarrow (x+3)^{n-1}\left(1+\left(\frac{x+2}{x+3}\right)+\left(\frac{x+2}{x+3}\right)^2+\cdots+\left(\frac{x+2}{x+3}\right)^{n-1}\right)$

$$\frac{1\cdot\left(\left(\frac{x+2}{x+3}\right)^n-1\right)}{\frac{x+2}{x+3}-1}\times(x+3)^{n-1}$$

$$\Rightarrow \frac{\left(\left(\frac{x+2}{x+3}\right)^n-1\right)}{\frac{x+2-x-3}{x+3}}\times(x+3)^{n-1}$$

$$\Rightarrow \left(1-\left(\frac{x+2}{x+3}\right)^n\right)(x+3)^n$$

Put $x=1$

$=4^n-3^n$

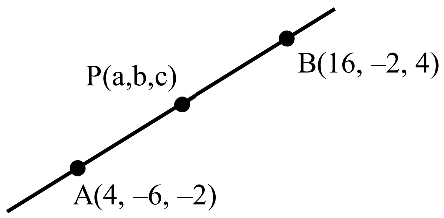
$=\beta^n-\gamma^n$

$\Rightarrow \beta=4,\gamma=3$

$\beta^2+\gamma^2=25$

27. A line passes through $A(4,-6,-2)$ and $B(16,-2,4)$. The point $P(a,b,c)$, where a, b, c are non-negative integers, on the line AB lies at a distance of 21 units, from the point A. The distance between the points $P(a,b,c)$ and $Q(4,-12,3)$ is equal to ______.

Sol. **22**



Eq. of AB

$$\frac{x-4}{12}=\frac{y+6}{4}=\frac{z+2}{6}=\alpha.$$

$a=12\alpha+4$

$b=4\alpha-6$

$c=6\alpha-2$

Distance PA = 21

$$\sqrt{(12\alpha+4-4)^2+(4\alpha-6+6)^2+(6\alpha-2+2)^2}=21$$

$144\alpha^2+16\alpha^2+36\alpha^2=441$

$196\alpha^2=441$

$\alpha^2 = \frac{441}{196}$

$\alpha = \pm\frac{21}{14} \Rightarrow \pm\frac{3}{2}$ (a, b, c are non negative)

$\left(\alpha - \frac{8}{2}\right)(a, b, c) \rightarrow$ the

$a = 12\times\frac{3}{2} + 4 \Rightarrow 22$

$b = 4\times\frac{3}{2} - 6 \Rightarrow 0$

$c = 6\times\frac{3}{2} - 2 \Rightarrow 7$

$P(22, 0, 7), \quad Q(4, -12, 3)$

$PQ = \sqrt{(18)^2 + (12)^2 + (4)^2}$

$PQ = \sqrt{324 + 144 + 16}$

$PQ = 22$

28. Let $y = y(x)$ be the solution of the differential equation

$\sec^2 x dx + \left(e^{2y}\tan^2 x + \tan x\right)dy = 0, 0 < x < \frac{\pi}{2}, y\left(\frac{\pi}{4}\right) = 0$. If $y\left(\frac{\pi}{6}\right) = \alpha$. then $e^{8\alpha}$ is equal to ______.

Sol. **9**

$\sec^2 x dx + \left(e^{2y}\tan^2 x + \tan x\right)dy = 0$

Put $\tan x = t$

$\sec^2 x dx = dt$

$dt + (e^{2y} t^2 + t)dy = 0$

$\frac{1}{t^2}\times\frac{dt}{dy} + \frac{1}{t} = -e^{2y}$

Put $\frac{1}{t} = u$

$-\frac{1}{t^2}\frac{dt}{dy} = \frac{du}{dy}$

$-\frac{du}{dy} + u = -e^{2y}$

$\frac{du}{dy} - u = e^{2y}$

$I.f. = e^{-\int dy} = e^{-y}$

$u \cdot e^{-y} = \int e^{-y} e^{2y} dy$

$$\frac{1}{\tan x\, e^y} = e^y + C$$

$$x = \frac{\pi}{4}, y = 0$$

Now $\quad 1 = 1 + C \quad \boxed{C = 0}$

$$x = \frac{\pi}{6} \qquad y = \alpha$$

$$\frac{\sqrt{3}}{e^{\alpha}} = e^{\alpha}$$

$$= e^{2\alpha} = \sqrt{3}$$

$$\boxed{e^{8\alpha} = 9}$$

29. $\left|\frac{120}{\pi^3}\int_0^{\pi}\frac{x^2 \sin x \cos x}{\sin^4 x + \cos^4 x}dx\right|$ is equal to ________.

Sol. **15**

$$\left|\frac{120}{\pi^3}\int_0^{\pi}\frac{x^2 \sin x \cos x}{\sin^4 x + \cos^4 x}dx\right|$$

$$\Rightarrow \left|\frac{120}{\pi^3}\int_0^{\pi/2}\frac{\sin x \cos x}{\sin^4 x + \cos^4 x}\left(x^2 - (\pi - x)^2\right)dx\right|$$

$$\Rightarrow \left|\frac{120}{\pi^3}\int_0^{\pi/2}\frac{\sin x . \cos x}{\sin^4 x + \cos^4 x}\left(2\pi x - \pi^2\right)dx\right|$$

$$\Rightarrow \left|\frac{120}{\pi^3}\left[2\pi\int_0^{\pi/2}\frac{x \sin x . \cos x}{\sin^4 x + \cos^4 x}dx - \pi^2\int_0^{\pi/2}\frac{\sin x . \cos x}{\sin^4 x + \cos^4 x}dx\right]\right|$$

$$\Rightarrow \left|\frac{120}{\pi^3}\left[2\pi \times \frac{\pi}{4}\int_0^{\pi/2}\frac{\sin x . \cos x}{\sin^4 x + \cos^4 x}dx - \pi^2\int_0^{\pi/2}\frac{\sin x . \cos x}{\sin^4 x + \cos^4 x}dx\right]\right|$$

$$\Rightarrow \left|\frac{120}{\pi^3}\left[\frac{-\pi^2}{2}\int_0^{\pi/2}\frac{\sin x . \cos x}{\sin^4 x + \cos^4 x}dx\right]\right|$$

$$\Rightarrow \left|\frac{120}{\pi^3}\left[\frac{-\pi^2}{2}\int_0^{\pi/2}\frac{\sin x . \cos x}{1 - 2\sin^2 x \cos^2 x}dx\right]\right|$$

$$\Rightarrow \left|\frac{120}{\pi^3}\left[\frac{-\pi^2}{2}\int_0^{\pi/2}\frac{\frac{1}{2}\sin 2x}{1 - \frac{1}{2}\sin^2 2x}dx\right]\right|$$

$$\Rightarrow \left|\frac{120}{\pi^3}\left[\frac{-\pi^2}{2}\int_0^{\pi/2}\frac{\sin 2x}{2 - \sin^2 2x}dx\right]\right|$$

$$\Rightarrow \left|\frac{120}{\pi^3}\left[\frac{-\pi^2}{2}\int_0^{\pi/2}\frac{\sin 2x}{1+\cos^2 2x}dx\right]\right|$$

take $\cos 2x = t,\ -2\sin 2x dx = dt$

$$\Rightarrow \left|\frac{120}{\pi^3}\left[\frac{-\pi^2}{2}\int_1^{-1}\frac{-\frac{1}{2}dt}{1+t^2}\right]\right|$$

$$\Rightarrow \left|\frac{120}{\pi^3}\left[\frac{-\pi^2}{2}\int_{-1}^{1}\frac{\frac{1}{2}dt}{1+t^2}\right]\right|$$

$$\Rightarrow \left|\frac{120}{\pi^3}\times\left[\frac{-\pi^2}{4}\left(\tan^{-1}(t)\right)_{-1}^{1}\right]\right|$$

$$\Rightarrow \left|\frac{120}{\pi^3}\times\left[\frac{-\pi^2}{4}\times\frac{\pi}{2}\right]\right|$$

$\Rightarrow 15$

30. If $\lim\limits_{x\to 0}\frac{ax^2e^x - b\log_e(1+x)+cxe^{-x}}{x^2\sin x}=1$, then $16\left(a^2+b^2+c^2\right)$ is equal to

Sol. **81**

$$\lim_{x\to 0}\frac{ax^2e^x - b\log_e(1+x)+cxe^{-x}}{x^2\sin x}=1$$

$$\lim_{x\to 0}\frac{ax^2\left(1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}...\right)-b\left(x-\frac{x^2}{2}+\frac{x^3}{3}+...\right)+cx\left(1-x+\frac{x^2}{2!}\right)}{x^2\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}+...\right)}$$

$$\lim_{x\to 0}\frac{(-b+c)x+x^2\left(a+\frac{b}{2}-c\right)+\left(a-\frac{b}{3}+\frac{c}{2}\right)x^3+...}{x^3\left(1-\frac{x^2}{3!}+\frac{x^5}{5!}+...\right)}$$

$-b + c = 0$

$\boxed{c=b}$

$a+\frac{b}{2}-c=0$

$a+\frac{b}{2}-b=0$

$\boxed{a=\frac{b}{2}}$

$a-\frac{b}{3}+\frac{c}{2}=1$

$$\frac{b}{2}-\frac{b}{3}+\frac{b}{2}=1 \qquad \Rightarrow \frac{3b-2b+3b}{6}=1$$

$$4b = 6 \qquad \boxed{c=\frac{3}{2}} \qquad a=\frac{b}{2}$$

$$\boxed{b=\frac{3}{2}} \qquad a=\frac{3}{2\times 2}\Rightarrow \frac{3}{4}$$

$$\boxed{a=\frac{3}{4}}$$

$$16\left(a^2+b^2+c^2\right)$$

$$16\left(\frac{9}{16}+\frac{9}{4}+\frac{9}{4}\right)$$

$$16\left(\frac{9+36+36}{16}\right)\Rightarrow 81$$

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