JEE MAIN 2024
Paper with Solution

MATHS | 30th January 2024 _ Shift-1

SECTION – A

1. A line passing through the point $A(9,0)$ makes an angle of 30º with the positive direction of x-axis. If this line is rotated about A through an angle of 15º in the clockwise direction, then its equation in the new position is:

(1) $\frac{y}{\sqrt{3}-2}+x=9$ (2) $\frac{x}{\sqrt{3}-2}+y=9$ (3) $\frac{x}{\sqrt{3}+2}+y=9$ (4) $\frac{y}{\sqrt{3}+2}+x=9$

Sol. **1**

$y - 0 = \tan 15^\circ (x - 9)$

$\Rightarrow y=\left(2-\sqrt{3}\right)\left(x-9\right)$

$\Rightarrow \frac{y}{2-\sqrt{3}}=x-9$

$\Rightarrow x+\frac{y}{\sqrt{3}-2}=9$

2. Let S_n denote the sum of first n terms of an arithmetic progression. If $S_{20}=790$ and $S_{10}=145$, then $S_{15}-S_5$ is:

(1) 395 (2) 390 (3) 405 (4) 410

Sol. **1**

$S_{20} = 790$

$\Rightarrow 10\left[2a+19d\right]=790$

$\Rightarrow 2a + 19d = 79 \ldots (1)$

$S_{10} = 145$

$\Rightarrow 5\left[2a+9d\right]=145$

$\Rightarrow 2a + 9d = 29 \ldots (2)$

By (1) and (2)

$10d = 50$

$d = 5$

$\Rightarrow a = -8$

$S_{15}=\frac{15}{2}\left[-16+14\times 5\right]$

$S_{15}=\frac{15}{2}\left[-16+54\right]$

$=\frac{15}{2}\times 54=405$

$S_5 = 10$

$S_{15} - S_5 = 405 - 10 = 395$

3. If $z = x + iy, xy \neq 0$, satisfies the equation $z^2 + i\bar{z} = 0$, then $\left|z^2\right|$ is equal to:

(1) 9 (2) 1 (3) 4 (4) $\frac{1}{4}$

Sol. **2**

$z = x + iy$

$z^2 = -i\bar{z}$

$\Rightarrow \left|z^2\right| = \left|-i\bar{z}\right|$

$\Rightarrow |z|^2 = |-i||\bar{z}|$

$\Rightarrow |z|^2 = |z| \Rightarrow |z| = 0$

$|z| = 1$

$\Rightarrow |z|^2 = 1$

$|Z^2| = 1$

4. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ be two vectors such that $|\vec{a}| = 1$, $\vec{a} \cdot \vec{b} = 2$ and $|\vec{b}| = 4$. If $\vec{c} = 2(\vec{a} \times \vec{b}) - 3\vec{b}$, then the angle between $\vec{b}$ and $\vec{c}$ is equal to:

(1) $\cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (2) $\cos^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ (3) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ (4) $\cos^{-1}\left(\frac{2}{3}\right)$

Sol. **3**

$\vec{c} = 2\left(\vec{a} \times \vec{b}\right) - 3\vec{b}$

$\vec{b} \cdot \vec{c} = 2\left(\vec{a} \times \vec{b}\right) \cdot \vec{b} - 3\vec{b} \cdot \vec{b}$

$\Rightarrow \left|\vec{b}\right|\left|\vec{c}\right|\cos\theta = 0 - 3\left|\vec{b}\right|^2$

$\left|\vec{c}\right|\cos\theta = -12$

$\left|\vec{c}\right|^2 = 4\left|\vec{a} \times \vec{b}\right|^2 + 9\left|\vec{b}\right|^2$

$= 4\left(\left|\vec{a}\right|^2\left|\vec{b}\right|^2 - \left(\vec{a} \cdot \vec{b}\right)^2\right) + 144$

$= 4(16 - 4) + 144$

$\left|\vec{c}\right|^2 = 48 + 144 = 192$

$\left|\vec{c}\right| = 8\sqrt{3}$

$\cos\theta = \frac{-12}{8\sqrt{3}} = \frac{-3}{2\sqrt{3}}$

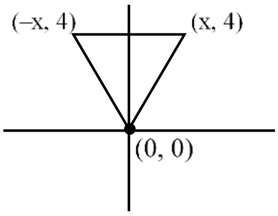
$\cos\theta = \frac{-\sqrt{3}}{2}$

$\theta = \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$

5. The maximum area of a triangle whose one vertex is at (0, 0) and the other two vertices lie on the curve $y = -2x^2 + 54$ at points (x, y) and (–x, y), where y > 0, is:

(1) 88 (2) 122 (3) 92 (4) 108

Sol. **4**



$A = xy$

$A = x(-2x^2 + 54)$

$A = -2x^3 + 54x$

$\frac{dA}{dx} = -6x^2 + 54 = 0$

$x = 3 \quad x = -3$

$A_{max} = 3(-2 \times 9 + 54)$

$3(54 - 18) = 108$

6. The value of $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{n^3}{(n^2+k^2)(n^2+3k^2)}$ is:

(1) $\frac{(2\sqrt{3}+3)\pi}{24}$ (2) $\frac{13\pi}{8(4\sqrt{3}+3)}$ (3) $\frac{13(2\sqrt{3}-3)\pi}{8}$ (4) $\frac{\pi}{8(2\sqrt{3}+3)}$

Sol. **2**

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{n^3}{(n^2+k^2)(n^2+3k^2)}$$

$$\Rightarrow \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \frac{1}{\left(1+\frac{k^2}{n^2}\right)\left(1+\frac{3k^2}{n^2}\right)}$$

$$\Rightarrow \int_0^1 \frac{dx}{(1+x^2)(1+3x^2)} = \frac{1}{2}\int_0^1 \frac{3(1+x^2)-(1+3x^2)}{(1+x^2)(1+3x^2)}dx$$

$$\Rightarrow \frac{1}{2}\left[\int_0^1 \frac{3dx}{1+3x^2} - \int_0^1 \frac{dx}{1+x^2}\right] = \frac{13\pi}{8(4\sqrt{3}+3)}$$

7. Let $g:\mathbf{R}\to\mathbf{R}$ be a non constant twice differentiable function such that $g'\left(\frac{1}{2}\right)=g'\left(\frac{3}{2}\right)$. If a real valued function f is defined as $f(x)=\frac{1}{2}[g(x)+g(2-x)]$, then

(1) $f''(x)=0$ for atleast two x in (0, 2)

(2) $f''(x)=0$ for exactly one x in (0, 1)

(3) $f''(x)=0$ for no x in (0, 1)

(4) $f'\left(\frac{3}{2}\right)+f'\left(\frac{1}{2}\right)=1$

Sol. **1**

$$g'\left(\frac{1}{2}\right)=g'\left(\frac{3}{2}\right)$$

$$f(x)=\frac{1}{2}\,(g(x)+g(2-x))$$

$$f'(x)=\frac{1}{2}\,(g'(x)-g'(2-x))$$

Now $f'\left(\frac{1}{2}\right)=\frac{1}{2}\left(g'\left(\frac{1}{2}\right)-g'\left(\frac{3}{2}\right)\right)=0$

$$f'(1)=\frac{1}{2}\left(g'(1)-g'(1)\right)=0$$

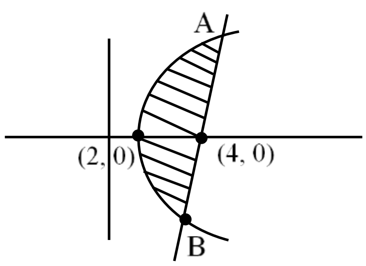
$$f'\left(\frac{3}{2}\right)=\frac{1}{2}\left(g'\left(\frac{3}{2}\right)-g'\left(\frac{1}{2}\right)\right)=0$$

$\Rightarrow f''(x)=0$ for at least two x in (0, 2)

8. The area (in square units) of the region bounded by the parabola $y^2=4(x-2)$ and the line $y = 2x - 8$, is:

(1) 8 (2) 9 (3) 6 (4) 7

Sol. **2**



$y^2 = 4x - 8$
$y = 2x - 8$
$\Rightarrow y^2 = 2(y + 8) - 8$
$\Rightarrow y^2 - 2y - 8 = 0$
$y = 4 \quad y = -2$

$$\int_{-2}^{4}\left(\frac{y+8}{2}-\frac{y^2+8}{4}\right)dy$$

$$\Rightarrow \int_{-2}^{4}\frac{y}{2}dy - \int_{-2}^{4}\frac{y^2}{4}dy + \int_{-2}^{4}2dy$$

$$\Rightarrow \left[\frac{y^2}{4}\right]_{-2}^{4} - \frac{1}{4}\left[\frac{y^3}{3}\right]_{-2}^{4} + 2[y]_{-2}^{4} = 9$$

9. Let y = y(x) be the solution of the differential equation $\sec x\, dy + \{2(1-x)\tan x + x(2-x)\}dx = 0$ such that y(0) = 2. Then y(2) is equal to:

(1) 2 (2) 2[1 – sin(2)] (3) 2{sin(2) + 1} (4) 1

Sol. **1**

$$dy = \frac{2(x-1)\tan x + x(x-2)}{\sec x}dx$$

$$\Rightarrow \int dy = \int 2(x-1)\sin x + (x^2-2x)\cos x$$

$$\Rightarrow y = 2[(x-1)(-\cos x)-(1)(-\sin x)] + [(x^2-2x)(\sin x)-(2x-2)(-\cos x)+(2)(-\sin x)] + c$$

y(0) = 2 [1] + [–2] + C
y(0) = 2
C = 2
y(2) = 2(1 + sin2) – 2sin2 = 2

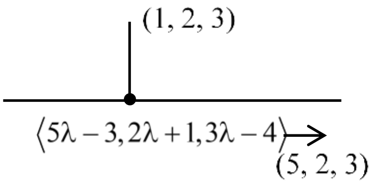
10. Let (α,β,γ) be the foot of perpendicular from the point (1, 2, 3) on the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$. Then $19(\alpha+\beta+\gamma)$ is equal to:

(1) 102 (2) 101 (3) 99 (4) 100

Sol. **2**

$$\vec{r} = \langle -3,1,-4\rangle + \lambda\langle 5,2,3\rangle$$

Foot of $\perp^r$ $\langle 5\lambda-3, 2\lambda+1, 3\lambda-4\rangle$



$$\langle 5\lambda-4, 2\lambda-1, 3\lambda-7\rangle\cdot\langle 5,2,3\rangle = 0$$

$\Rightarrow 25\lambda - 20 + 4\lambda - 2 + 9\lambda - 21 = 0$

$38\lambda = 43$

$\lambda = \frac{43}{38}$

$\left\langle \frac{101}{38}, \frac{124}{38}, \frac{-23}{38} \right\rangle \Rightarrow 101$

11. Two integers x and y are chosen with replacement from the set $\{0,1,2,3,\ldots,10\}$. Then the probability that $|x-y|>5$, is:

(1) $\frac{30}{121}$ (2) $\frac{62}{121}$ (3) $\frac{60}{121}$ (4) $\frac{31}{121}$

Sol. **1**

$|x-y|>5$

$\Rightarrow x = 0$ y = 6, 7, 8, 9, 10

$\Rightarrow x = 1$ y = 7 8 9, 10

$\Rightarrow x = 2$ y = 8, 9, 10

$\Rightarrow x = 3$ y = 9, 10

$\Rightarrow x = 4$ y = 10

$P(A) = \frac{30}{121}$

12. If the domain of the function $f(x)=\cos^{-1}\left(\frac{2-|x|}{4}\right)+\{\log_e(3-x)\}^{-1}$ is $[-\alpha,\beta)-\{\gamma\}$, then $\alpha+\beta+\gamma$ is equal to:

(1) 12 (2) 9 (3) 11 (4) 8

Sol. **3**

$f(x) = \cos^{-1}\left(\frac{2-|x|}{4}\right)+\frac{1}{\ln(3-x)}$

$f(x) = -1 \le \frac{2-|x|}{4} \le 1 \quad x < 3 - \{2\}$

$\Rightarrow -6 \le x \le 6 \quad x \in [-6,3)-\{2\}$

$\alpha + \beta + \gamma = 11$

13. Consider the system of linear equations $x+y+z=4\mu,\ x+2y+2\lambda z=10\mu,\ x+3y+4\lambda^2 z=\mu^2+15$, where $\lambda,\mu \in \mathbf{R}$. Which one of the following statements is NOT correct?

(1) The system has unique solution if $\lambda \ne \frac{1}{2}$ and $\mu \ne 1,15$

(2) The system is inconsistent if $\lambda = \frac{1}{2}$ and $\mu \ne 1$

(3) The system has infinite number of solutions if $\lambda = \frac{1}{2}$ and $\mu = 15$

(4) The system is consistent if $\lambda \ne \frac{1}{2}$

Sol. **2**

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2\lambda \\ 1 & 3 & 4\lambda^2 \end{vmatrix} = (2\lambda - 1)^2$$

For unique solution $\Delta \neq 0 \Rightarrow \lambda \neq \frac{1}{2}$

If $\lambda = \frac{1}{2}$, $\begin{vmatrix} 4\mu & 1 & 1 \\ 10\mu & 2 & 1 \\ \mu^2 + 15 & 3 & 1 \end{vmatrix} = 0$

$(\mu - 15)(\mu - 1) = 0$ Infinite solution.

14. If the circles $(x+1)^2 + (y+2)^2 = r^2$ and $x^2 + y^2 - 4x - 4y + 4 = 0$ intersect at exactly two distinct points, then

(1) $5 < r < 9$ (2) $0 < r < 7$ (3) $3 < r < 7$ (4) $\frac{1}{2} < r < 7$

Sol. **3**

for intereaction.

$|r_1 - r_2| < C_1C_2 < r_1 + r_2$

$|r - 2| < 5 < r + 2$

$r > 3$ …(1)

$-5 < r - 2 < 5$

$-3 < r < 7$ …(2)

By (1) and (2)

$3 < r < 7$

15. If the length of the minor axis of an ellipse is equal to half of the distance between the foci, then the eccentricity of the ellipse is:

(1) $\frac{\sqrt{5}}{3}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{1}{\sqrt{3}}$ (4) $\frac{2}{\sqrt{5}}$

Sol. **4**

$2b = ae$

$\frac{b}{a} = \frac{e}{2}$

$e^2 = 1 - \frac{b^2}{a^2}$

$\Rightarrow \frac{5e^2}{4} = 1 \Rightarrow e = \frac{2}{\sqrt{5}}$

16. Let M denote the median of the following frequency distribution

Class	0 – 4	4 – 8	8 – 12	12 – 16	16 – 20
Frequency	3	9	10	8	6

Then 20M is equal to:

(1) 416 (2) 104 (3) 52 (4) 208

Sol. **4**

$$M = \ell + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

$$M = 8 + \left(\frac{18-12}{10}\right) \times 4$$

$$M = 8 + \frac{24}{10} = 10.4$$

20M = 208

17. If $f(x) = \begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3+\sin^2 2x \\ 3+2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3+2\sin^4 x & \sin^2 2x \end{vmatrix}$, then $\frac{1}{5}f'(0) =$ is equal to:

(1) 0 (2) 1 (3) 2 (4) 6

Sol. **1**

$$f(x) = \begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3+\sin^2 2x \\ 3+2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3+2\sin^4 x & \sin^2 2x \end{vmatrix}$$

f '(0) = 0 + 0 + 0 = 0

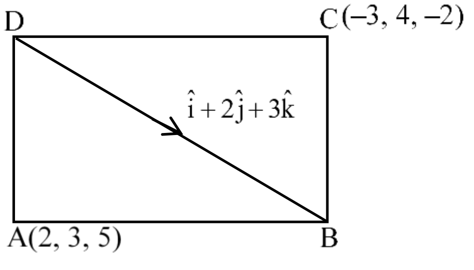
f '(0) = 0

$$\frac{1}{5} \cdot f'(0) = 0$$

18. Let A(2, 3, 5) and C(–3, 4, –2) be opposite vertices of a parallelogram ABCD. If the diagonal $\overrightarrow{BD} = \hat{i} + 2\hat{j} + 3\hat{k}$, then the area of the parallelogram is equal to:

(1) $\frac{1}{2}\sqrt{410}$ (2) $\frac{1}{2}\sqrt{474}$ (3) $\frac{1}{2}\sqrt{586}$ (4) $\frac{1}{2}\sqrt{306}$

Sol. **2**



$$\overrightarrow{CA} = 5\hat{i} - \hat{j} + 7\hat{k} = \vec{d}_1$$

$$\overrightarrow{BD} = \hat{i} + 2\hat{j} + 3\hat{k} = \vec{d}_2$$

$$\frac{1}{2}\left|\vec{d}_1 \times \vec{d}_2\right| = \frac{1}{2}\sqrt{474}$$

19. If $2\sin^3 x + \sin 2x \cos x + 4\sin x - 4 = 0$ has exactly 3 solutions in the interval $\left[0, \frac{n\pi}{2}\right], n \in N$, then the roots of the equation $x^2 + nx + (n-3) = 0$ belong to:

(1) $(0, \infty)$ (2) $(-\infty, 0)$ (3) $\left(-\frac{\sqrt{17}}{2}, \frac{\sqrt{17}}{2}\right)$ (4) Z

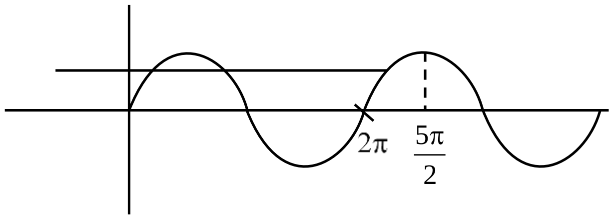
Sol. 2

$2\sin^3 x + 2\sin x \cos^2 x + 4\sin x - 4 = 0$

$\Rightarrow 2\sin^3 x + 2\sin x(1-\sin^2 x) + 4\sin x - 4 = 0$

$\Rightarrow 2\sin^3 x + 6\sin x - 2\sin^3 x - 4 = 0$

$\sin x = \frac{2}{3}$



$\left[0, \frac{5\pi}{2}\right]$

$x^2 + 5x + 2 = 0$

$\alpha, \beta = \frac{-5 \pm \sqrt{17}}{2}$ both negative hence $(-\infty, 0)$

20. Let $f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to \mathbf{R}$ be a differentiable function such that $f(0) = \frac{1}{2}$. If the $\lim_{x \to 0} \frac{x\int_0^x f(t)dt}{e^{x^2} - 1} = \alpha$, then $8\alpha^2$ is equal to:

(1) 16 (2) 2 (3) 1 (4) 4

Sol. 2

$$\lim_{x \to 0} \frac{x\int_0^x f(t)dt}{e^{x^2} - 1}$$

$$\Rightarrow \lim_{x \to 0} \frac{\int_0^x f(t)dt + xf(x)}{2xe^{x^2}}$$

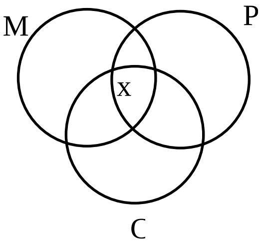
$$\Rightarrow \lim_{x \to 0} \frac{f(x) + f(x) + xf'(x)}{2e^{x^2} + 4x^2e^{x^2}} = \frac{2f(0)}{2} = \frac{1}{2} = \alpha$$

$8 \times \frac{1}{4} = 2$

SECTION – B

21. A group of 40 students appeared in an examination of 3 subjects - Mathematics, Physics and Chemistry. It was found that all students passed in atleast one of the subjects, 20 students passed in Mathematics, 25 students passed in Physics, 16 students passed in Chemistry, atmost 11 students passed in both Mathematics and Physics, atmost 15 students passed in both Physics and Chemistry, atmost 15 students passed in both Mathematics and Chemistry. The maximum number of students passed in all the three subjects is_____

Sol. **10**

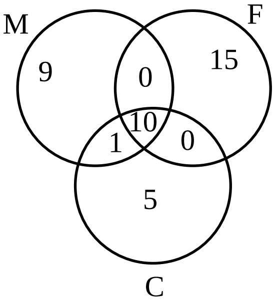


$11 - x \geq 0$

$x \leq 11$

x = 11 does not satisfy the data.

x = 10



Hence maximum number of students passed in all the three subjects is 10.

22. If d_1 is the shortest distance between the lines $x+1=2y=-12z$, $x=y+2=6z-6$ and d_2 is the shortest distance between the lines $\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}$, $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$, then the value of $\frac{32\sqrt{3}d_1}{d_2}$ is :

Sol. **16**

$L_1 : \frac{x+1}{1}=\frac{y}{\frac{1}{2}}=\frac{z}{-1/12}$ $\quad L_2 : \frac{x}{1}=\frac{y+2}{1}=\frac{z-1}{1/6}$

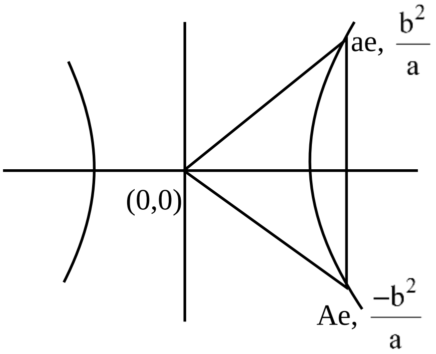
$$d_1 = \left|\frac{(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)}{|(\vec{b}_1\times\vec{b}_2)|}\right| = 2$$

Similarly $d_2 = \frac{12}{\sqrt{3}}$

$$\frac{d_1 32\sqrt{3}}{d_2} = \frac{2\times 32\times\sqrt{3}\times\sqrt{3}}{12} = 16$$

23. Let the latus ractum of the hyperbola $\frac{x^2}{9}-\frac{y^2}{b^2}=1$ subtend an angle of $\frac{\pi}{3}$ at the centre of the hyperbola. If b^2 is equal to $\frac{l}{m}(1+\sqrt{n})A$, where l and m are co-prime numbers, then $l^2+m^2+n^2$ is equal to ______.

Sol. **182**



$\tan 30^\circ = \frac{\frac{b^2}{a}}{ae} = \frac{b^2}{a^2 e} = \frac{1}{\sqrt{3}}$

$e = \frac{\sqrt{3}b^2}{a^2}$

$e^2 = 1 + \frac{b^2}{9} \Rightarrow 1 + \frac{b^2}{9} = \frac{3b^4}{81}$

$b^4 = 3b^2 + 27$

$b^4 - 3b^2 - 27 = 0$

$b^2 = \frac{3}{2}(1+\sqrt{13})$

$\ell = 3$

$m = 2$

$n = 13$

$\ell^2 + m^2 + n^2 = 182$

24. Let $A=\{1,2,3,\ldots,7\}$ and let P(A) denote the power set of A. If the number of functions $f: A \to P(A)$ such that $a \in f(a), \forall a \in A$ is m^n, m and $n \in N$ and m is least, then $m+n$ is equal to ________.

Sol. **44**

$f: A \to P(A)$ $\quad\quad a \in f(x)$

a will connect with subset which contain a hence total $(2^6)^7 = 2^{42}$

$2 + 42 = 44$

25. The value of $9\int_0^9 \left[\sqrt{\frac{10x}{x+1}}\right]dx$, where [t] denotes the greatest integer less than or equal to t, is _____.

Sol. **155**

$$9\times\int_0^9\left[\sqrt{\frac{10x}{x+1}}\right]dx$$

$$\Rightarrow 9\times\left(\int_0^{1/9}0\cdot dx+\int_{1/9}^{2/3}1\cdot dx+\int_{2/3}^{9}2\cdot dx\right)$$

$$\Rightarrow 9\times\left(\left[\frac{2}{3}-\frac{1}{9}\right]+2\left[9-\frac{2}{3}\right]\right)$$

$$\Rightarrow 9\times\left(\frac{6-1}{9}+2\cdot\frac{25}{3}\right)=9\left(\frac{5}{9}+\frac{50}{3}\right)=9\left(\frac{5+150}{9}\right)=155$$

26. Number of integral terms in the expansion of $\left\{7^{\left(\frac{1}{2}\right)}+11^{\left(\frac{1}{6}\right)}\right\}^{824}$ is equal to ________.

Sol. **138**

$$T_{r+1}={}^{824}C_r\,(7)^{\frac{824-r}{2}}\,(11)^{\frac{r}{6}}$$

$$\Rightarrow {}^{824}C_r\,(7)^{412-\frac{r}{2}}(11)^{\frac{r}{6}}$$

r = 0, 6, 12, …… 822

total 138

27. Let y = y(x) be the solution of the differential equation $(1-x^2)dy=\left[xy+(x^3+2)\sqrt{3(1-x^2)}\right]dx$, $-1<x<1, y(0)=0$. If $y\left(\frac{1}{2}\right)=\frac{m}{n}$, m and n are co-prime numbers, then $m+n$ is equal to

Sol. **97**

$$\frac{dy}{dx}=\frac{xy}{1-x^2}+\frac{(x^3+2)\sqrt{3(1-x^2)}}{1-x^2}$$

$$\frac{dy}{dx}-\frac{x}{1-x^2}y=\frac{(x^3+2)\sqrt{3}}{\sqrt{1-x^2}}$$

$$e^{\int\frac{-x}{1-x^2}dx}\qquad 1-x^2=t$$

$$-2x\,dx=dt$$

$$e^{\frac{1}{2}\int\frac{dt}{t}}=e^{\frac{1}{2}\ln\left(1-x^2\right)}=\sqrt{1-x^2}$$

$$y\cdot\sqrt{1-x^2}=\sqrt{3}\int x^3+2dx$$

$$y\cdot\sqrt{1-x^2}=\sqrt{3}\left(\frac{x^4}{4}+2x\right)+C$$

C = 0

$$y\sqrt{1-x^2}=\sqrt{3}\left(\frac{x^4}{4}+2x\right)$$

$$y\frac{\sqrt{3}}{2}=\sqrt{3}\left(\frac{1}{64}+1\right)$$

$$y=\frac{65}{32}$$

m + n = 65 + 32 = 97

28. Let $\alpha,\beta\in\mathbf{N}$ be roots of the equation $x^2-70x+\lambda=0$, where $\frac{\lambda}{2},\frac{\lambda}{3}\notin\mathbf{N}$. If λ assumes the minimum possible value, then $\frac{(\sqrt{\alpha-1}+\sqrt{\beta-1})(\lambda+35)}{|\alpha-\beta|}$ is equal to:

Sol. **60**

$\alpha + \beta = 70$ $\quad$ $\alpha\beta = \lambda$

$\frac{\lambda}{2},\frac{\lambda}{3}\notin N$

$\alpha = 5$ $\quad$ $\beta = 65$

$$\frac{(\sqrt{\alpha-1}+\sqrt{\beta-1})(\lambda+35)}{|\alpha-\beta|}$$

$$\Rightarrow \frac{(2+8)(360)}{60} = 60$$

29. If the function

$$f(x)=\begin{cases}\frac{1}{|x|}, & |x|\geqslant 2\\ ax^2+2b, & |x|<2\end{cases}$$

Is differentiable on R, then 48(a + b) is equal to _______.

Sol. **15**

$$f(x)=\begin{cases}-\frac{1}{x} & -\infty<x\le -2\\ ax^2+2b & -2<x<2\\ \frac{1}{x} & 2\le x<\infty\end{cases}$$

$$\Rightarrow \quad \frac{1}{2} = 4a + 2b \qquad \ldots(1)$$

$$\Rightarrow \quad 4a = -\frac{1}{4}$$

$$a = \frac{-1}{16} \Rightarrow \frac{1}{2}=-\frac{1}{4}+2b$$

$$\Rightarrow 2b=\frac{1}{2}+\frac{1}{4}$$

$$\Rightarrow b = \frac{3}{8}$$

$$48\,(a + b) = 28\left(\frac{-1}{16}+\frac{3}{8}\right) = 15$$

30. Let $\alpha = 1^2 + 4^2 + 8^2 + 13^2 + 19^2 + 26^2 + \ldots$ upto 10 terms and $\beta = \sum_{n=1}^{10} n^4$. If $4\alpha - \beta = 55k + 40$, then k is equal to _______.

Sol. **353**

$$\alpha = 1^2 + 4^2 + 8^2 + \cdots 10 \text{ terms}$$

$$\alpha = \frac{1}{4}\sum_{r=1}^{10}\left(r^2 + 3r - 2\right)^2$$

$$\alpha = \frac{1}{4}\left[\sum_{r=1}^{10} r^4 + 9r^2 + 4 - 4r^2 - 12r + 6r^3\right]$$

$$4\alpha = \sum_{r=1}^{10} r^4 + 6\sum_{r=1}^{10} r^3 + 5\sum_{r=1}^{10} r^2 - 12\sum_{r=1}^{10} r + \sum_{r=1}^{10} 4$$

$$4\alpha - \beta = 6\left(\frac{r(r+1)}{2}\right)^2 + 5r\frac{(r+1)(2r+1)}{6} - 12\frac{r(r+1)}{2} + 40$$

$$= 6\left(\frac{10 \times 11}{2}\right)^2 + 5 \times \frac{10 \times 11 \times 21}{6} - 55 \times 12 + 40$$

$\Rightarrow$ $6 \times 55 \times 55 + 55 \times 35 - 55 \times 12 + 40$

$\Rightarrow$ $55\,(330 + 35 - 12) + 40$

$55\,(353) + 40$

$k = 353$

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(2022)
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