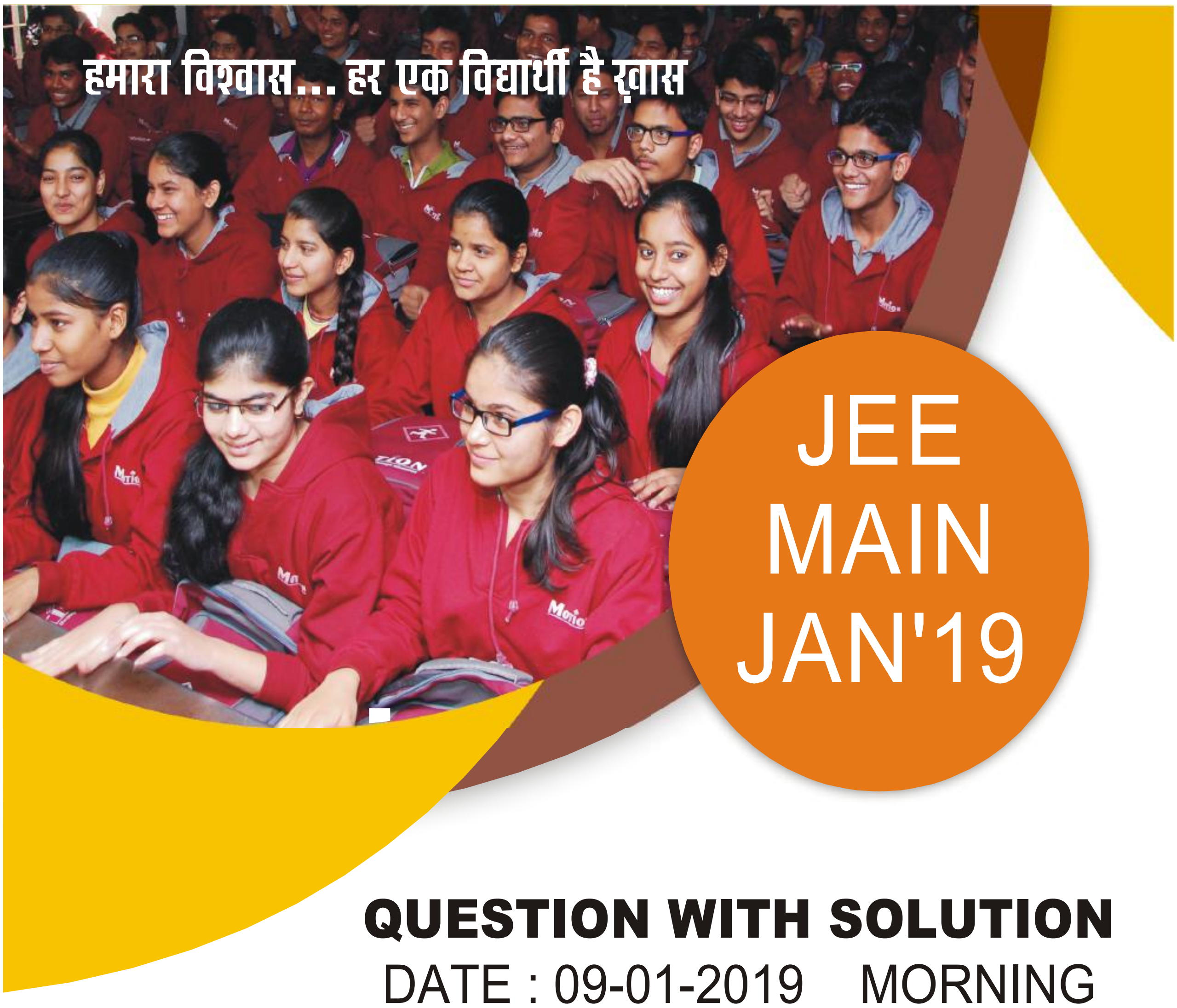


JEE MAIN JAN'19

QUESTION WITH SOLUTION

DATE : 09-01-2019 _ MORNING

[MATHEMATICS] 09-01-2019_Morning

1. let A = $\left\{\theta \in \left(-\frac{\pi}{2}, \pi\right) : \frac{3+2i\sin\theta}{1-2i\sin\theta} \text{is purely imaginary}\right\}$. Then the sum of the elements in A is :;

(A) $\frac{3\pi}{4}$ (B) π (C) $\frac{2\pi}{3}$ (D) $\frac{5\pi}{6}$

Sol. **C**

$\frac{3+2i\sin\theta}{1-2i\sin\theta} \in$ Pure Imaginary

$\Rightarrow \text{Re}\left(\frac{3+2i\sin\theta}{1-2i\sin\theta}\right) = 0$

$3 - 4\sin^2\theta = 0$

$\sin\theta = \pm\frac{\sqrt{3}}{2}$

$\theta \in \left\{-\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}\right\}$

2. for $x \in R - \{0,1\}$. let $f_1(x) = \frac{1}{x}$, $f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1-x}$ be three given functions. if a function, J(x) satisfies $(f_2\ J.f_1)(x) = f_3(x)$ then J(x) is equal to :

(A) $f_1(x)$ (B) $f_3(x)$ (C) $f_2(x)$ (D) $\frac{1}{x}f_3(x)$

Sol. **B**

$f_2(J(f_1(x)) = f_3(x)$

$1 - J\left(\frac{1}{x}\right) = \frac{1}{1-x}$

$J\left(\frac{1}{x}\right) = 1 - \frac{1}{1-x}$

$J\left(\frac{1}{x}\right) = \frac{1-x-1}{1-x}$

$J(x) = \frac{1/x}{1/x - 1}$

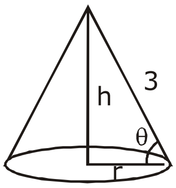
$J(x) = \frac{1}{1-x}$

$\Rightarrow J(x) = f_3(x)$

3. The maximum volume (in cu. m) of the right circular cone having slant height 3m is :

(A) $\frac{4}{3}\pi$ (B) 6π (C) $2\sqrt{3}\pi$ (D) $3\sqrt{3}\pi$

Sol. C



$h^2 + r^2 = g$

$v = \frac{1}{3}\pi r^2 h$

$\Rightarrow \quad v = \frac{1}{3}\pi\cos^2\theta\sin\theta$

$\frac{v}{9\pi} = \cos^2\theta\sin\theta$

$\Rightarrow \quad \frac{v^2}{81\pi^2} = \cos^4\theta \sin^2\theta$

$\frac{\frac{c^2}{2} + \frac{c^2}{2} + \sin^2\theta}{3} \geq \sqrt[3]{\frac{v^2}{81\pi^2 4}}$

$\frac{v^2}{4.81\pi^2} \leq \sqrt[3]{\frac{v^2}{81\pi^2 4}}$

$\frac{v^3}{4.81\pi^2} \leq \frac{1}{27}$

$v^2 \leq \frac{4.81\pi^2}{27}$

$v^2 \leq 2\sqrt{3}\,\pi$

4. If y = y(x) is the solutions of the differential equation, $x\frac{dy}{dx} + 2y = x^2$ satisfying y (1) = 1, then $y\left(\frac{1}{2}\right)$ is equal to :

(A) $\frac{13}{16}$ (B) $\frac{1}{4}$ (C) $\frac{7}{64}$ (D) $\frac{49}{16}$

Sol. **D**

$x\frac{dy}{dx} + 2y = x^2 \qquad \frac{dy}{dx} + \left(\frac{2}{x}\right)y = x \mid_{LDE}$

$$IF = e^{\int \frac{2}{x}dx} = x^2$$

$$yx^2 = \int x^3 dx$$

$$yx^2 = \frac{x^4}{4} + C$$

$$C = \frac{3}{4}$$

$$yx^2 = \frac{x^4}{4} + \frac{3}{4}$$

$$y(1/2) \Rightarrow y\ \frac{1}{4} = \frac{1}{64} + \frac{3}{4}$$

$$y = \frac{1}{16} + 3$$

$$y = \frac{49}{16}$$

5. If $\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2}\left(x > \frac{3}{4}\right)$ then x is equal to :

(A) $\frac{\sqrt{145}}{12}$ (B) $\frac{\sqrt{145}}{10}$ (C) $\frac{\sqrt{146}}{12}$ (D) $\frac{\sqrt{145}}{11}$

Sol. **A**

$$\cos^{-1}\left(\frac{2}{3x}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{3}{4x}\right)$$

$$\cos\left[\cos^{-1}\left(\frac{2}{3x}\right)\right] = \cos\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{3}{3x}\right)\right]$$

$$\frac{2}{3x} = \sin\left(\cos^{-1}\frac{3}{4\pi}\right)$$

$$\frac{2}{3x} = \cos\left(\sin^{-1}\sqrt{1-\left(\frac{3}{4\pi}\right)^2}\right)$$

$$\left(\frac{2}{3x}\right)^2 = 1 - \left(\frac{3}{4x}\right)^2$$

$\frac{4}{9x^2} + \frac{9}{16x^2} = 1 \qquad \Rightarrow x^2 = \frac{4}{9} + \frac{9}{16}$

$\Rightarrow x^2 = \frac{64+81}{16.9}$

$\Rightarrow x^2 = \sqrt{\frac{145}{12}}$

6. Three circles of radii a,b,c (a < b < c) touch each other externally. If they have x - axis as a common tangent, then :

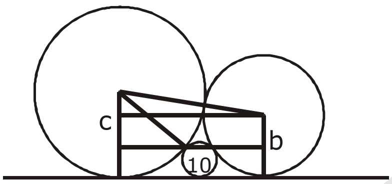
(A) a,b,c are in A.P

(B) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are In A.P.

(C) $\frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{c}}$

(D) $\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$

Sol. **D**



$\sqrt{(b+c)^2 - (c-b)^2} = \sqrt{(c+a)^2 - (c-a)^2} + \sqrt{(b+a)^2 - (b-a)^2}$

$\sqrt{2bc} = \sqrt{2ac} + \sqrt{2ab}$

$\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$

7. Two cards are drawn successively with replacement from a well - shuffled deck of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards. Then P(X = 1) + (P(X = 2) equals :

(A) 52/169 (B) 24/169 (C) 25/169 (D) 49/169

Sol. **C**

P (x = 1) + P (x = 2)

$= 2\frac{4}{52} \times \frac{48}{52} + \frac{4}{52}.\frac{4}{52}$

$= \frac{400}{52.52} = \frac{25}{169}$

8. Consider the set of all lines px + qy + r = 0 such that 3 p + 2q + 4r = 0 . Which one of the following statements is true ?

(A) The lines are concurrent at the point $\left(\frac{3}{4}, \frac{1}{2}\right)$

(B) The lines are all parallel.

(C) Each line passses through the origin

(D) the lines are not concurrent.

Sol. **A**

px + qy + r = 0

$\frac{3}{4}P + \frac{2}{4}q + r = 0$

$\Rightarrow$ Line Pass $\left(\frac{3}{4}, \frac{1}{2}\right)$

9. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to :

(A) 6 (B) 4 (C) 8 (D) 14

Sol. **C**

$\left\{\frac{2^{403}}{15}\right\}$

$= \left\{\frac{8(2^4)^{100}}{15}\right\}$

$\left\{\frac{8(15+1)^{100}}{15}\right\}$

$\Rightarrow$ k = 8

10. consider a class of 5 girls and 7 boys. the number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team is :

(A) 200 (B) 300 (C) 500 (D) 350

Sol. **B**

5 G & 7 B

↓

2 G + 3 B

${}^5C_2.{}^7C_3 - {}^5C_2.{}^2C_2.{}^5C_1$

$= {}^5C_2\left\{{}^7C_3 - {}^5C_1\right\}$

= 10 {35 – 5}

= 300

11. The equation fo the line passing through (–4,3,1), parallel to the plane x + 2y – z – 5 = 0 and intersecting the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$ is :

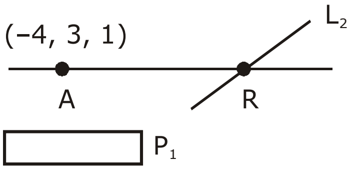
(A) $\frac{x-4}{2} = \frac{y+3}{1} = \frac{z+1}{4}$

(B) $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

(C) $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$

(D) $\frac{x+4}{1} = \frac{y-3}{1} = \frac{z-1}{3}$

Sol. **B**



Let R on L_2 ; $(-1 - 3t, 3 + 2t, 2 - t)$

for t $V_L . n_p = 0$

$\overline{AR}.\overline{n}_p = 0$

$(3 - 3t).1 + 2t.2 + (1 - t)(-1) = 0$

$3 - 3t + 4t - 1 + t = 0$

$t = -1$

$V_L = <6, -2, 2>$

12. 5 students of a class have an average height 150 cm and variance 18 cm^2. A new student, whose height is 156 cm, joined them. The variance (in cm^2) of the height of these six students is :

(A) 16 (B) 20 (C) 22 (D) 18

Sol. **B**

Let students are s_1, s_2, s_3, s_4, s_5

Given avg. high

$$\bar{x} = \frac{s_1 + s_2 + s_3 + s_4 + s_5}{5} = 15$$

$$\sum s_i = 750$$

& Variance

$$\frac{\sum (s_i)^2}{5} - (\bar{x})^2 = 18$$

$$\sum (s_i)^2 = 112590$$

hight of new student is 156

Now new variance

$$= \frac{112590 + (156)^2}{6} - \frac{(750 + 156)}{6} = 20$$

13. For $x \neq n\pi + 1$, $n \in N$ (the set of the natural numbers), the integral

$$\int x\sqrt{\frac{2\sin(x^2-1) - \sin 2(x^2-1)}{2\sin(x^2-1) + \sin 2(x^2-1)}}\,dx$$ is equal to :

(where c is a constant of integration.

(A) $\log_e\left|\sec\left(\frac{x^2-1}{2}\right)\right| + C$

(B) $\log_e\left|\frac{1}{2}\sec^2(x^2-1)\right| + C$

(C) $\frac{1}{2}\log_e\left|\sec^2\left(\frac{x^2-1}{2}\right)\right| + C$

(D) $\frac{1}{2}\log_e\left|\sec(x^2-1)\right| + C$

Sol. **A**

Let $x^2 - 1 = \theta \Rightarrow 2x\,dx = d\theta$

$$\frac{1}{2}\int\sqrt{\frac{2\sin\theta - \sin 2\theta}{2\sin\theta + \sin 2\theta}}\,d\theta$$

$$\frac{1}{2}\int\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$$= \frac{1}{2}\int |\tan\theta/2|\,d\theta$$

$$= \ln\left|\sec\theta/2\right| + C$$

$$= \ln\left|\sec\left(\frac{x^2-1}{2}\right) + C\right|$$

14. Let f : R $\rightarrow$ R be a function defined as

$$f(x) = \begin{cases} 5, & \text{if } x \le 1 \\ a + bx, & \text{if} 1 < x < 3 \\ b + 5x, & \text{if } 3 \le x < 5 \\ 30, & \text{if } x \ge 5 \end{cases}$$ Then, f is :

(A) Continuous if a = 5 and b = 5
(B) not continuous for any values of a and b
(C) continuous if a = 0 and b = 5
(D) continuous if a = – 5 and b = 10

Sol. **B**

x = 1 $\quad 5 = 5 = a + b \Rightarrow a + b = 5$
x = 3 $\quad a + 3b = b + 15 = b + 15 \Rightarrow a + 2b = 15$
x = 5 $\quad b + 25 = 30 = 30$
$\Rightarrow \quad b = 5$

15. Let $0 < \theta < \frac{\pi}{2}$. If the eccentricity of the hyperbola $\frac{x^2}{\cos^2\theta} - \frac{y^2}{\sin^2\theta} = 1$ is greater than 2, then the length of its latus rectum lies in the interval :

(A) (3/2, 2] (B) (3, ∞) (C) (1,3/2] (D) (2,3]

Sol. **B**

$e_H > 2$

$$\Rightarrow \sqrt{1 + \frac{\sin^2\theta}{\cos^2\theta}} > 2$$

$\sec\theta > 2$

$0 < \cos\theta < \frac{1}{2}$

$\frac{\pi}{3} < \theta < \frac{\pi}{2}$

Now

$$LR = \frac{2b^2}{a} = \frac{(2b)^2}{2a} = \frac{(\sin\theta)^2}{2\cos\theta}$$

$LR = 2\frac{\sin^2\theta}{\cos\theta}$

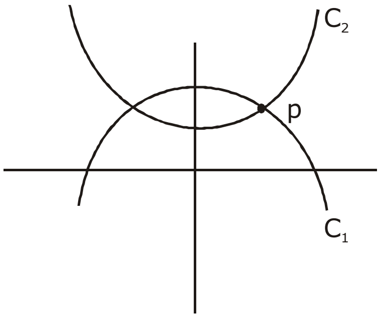
$LR = 2\tan\theta\sin\theta$

$\Rightarrow\ LR \in (3, \infty)$

16. if θ denotes the acute angle between the curves, $y = 10-x^2$ and $y = 2 + x^2$ at a point of their intersection, then $|\tan\theta|$ is equal to :

(A) $\frac{4}{9}$ (B) $\frac{8}{17}$ (C) $\frac{8}{15}$ (D) $\frac{7}{17}$

Sol. **C**



$C_1 : y = 10 - x^2$

$C_1 : y = 2 + x^2$

for P.O.I

$10 - x^2 = 2 + x^2$

$x^2 = 4$

$x = \pm 2$

P : (2, 6)

$\tan\theta = \left|\frac{-2.2-2.2}{1-(2.2).(2.2)}\right|$

$\tan\theta = \frac{8}{15}$

17. Let $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a}\times\vec{c}\times\vec{b} = \vec{0}$ and $\vec{a}.\vec{c} = 4$, then $|\vec{c}|^2$ is equal to :

(A) $\frac{19}{2}$ (B) 8 (C) $\frac{17}{2}$ (D) 9

Sol. **A**

$\bar{a}\times\bar{c}+\bar{b} = 0$

$\bar{a}\times(\bar{a}\times\bar{c}+\bar{b}) = 0$

$(\bar{a}.\bar{c})\bar{a} - a^2\bar{c} + \bar{a}\times\bar{b} = 0$

$\bar{c} = \frac{4\bar{a}+\bar{a}\times\bar{b}}{|\bar{a}|^2}$

$\bar{c} = \frac{<4, -4, 0> + <-1, -1, 2>}{2}$

$\bar{c} = \dfrac{<+3, -5, 2>}{2}$

$|\bar{c}|^2 = \dfrac{9+25+4}{4}$

$|\bar{c}|^2 = \dfrac{19}{2}$

18. If a,b and c be three distinct real numbers in G.P. and a + b + c = xb, then x cannot be :
(A) −2 (B) −3 (C) 2 (D) 4

Sol. **C**

a, b, c are in G.P.

$\dfrac{b}{r}, b, br$ are in G.P.

Now $\dfrac{b}{r} + b + br = xb$

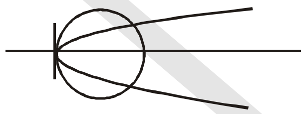
$x - 1 = r + \dfrac{1}{r}$

$x \geq 3$ or $x \leq -1$

19. Equation of a common tangent to the circle $x^2 + y^2 - 6x = 0$ and the parabola, $y^2 = 4x$, is :
(A) $2\sqrt{3}y = -x - 12$ (B) $2\sqrt{3}y = 12x + 1$ (C) $\sqrt{3}y = x + 3$ (D) $\sqrt{3}y = 3x + 1$

Sol. **C**

$C_1 : x^2 + y^2 - 6x = 0$
$P : y^2 = 4x$



$C_1 : (x-3)^2 + y^2 = 3^2$
$P : y^2 = 4x$

$T \mid C_1 \Rightarrow y = m(x - 3) \pm 3\sqrt{1+m^2}$

$T \mid P \Rightarrow y = m x + \dfrac{1}{m}$

$\Rightarrow \dfrac{1}{m} = -3m + 3\sqrt{1+m^2}$

$\left(\dfrac{1}{m} + 3m\right)^2 = 9 + 9m^2$

$\dfrac{1}{m^2} + 9m^2 + 6 = 9 + 9m^2$

$m^2 = \frac{1}{3}$

$m = \pm \frac{1}{\sqrt{3}}$

T : $y = \pm \frac{x}{\sqrt{3}} + \sqrt{3}$

20. The plane through the intersection of the planes x +y + z = 1 and 2x + 3y – z + 4 = 0 and parallel to y - axis also passes through the point :

(A) (–3,1,1) (B) (3,2,1) (C) (3,3, – 1) (D) (–3,0,–1)

Sol. **B**

P : $P_1 + \lambda P_2 = 0$

P : $(x + 2\lambda)$ x+ $(1 + 3\lambda)$ y + $(1 - \lambda)$ z + $(-1 + 4\lambda)$ = 0

for λ :

$\overline{n_p}.\hat{j} = 0$

$1 + 3\lambda = 0 \Rightarrow \lambda = -1/3$

P : $\pm\frac{1}{3}x + \frac{4}{3}z - \frac{7}{3} = 0$

P : x + 4z – 7 = 0

Now check options

21. $\lim\limits_{y\to 0} \frac{\sqrt{1+\sqrt{1+y^4}} - \sqrt{2}}{y^4}$

(A) exists and equals $\frac{1}{4\sqrt{2}}$ (B) does not exist

(C) exsits and equals $\frac{1}{2\sqrt{2}}$ (D) exists and equals $\frac{1}{2\sqrt{2}\left(\sqrt{2}+1\right)}$

Sol. **A**

$$\lim_{y\to 0} \frac{\sqrt{1+\sqrt{1+y^4}} - \sqrt{2}}{y^4}$$

$$\lim_{y\to 0} \left(\frac{1+\sqrt{1+y^4} - 2}{y^4}\right)\left(\frac{1}{\sqrt{1+\sqrt{1+y^4}} + \sqrt{2}}\right)$$

$$\lim_{y\to 0} \frac{1+y^4-1}{y^4}\left(\frac{1}{\sqrt{1+(y)^4}+1}\right)\left(\frac{1}{\sqrt{1+\sqrt{1+y^4}} + \sqrt{2}}\right)$$

$$= \frac{1}{2}.\frac{1}{2\sqrt{2}} = \frac{1}{4\sqrt{2}}$$

22. If A = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then the matrix A^{-50} when $\theta = \frac{\pi}{12}$, is equal to :

(A) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (B) $\begin{bmatrix} \frac{1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ (C) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (D) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

Sol. **A**

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$A^3 = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$A^n = \begin{bmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{bmatrix}$$

$$A^{-50} = \begin{bmatrix} \cos(-50)\theta & -\sin(-50)\theta \\ \sin(-50)\theta & \cos(-50)\theta \end{bmatrix}$$

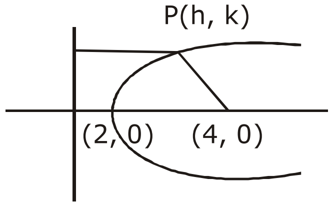
$$A^{-50} = \begin{bmatrix} \cos\frac{50\pi}{12} & \sin\frac{50p}{12} \\ -\sin\frac{50\pi}{12} & \cos\frac{50\pi}{12} \end{bmatrix}$$

$$A^{-50} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

23. Axis of a parabola lies along x - axis. If its vertex and focus are at distances 2 and 4 respectively from the origin, on the positive x - axis then which of the following points does not lie on it ?

(A) (8,6) (B) $(5, 2\sqrt{6})$ (C) $(4, -4)$ (D) $(6, 4\sqrt{2})$

Sol. **A**



$y^2 = 8(x - 2)$

24. Let $a_1, a_2, ...,a_{30}$ be an A.P., $S = \sum_{i=1}^{30} a_i$ and $T = \sum_{i=1}^{15} a_{(2i-1)}$. If $a_5 = 27$ and $S - 2T = 75$, then a_{10} is equal to :

(A) 42 (B) 47 (C) 57 (D) 52

Sol. **D**

S – 27 = 75

$$\frac{30}{2}[2a_1 + 29d] - 2\left\{\frac{15}{2}\right\}[2a_1 + (14)2d] = 75$$

15 d = 75

d = 5 ... (i)

& $a_5 = 27$

$a_1 + 4d = 27$ ...(2)

$a_1 = 7$

Now $a_{10} = a_1 + 9d$

= 7 + 45 = 52

25. the system of linear equations

x + y + z = 2

2x + 3y + 2z = 5

$2x + 3y + (a^2 - 1)z = a + 1$

(A) has a unique solutions for $|a| = \sqrt{3}$ (B) is inconsistent when $|a| = \sqrt{3}$

(C) has infinitely many solutions for a = 4

(D) is inconsistent when a = 4

Sol. **B**

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 3 & a^2 - 1 \end{vmatrix}$$

$\Rightarrow 3a^2 - 3 - 6 - 2a^2 + 2 + 4 + 2a^2 - 2 - 4 = 0$

$\Rightarrow 3(a^2 - 3) = 0$

$\Rightarrow a^2 = 3 \Rightarrow |a| = \sqrt{3}$

$\Rightarrow$ in constant

$\Delta = 0$

26. Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to :

(A) 512 (B) –512 (C) 256 (D) –256

Sol. **D**

$x^2 + 2x + 1 = -1$

$(x + 1)^2 = i^2$

x = – (1 + i) – (1 – i)

$$\alpha^{15} + \beta^{15} = 2^{15/2}\left\{\frac{(-1+i)^{15}}{\sqrt{2}} + \frac{(-1-i)^{15}}{\sqrt{2}}\right\}$$

$$= 2^{15/2}\left\{2\cos\left(\frac{3\pi}{4}.15\right)\right\}$$

$$2^{15/2}\left\{-2\cos\left(\frac{45\pi}{4}\right)\right\}$$

$= -2^{15/2}.2.2^{1/2} = -256$

27. If the Boolean expression $(p \oplus q) \wedge (\sim p \odot q)$ is equivalent to $p \wedge q$, where $\oplus$, $\odot \in \{\wedge, \vee\}$, then the ordered pair $(\oplus, \odot)$ is :

(A) $(\wedge, \wedge)$ (B) $(\wedge, \vee)$ (C) $(\vee, \vee)$ (D) $(\vee, \wedge)$

Sol. B

p	q	~ p	p ^ q	~ p v q	(p ^ q) ^ (~ p v q)
T	T	F	T	T	T
T	F	F	F	F	F
F	T	T	F	T	F
F	F	T	F	T	F

28. The value of $\int_0^{\pi} |\cos x|^3 dx$ is :

(A) $\frac{4}{3}$ (B) 0 (C) $-\frac{4}{3}$ (D) $\frac{2}{3}$

Sol. **A**

$$\int_0^{\pi} |\cos x|^3 dx$$

$$\int_0^{\pi/2} (\cos x)^3 dx - \int_{\pi/2}^{\pi} (\cos x)^3 dx$$

$$= \frac{2}{3} - \left(-\frac{2}{3}\right)$$

$$= \frac{4}{3}$$

29. For any $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$, the expression $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals :

(A) $13 - 4\cos^2\theta + 6\cos^4\theta$ (B) $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$

(C) $13 - 4\cos^6\theta$ (D) $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$

Sol. **C**

$$= 3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$$

$$= \; = 3\{1 - \sin 2\theta\}^2 + 6\{1 + \sin 2\theta\} + 4\sin^6\theta$$

$$= 9 + 3\sin^2 2\theta + 4\sin^6\theta$$

$$= 9 + 4\sin^2\theta\{3\cos^2\theta + \sin^4\theta\}$$

$$= 9 + 4(1 - c^2)\{3c^2 + (1 - c^2)^2\}$$

$$= 9 + 4(1 - c^2)(3c^2 + c^2 - 2c^2 + 1)$$

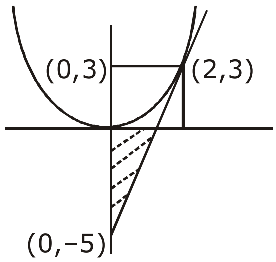
$$= 13 - 4\cos^6\theta$$

30. The area (in sq. units) bounded by the parabola $y = x^2 - 1$, the tangent at the point (2,3) to it and the y - axis is :

(A) $\frac{14}{3}$ (B) $\frac{32}{3}$ (C) $\frac{56}{3}$ (D) $\frac{8}{3}$

Sol. **D**

C : $y = x^2 - 1$



$T_{(2,3)}$: $y - 3 = 4(x - 2)$

$y - 4x + 5 = 0$

$A = \frac{1}{2}2.8 - \int_{-1}^{3} x\,dy$

$A = 8 - \int_{-1}^{3} \sqrt{1+y}\,dy$

$A = 8 - \frac{23}{3}(1+y)^{3/2} \Big|_{-1}^{3}$

$= 8 - 2/3\ \{2^3\}$

$= \left|8 - \frac{16}{3}\right| = 8/3$