JEE MAIN JAN'19

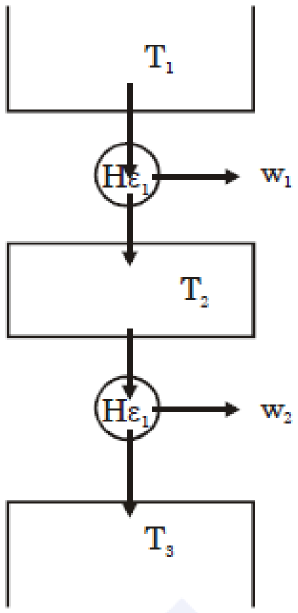
QUESTION WITH SOLUTION

DATE : 09-01-2019 _ EVENING

1. Two Carnot engines A and B are operated in series. The first one, A, receives heat at T_1 (= 600 k) and rejects to a reservoir at temperature T_2. The second engine B receives heat rejected by the first engine and in turn, rejects to a heat reservoir at T_3 (=400 K). Calculate the temperature T_2 if the work outputs of the two engines are equal:

(A) 600 K (B) 300 K (C) 500 K (D) 400 K

Sol. **C**



$w_1 = w_2$

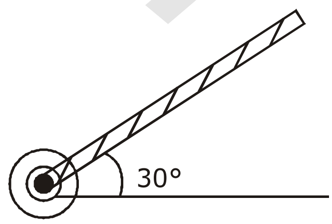
$\Delta u_1 = \Delta u_2$

$T_3 - T_2 = T_2 - T_1$

$2T_2 = T_1 + T_3$

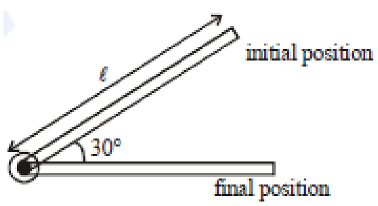
$T_2 = 500$ K

2. A rod of length 50 cm is pivoted at one end. it is rasied such that if makes an angle of 30º from the horizontal as shown and released from rest. Its angular speed when it passes through the horizontal (in rads^{-1}) will be (g = 10 ms^{-2})



(A) $\frac{\sqrt{30}}{2}$ (B) $\sqrt{30}$ (C) $\sqrt{\frac{30}{2}}$ (D) $\frac{\sqrt{20}}{3}$

Sol. **B**



Work done by gravity from initial to final position is,

$W = mg\frac{\ell}{2}\sin 30^o$

$= \frac{mg\ell}{4}$

According to work energy theorem

$W = \frac{1}{2}I\omega^2$

$\Rightarrow \frac{1}{2}\frac{m\ell^2}{3}\omega^2 = \frac{mg\ell}{4}$

$\omega = \sqrt{\frac{3g}{2\ell}} = \sqrt{\frac{3\times 10}{2\times 0.5}}$

$\omega = \sqrt{30}\,\text{rad/sec}$

3. The pitch and the number of divisions, on the circular scale, for a given screw gauge are 0.5 mm and 100 respectively. When the screw gauge is fully tightened without any object, the zero of its circular scale lies 3 divisions below the mean line.
The readings of the main scale and the circular scale, fore a thin sheet, are 5.5 mm and 48 respectively, the thickness of this sheet is:

(A) 5.950 mm (B) 5.740 mm (C) 5.755 mm (D) 5.725 mm

Sol. **D**

$LC = \frac{\text{Pitch}}{\text{No of division}}$

$LC = 0.5 \times 10^{-2}$ mm
+ve error = $3 \times 0.5 \times 10^{-2}$ mm
= 1.5×10^{-2} mm = 0.015 mm
reading = MSR + CSR – (+ve error)
= 5.5 mm + $(48 \times 0.5 \times 10^{-2})$ – 0.015
= 5.5 + 0.24 – 0.015 = 5.725mm

4. The position co-ordinates of a particle moving in a 3-D coordinate system is given by
$x = a\cos\omega t$
$y = a\sin\omega t$
and $z = a\omega t$
The speed of the particle is:

(A) $\sqrt{3}a\omega$ (B) $2a\omega$ (C) $a\omega$ (D) $\sqrt{2}a\omega$

Sol. **D**

$V_x = -a\omega\sin\omega t$ $\Rightarrow v_y = a\omega\cos\omega t$

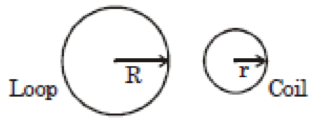
$V_z = A\omega$ $\Rightarrow v = \sqrt{v_x^2 + v_y^2 + v_z^2}$

$v = \sqrt{2}a\omega$

5. One of the two identical conducting wires of length l is bent in the form of a circular loop and the other one into a circular coil of N identical turns. if the same current is passed in both, the ratio of the magnetic field at the central of the loop (B_L) to that at the centre of the coil (B_C), i.e. $\frac{B_L}{B_C}$ will be :

(A) $\frac{1}{N}$ (B) $\frac{1}{N^2}$ (C) N (D) N^2

Sol. **B**



$L = 2\pi R \quad L = N \times 2\pi r$

$R = Nr$

$B_L = \frac{\mu_0 i}{2R} \quad B_C = \frac{\mu_0 Ni}{2r}$

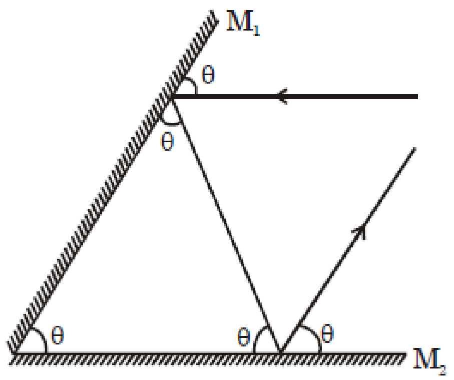
$B_C = \frac{\mu_0 N^2 i}{2R}$

$\frac{B_L}{B_C} = \frac{1}{N^2}$

6. Two plane mirrors are inclined to each other such that a ray of light incident on the first mirror (M_1) and parallel to the second mirror (M_2) is finally reflected from the second mirror (M_2) parallel to the first mirror (M_1). The angle between the two mirror will be :

(A) 75º (B) 45º (C) 60º (D) 90º

Sol. **C**



Assuming angles between two mirrors be θ as per geometry,
sum of anlges of Δ
$3\theta = 180º$
$\theta = 60º$

7. A force acts on a 2 kg object so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds ?

(A) 875 J (B) 900 J (C) 950 J (D) 850 J

Sol. **B**

$x = 3t^2 + 5$

$V = \frac{dx}{dt}$

$v = 6t + 0$

at $t = 0 \quad v = 0$

$t = 5$ sec $v = 30$ m/s

W.D. = ΔKE

W.D. = $\frac{1}{2}mv^2 - 0 = \frac{1}{2}(2)(30)^2 = 900J$

8. In a car race on straight road, car A takes a time t less than car B at the finish and passes finishing point with a speed 'υ' more than that of car B. Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively. Then 'υ' is equal to :

(A) $\sqrt{2a_1a_2}\,t$ (B) $\frac{a_1 + a_2}{2}t$ (C) $\sqrt{a_1a_2}\,t$ (D) $\frac{2a_1a_2}{a_1 + a_2}t$

Sol. **C**

For A & B let time taken by A is t_0

$V_A - V_B = V = (a_1 - a_2)t_0 - a_2t$ ...(A)

$x_B = x_A = \frac{1}{2}a_1t_0^2 = \frac{1}{2}a_2(t_0 + t)^2$

$\Rightarrow \sqrt{a_1t_0} = \sqrt{a_2}(t_0 + t)$

$\Rightarrow \left(\sqrt{a_2} - \sqrt{a_2}\right)t_0 = \sqrt{a_2}\,t$ ...(B)

putting t_0 in equation

$v = (a_1 - a_2)\frac{\sqrt{a_2}\,t}{\sqrt{a_1} - \sqrt{a_2}} - a_2t$

$= \left(\sqrt{a_1} + \sqrt{a_2}\right)\sqrt{a_2}\,t - a_2t \Rightarrow v = \sqrt{a_1a_2}\,t$

$\Rightarrow \sqrt{a_1a_2}\,t + a_2t - a_2t$

9. A particle is executing simple harmonic motion (SHM) of amptitude A, along the x-axis, about x = 0. When its potential Energy (PE) equals kinetic energy (KE), the position of the particle will be :

(A) $\frac{A}{2\sqrt{2}}$ (B) $\frac{A}{2}$ (C) A (D) $\frac{A}{\sqrt{2}}$

Sol. **D**

Potential energy (U) = $\frac{1}{2}kx^2$

Kinetic energy (K) = $\frac{1}{2}KA^2 - \frac{1}{2}kx^2$

According to the question, U = k

$\therefore \frac{1}{2}kx^2 = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$

$x = \pm\frac{A}{\sqrt{2}}$

$\therefore$ Correct answer is (D)

10. The energy required to take a satellite to a height 'h' above Earth surface (radius of Earth $= 6.4 \times 10^3$ km) is E_1 and kinetic energy required for the satellite to be in a circular orbit at this height is E_2. The value of h for which E_1 and E_2 are equal, is :

(A) 1.28×10^4km (B) 6.4×10^3km (C) 3.2×10^3km (D) 1.6×10^3km

Sol. **C**

$U_{surface} + E_1 = U_h$

KE of satelite is zero at earth surface & at height h

$-\frac{GM_em}{R_e} + E_1 = -\frac{GM_em}{(Re + h)}$

$E_1 = GM_e m\left(\frac{1}{R_e} - \frac{1}{R_e + h}\right)$

$E_1 = \frac{GM_e m}{(R_e + h)} \times \frac{h}{R_e}$

Gravitational attraction $F_G = ma_C = \frac{mv^2}{(R_e + h)}$

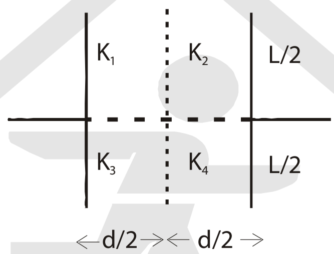
$E_2 \Rightarrow \frac{GM_e m}{(R_e + h)}$

$mv^2 = \frac{mv^2}{2} = \frac{GM_e m}{2(R_e + h)}$

$E_1 = E_2$

$\frac{h}{R_e} = \frac{1}{2} \Rightarrow h = \frac{R_e}{2} = 32000\text{km}$

11. A parallel plate capacitor with square plates is filled with four dielectrics of dielectric constants K_1, K_2, K_3, K_4 arranged as shown in the figure. The effective dielectric constant K will be :



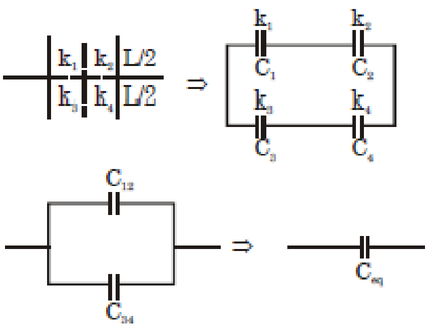
(A) $K = \frac{(K_1 + K_2)(K_3 + K_4)}{K_1 + K_2 + K_3 + K_4}$

(B) $K = \frac{(K_1 + K_4)(K_2 + K_3)}{2(K_1 + K_2 + K_3 + K_4)}$

(C) $K = \frac{(K_1 + K_2)(K_3 + K_4)}{2(K_1 + K_2 + K_3 + K_4)}$

(D) $K = \frac{(K_1 + K_3)(K_2 + K_4)}{K_1 + K_2 + K_3 + K_4}$

Sol. **D**



$$C_{12} = \frac{C_1 C_2}{C_1 + C_2} = \frac{\frac{k_1 \in_0 \frac{L}{2} \times L}{d/2} \frac{k_2\left[\in_0 \frac{L}{2} \times L\right]}{d/2}}{(K_1 + K_2)\left[\frac{\in_0 \cdot \frac{L}{2} \times L}{d/2}\right]}$$

$$C_{12} = \frac{k_1k_2}{k_1+k_2}\frac{\epsilon_0 L^2}{d}$$

in the same way we get, $C_{34} = \frac{k_3k_4}{k_3+k_4}\frac{\epsilon_0 L^2}{d}$

$$\therefore C_{eq} = C_{12} + C_{34} = \left[\frac{k_1k_2}{k_1+k_2} + \frac{k_3k_4}{k_3+k_4}\right]\frac{\varepsilon_0 L^2}{d} \quad ...(i)$$

Now if $k_{eq} = k$, $C_{eq} = \frac{k \epsilon_0 L^2}{d}$ ....(ii)

on comparing equation (i) to equation (ii), we get

$$k_{eq} = \frac{k_1k_2(k_3+k_4)+k_3k_4(k_1+k_2)}{(k_1+k_2)(k_3+k_4)}$$

This does not match with any of the options so probably they have assumed the wrong combination

$$C_{13} = \frac{k_1 \epsilon_0 L\frac{L}{2}}{d/2} + k_3 \epsilon_0 \frac{L\frac{L}{2}}{d/2}$$

$$= (k_1 + k_3)\frac{\epsilon_0 L^2}{d}$$

$$C_{24} = (k_2 + k_4)\frac{\epsilon_0 L^2}{d}$$

$$C_{eq} = \frac{C_{13}C_{24}}{C_{13}C_{24}} = \frac{(k_1+k_3)(k_2+k_4)}{(k_1+k_2+k_3+k_4)}\frac{\epsilon_0 L^2}{d}$$

$$= \frac{k \epsilon_0 L^2}{d}$$

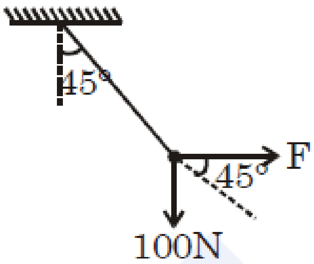
$$k = \frac{(k_1+k_3)(k_2+k_4)}{(k_1+k_2+k_3+k_4)}$$

However this is one of the four options.
it must be a "Bonus" logically but of the given options probably they might go with (D)

12. A mass of 10 kg is suspended vertically by a rope from the roof. When a horizontal force is applied on the rope at some point, the rope deviated at an angle of 45° at the roof point. If the suspended mass is at equilibrium, the magnitude of the force applied is ($g=10\ ms^{-2}$)

(A) 70 N (B) 200 N (C) 100 N (D) 140 N

Sol. **C**



at equation

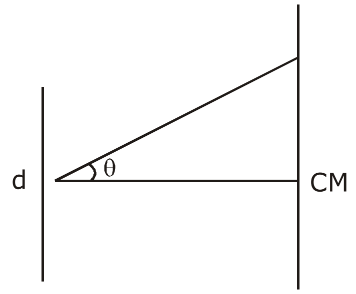
$$\tan 45^o = \frac{100}{F}$$

F = 100 N

13. In a Young's double slit experiment, the slits are placed 0.320 mm apart. Light of wavelength λ = 500 nm is incident on the slits. The total number of bright fringes that are observed in the angular range $-30^\circ \le \theta \le 30^\circ$ is :

(A) 321 (B) 640 (C) 320 (D) 641

Sol. **D**



Pam difference

$d\sin\theta = n\lambda$

where d = seperation of slits

λ = wave length

n = no. of maximas

$0.32 \times 10^{-3} \sin 30 = n \times 500 \times 10^{-9}$

n = 320

Hence total no. of maximas observed in angular range –30º is

maximas = 320 + 1 + 320 = 641

14. At a given instant, say t= 0, two radioactive substances A and B have equal activities. The ratio $\frac{R_B}{R_A}$ of their activities after time t itself decays with time t as e^{-3t}. If the half-life of A is ln2, the half-life of B is :

(A) 4ln2 (B) $\frac{\ln 2}{2}$ (C) $\frac{\ln 2}{4}$ (D) 2ln2

Sol. **C**

Half life of A = ℓn2

$t_{1/2} = \frac{\ell n 2}{\lambda}$

$\lambda_A = 1$

at t = 0 $R_A = R_B$

$N_A e^{-\lambda AT} = N_B E^{-\lambda BT}$

$N_A = N_B$ at t = 0

at t = t $\frac{R_B}{R_A} = \frac{N_0 e^{-\lambda_B T}}{N_0 e^{-\lambda_A t}}$

$e^{-(\lambda_B - \lambda_A)t} = e^{-t}$

$\lambda_B - \lambda_A = 3$

$\lambda_B = 3 + \lambda_A = 4$

$t_{1/2} = \frac{\lambda n 2}{\lambda_B} = \frac{\lambda n 2}{4}$

15. The magnetic field associated with a light wave is given, at the origin, by $B = B_0$ [sin(3.14×10^7)ct + sin(6.28×10^7)ct]. If this light falls on a silver plate having a work function of 4.7 eV, what will be the maximum kinetic energy of the photo electrons? ($c=3\times10^8 ms^{-1}$, $h = 6.6\times10^{-34}$ J-s)

(A) 6.82eV (B) 7.72 eV (C) 8.52 eV (D) 12.5 eV

Sol. **B**

$B = B_0\sin(\pi \times 10^7C)t + B_0\sin(2\pi \times 10^7C)t$ since there are two EM waves with different frequency, to get gmaximum kinetic energy we take the photon with higher frequency

$B_1 = B_0\sin(\pi \times 10^7C)t \quad V_1 = \frac{10^7}{2}\times C$

$B_2 = B_0\sin(2\pi \times 10^7C)t \quad V_2 = 10^7C$

where C is speed of light $C = 3 \times 10^8$ m/s

$v_2 > v_1$

So KE of photoelectron will be maximum for photon of higher energy.

$v_2 = > 10^7C$ Hz

$h\nu = \phi + KE_{max}$

energy of photon

$E_{ph} = 6.6 \times 10^{-34} \times 10^7 \times 3 \times 10^9$

$E_{ph} = 6.6 \times 3 \times 10^{-19}J$

$= \frac{6.6\times3\times10^{-19}}{1.6\times10^{-19}}$ eV = 12.375eV

$KE_{max} = E_{ph} - \phi$

$= 12.375 - 4.7 = 7.675$ eV ≈ 7.7 eV

16. Expression for time in terms of G (universal gravitational constant). h (Plank constant) and c (speed of light) is proportional to :

(A) $\sqrt{\frac{c^3}{Gh}}$ (B) $\sqrt{\frac{Gh}{c^3}}$ (C) $\sqrt{\frac{hc^5}{G}}$ (D) $\sqrt{\frac{Gh}{c^5}}$

Sol. **D**

$F = \frac{GM^2}{R^2} \Rightarrow G = [M^{-1}L^3T^{-2}]$

$E = h\nu \Rightarrow h = [ML^2T^{-1}]$

$C = [LT^{-1}]$

$T \propto G^xh^yC^z$

$[T] = [M^{-1}L^3T^{-2}]^x[ML^2T^{-1}]^y[LT^{-1}]z$

$[M^0L^0T^1] = [M^{-x+y}L^{3x+2y+z}T^{-2x-y-z}]$

On comparing the powers of M, L, T

$-x + y = 0 \Rightarrow x = y$

$3x + 2y + z = 0 \Rightarrow 5x + z = 0$ ....(i)

$-2x - y - z = 1 \Rightarrow 3x + z = -1$ ....(ii)

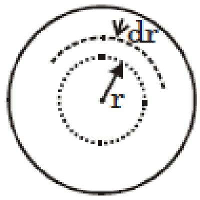
On solving (i) & (ii) $x = y = \frac{1}{2}$, $z = -\frac{5}{2}$

$t \propto \sqrt{\frac{Gh}{C^5}}$

17. Charge is distributed within a sphere of radius R with a volume charge density $\rho(r) = \frac{A}{r^2} e^{-2r/a}$, where A and a are constants. If Q is the total charge of this charge distribution, the radius R is:

(A) $a \log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$ (B) $a \log\left(1-\frac{Q}{2\pi aA}\right)$ (C) $\frac{a}{2} \log\left(1-\frac{Q}{2\pi aA}\right)$ (D) $\frac{a}{2} \log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$

Sol. **D**



$Q = \int \rho dv$

$= \int_0^R \frac{A}{r^2} e^{-2r/a}(4\pi r^2 dr$

$= \int_0^R \frac{A}{r^2} e^{-2r/a}(4\pi r^2 dr$

$= 4\pi A \int_0^R e^{-2r/a} dr$

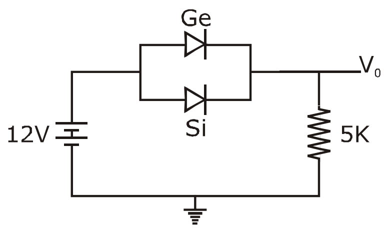
$= 4\pi A \left(\frac{e^{-2r/a}}{-\frac{2}{a}}\right)_0^R$

$= 4\pi A \left(-\frac{a}{2}\right)\left(e^{-2R/a} - 1\right)$

$Q = 2\pi aA(1-e^{2R/a})$

$R = \frac{a}{2} \log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$

18. Ge and Si diodes start conducting at 0.3 V and 0.7 V respectively. In the following figure if Ge diode connection are reversed, the value of V_0 changes by : (assume that the Ge diode has large breakdown voltage)



(A) 0.4 V (B) 0.8 V (C) 0.2 V (D) 0.6 V

Sol. **A**

Initially Ge & Si are both forward biased so current will effectivily pass through Ge diode with a drop of 0.3 V

if "Ge" is revesed then current will flow through "Si" diode hence an effective drop of (0.7 –0.3) = 0.4 V is observed.

19. In a communication system operating at wavelength 800 nm. only one percent of source frequency is available as signal bandwidth. The number of channels accommodated for transmitting TV signals of band width 6 MHz are (Take velocity of light c = 3×10^8m/s, h = 6.6×10^{-34} J–s)

(A) 6.25×10^5 (B) 4.87×10^5 (C) 3.86×10^6 (D) 3.75×10^6

Sol. **A**

$$f = \frac{3\times10^8}{8\times10^{-7}} = \frac{30}{8}\times10^{14}\text{Hz}$$

= 3.75×10^{14} Hz

1% of f = 0.0375×10^{14} MHz

= 3.75×10^{12} Hz = 3.75×10^6 MHz

number of channels = $\frac{3.75\times10^6}{6}$ = 6.25×10^5

$\therefore$ correct answer is (A)

20. The energy associated with electric field is (U_E) and with magnetic field is (U_B) for an electromagnetic wave in free space. Then :

(A) $U_E = \frac{U_B}{2}$ (B) $U_E < U_B$ (C) $U_E = U_B$ (D) $U_E > U_B$

Sol. **C**

Average energy density of magnetic field

$u_B = \frac{B_0^2}{2\mu_0}, B_0$ is maximum value of magetic field.

Average energy density of electric field,

$$u_E = \frac{\in_0 \in_0^2}{2}$$

Now, $\in_0 = CB_0, C^2 = \frac{1}{\mu_0 \in_0}$

$$U_E = \frac{\in_0}{2}\times C^2B_0^2$$

$$= \frac{\in_0}{2}\times\frac{1}{\mu_0 \in_0}\times B_0^2 = \frac{B_0^2}{2\mu_0} = u_B$$

$u_E = u_B$

Since energy density of electric & magnetic field is same, energy associated with equal volume will be equal.

21. A particle having the same charge as of electron moves in a circular path of radius 0.5 cm under the influence of a magnetic field of 0.5 T. If an electric field of 100 V/m makes it to move in a straight path, then the mass of the particle is (Given charge of electron = 1.6×10^{-19}C)

(A) 1.6×10^{-27}kg (B) 2.0×10^{-24}kg (C) 1.6×10^{-19} kg (D) 9.1×10^{-31} kg

Sol. **B**

$\frac{mv^2}{R} = qvB$

mv = qBR ...(i)

Path is straight line it qE = qvB

E = vB ....(ii)

From equation (i) & (ii)

$m = \frac{qB^2R}{E}$

$m = 2.0 \times 10^{-24}$ kg

22. A rod of mass 'M' and length '2L' is suspended at its middle by a wire. It exhibits torsional oscillations; If two masses each of 'm' are attached at distance 'L/2' from its centre on both sides, it reduces the oscillation frequency by 20%. The value of ratio m/M is close to -

(A) 0.17 (B) 0.77 (C) 0.57 (D) 0.37

Sol. **D**

Frequency of torsonal oscillations is given by

$f = \frac{k}{\sqrt{I}}$

$f_1 = \frac{k}{\sqrt{\frac{M(2L)^2}{12}}}$

$f_2 = \frac{k}{\sqrt{\frac{M(2L)^2}{12} + 2m\left(\frac{L}{2}\right)^2}}$

$f_2 = 0.8\, f_1$

$\frac{m}{M} = 0.375$

23. A power transmission line feeds input power at 2300 V to a step down transformer with its primary windings having 4000 turns. The output power is delivered at 230 V by the transformer. If the currents in the primary of the transformer is 5A and its efficiency is 90%, the output current would be -

(A) 35 A (B) 25 A (C) 50 A (D) 45 A

Sol. **D**

$\eta = \frac{P_{out}}{P_{in}} = \frac{V_S I_S}{V_P I_P}$

$\Rightarrow 0.9 = \frac{23 \times I_s}{230 \times 5}$

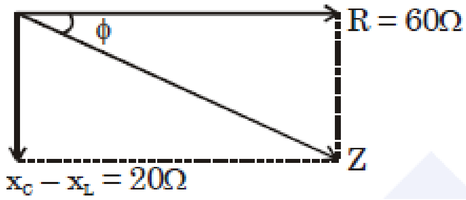
$\Rightarrow I_S = 45A$

24. A series AC circuit containing an inductor (20 mH), a capacitor (120 μF) and a resistor (60Ω) is driven by an AC source of 24V/50 Hz. The energy dissiapted in the circuit in 60 s is -
(A) 5.17×10^2 J (B) 3.30×10^3 J (C) 5.65×10^2 J (D) 2.26×10^3 J

Sol. **A**
R = 60Ω f = 50Hz, ω= 2πf = 100π

$X_C = \frac{1}{\omega C} = \frac{1}{100\pi \times 120 \times 10^{-6}}$

$X_C = 26.52\Omega$
$X_L = \omega L = 100\pi \times 20 \times 10^{-3} = 2\pi\Omega$
$X_C - x_L = 20.24 \approx 20$
$X_C - X_L = 20\Omega$



$z = \sqrt{R^2 + (x_C - x_L)^2}$

$z = 20\sqrt{10}\Omega$

$\cos\phi = \frac{R}{z} = \frac{3}{\sqrt{10}}$

$P_{avg} = VI\cos\phi,\ I = \frac{v}{z}$

$= \frac{v^2}{z}\cos\phi$
= 8.64 watt
Q = P.t = 8.64 × 60 = 5.18×10^2

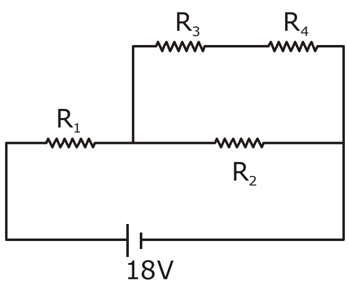
25. A 15 g mass of nitrogen gas is enclosed in a vessel at a temeprature 27°C. Amount of heat transferred to the gas, so that rms velocity of molecules is doubled, is about -
[Take R = 8.3 J/K mole]
(A) 10 kJ (B) 14 kJ (C) 0.9 kJ (D) 6 kJ

Sol. **A**
$Q = nC_V\Delta T$ as as in closed vessel

$Q = \frac{15}{28} \times \frac{5 \times R}{2} \times (4T - T)$

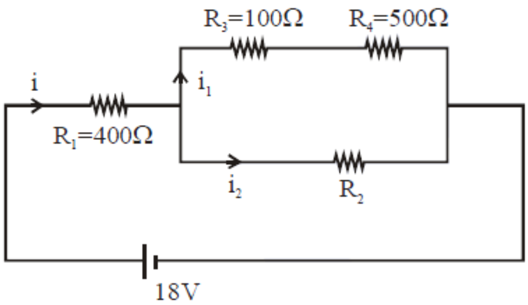
Q = 10000 J = 10 kJ

26. In the given circuit the internal resistance of the 18 V cell is negligible. If R_1 = 400 Ω, R_3 = 100 Ω and R_4 = 500 Ω and the reading of an ideal voltmeter across R_4 is 5 V, then the value of R_2 will be -



(A) 300 Ω (B) 550 Ω (C) 450 Ω (D) 230 Ω

Sol. **A**



$V_4 = 5V$

$i_1 = \frac{V_4}{R_4} = 0.01\ A$

$V_3 = i_1R_3 = 1V$

$V_3 + V_4 = 6V = V_2$

$V_1 + V_3 + V_4 = 18V$

$V_1 = 12V$

$i = \frac{V_1}{R_1} = 0.03\ Amp \quad V_2 = 6V$

$R_2 = \frac{V_2}{i_2} = \frac{6}{0.02} = 300\Omega$

27. A musician using an open flute of length 50 cm produces second harmonic sound waves. A person runs towards the musician from anotehr end of a hall at a speed of 10 km/h. If the wave speed is 330 m/s, the frequency heard by the running person shall be close to -

(A) 666 Hz (B) 333 Hz (C) 500 Hz (D) 753 Hz

Sol. **A**

Frequency of the sound produced by flute,

$f = 2\left(\frac{v}{2\ell}\right) = \frac{2\times 330}{2\times 0.5} = 660Hz$

Velocity of observer, $v_0 = 10 \times \frac{5}{18} = \frac{25}{9}$ m/s

$\therefore$ frequency detected by observer, f' =

$\left[\frac{v+v_0}{v}\right]f$

$\therefore f' = \left[\frac{\frac{25}{9}+330}{330}\right]660$

= 335.56 × 2 = 671.12

$\therefore$ closest answer is (A)

28. The top of a water tank is open to air and its water lavel is maintained. It is giving out 0.74 m^3 water per minute through a circular opening of 2 cm radius in its wall. The depth of the centre of the opening from the level of water in the tank is close to -
(A) 6.0 m (B) 4.8 m (C) 9.6 m (D) 2.9 m

Sol. **B**
In flow volume = outflow volume

$\Rightarrow \frac{0.74}{60} = \left(\pi \times 4 \times 10^{-4}\right) \times \sqrt{2gh}$

$\Rightarrow \sqrt{2gh} = \frac{74 \times 100}{240\pi}$

$\Rightarrow \sqrt{2gh} = \frac{740}{24\pi}$

$\Rightarrow 2gh = \frac{740 \times 740}{24 \times 24 \times 10} (\pi^2 = 10)$

$\Rightarrow h = \frac{74 \times 74}{2 \times 24 \times 24}$

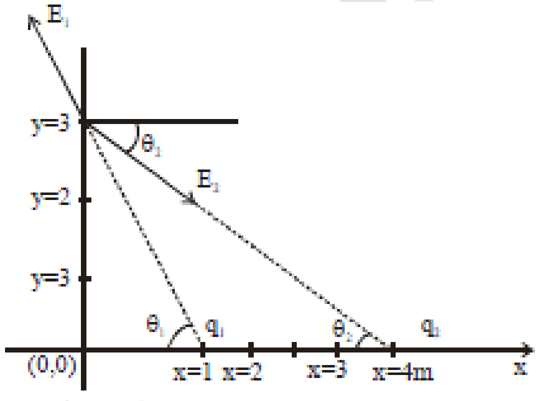
$\Rightarrow h \approx 4.8m$

29. Two point charge q_1 $\left(\sqrt{10}\mu C\right)$ and q_2 (–25 μC) are placed on the x-axis at x = 1 m and x = 4 m respectively. The electric field (in V/m) at a point y = 3 m on y-axis is -

$\left[\text{take} \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 Nm^2C^{-2}\right]$

(A) $\left(-63\hat{i} + 27\hat{j}\right) \times 10^2$ (B) $\left(-81\hat{i} + 81\hat{j}\right) \times 10^2$
(C) $\left(81\hat{i} - 81\hat{j}\right) \times 10^2$ (D) $\left(63\hat{i} - 27\hat{j}\right) \times 10^2$

Sol. **D**



Let $\vec{E}_1$ & $\vec{E}_2$ are the vaues of electric field due to q_1 & q_2 respectively magnitude of

$E_2 = \frac{1}{4\pi e_0} \frac{q_2}{r^2}$

$E_2 = \frac{9 \times 10^9 \times (25) \times 10^{-6}}{(4^2 + 3^2)}$ V/m

$E_2 = 9 \times 10^3$ V/m

$\therefore\ \vec{E}_2 = 9\times10^3(\cos\theta_2\hat{i} - \sin\theta_2\hat{j})$

$\therefore\ \tan\theta_2 = \dfrac{3}{4}$

$\therefore\ \vec{E}_2 = 9\times10^3\left(\dfrac{4}{5}\hat{i} - \dfrac{3}{5}\hat{j}\right) = (72\hat{i} - 54\hat{j})\times10^2$

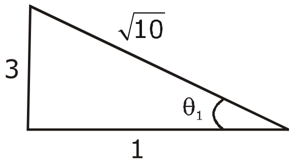
Magnitude of $E_1 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{\sqrt{10}\times10^{-6}}{(1^2+3^2)}$

$= (9 \times 10^9) \times \sqrt{10} \times 10^{-7}$

$= 9\sqrt{10} \times 10^2$

$\therefore\ \vec{E}_1 = 9\sqrt{10}\times10^2\left[\cos\theta_1(-\hat{i}) + \sin\theta_1\hat{j}\right]$

$\therefore\ \tan\theta_1 = 3$



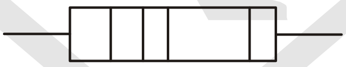
$E_1 = 9 \times \sqrt{10}\times10^2\left[\dfrac{1}{\sqrt{10}}(-\hat{i}) + \dfrac{3}{\sqrt{10}}\hat{j}\right]$

$E_1 = 9 \times 10^2\left[-\hat{i} + 3\hat{j}\right] = \left[-9\hat{i} + 27\hat{j}\right]10^2$

$\therefore\ \vec{E} = \vec{E}_1 + \vec{E}_2\ \ (63\hat{i} - 27\hat{j})\times10^2 \text{ v/m}$

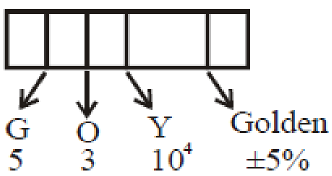
$\therefore$ correct answer is (D)

30. A carbon resistance has a following colour code. What is the value of the resistance ?



(A) 6.4 MΩ ± 5% (B) 5.3 MΩ ± 5% (C) 64 kΩ ± 10% (D) 530 kΩ ± 5%

Sol. **B**



$R = 53 \times 10^4 \pm 5\% = 530\ k\Omega \pm 5\%$