JEE MAIN JAN'19

QUESTION WITH SOLUTION

DATE : 10-01-2019 _ MORNING

[PHYSICS]

1. Using a nuclear counter the counter rate of emitted particles from a radioactive source is measured. At t = 0 it was 1600 counts per second and t = 8 seconds it was 100 counts per second. The count rate observed, as counts per second, at t = 6 seconds is close to -

(A) 360 (B) 150 (C) 400 (D) 200

Sol. **D**

at t = 0, $A_0 = \frac{dN}{dt}$ = 1600 C/s

at t = 8s, A = 100 C/s

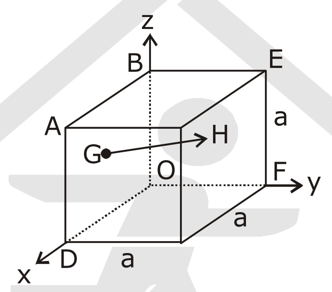
$\frac{A}{A_0} = \frac{1}{16}$ in 8 sec

Therefor half life is $t_{1/2}$ = 2 sec

$\therefore$ Activity at t = 6 will be $1600 \left(\frac{1}{2}\right)^3$

= 200 C/s

2. In the cube of side 'a' shown in the figure, the vector from the central point of the face ABOD to the central point of the face BEFO will be -



(A) $\frac{1}{2}a(\hat{j}-\hat{i})$ (B) $\frac{1}{2}a(\hat{i}-\hat{k})$ (C) $\frac{1}{2}a(\hat{j}-\hat{k})$ (D) $\frac{1}{2}a(\hat{k}-\hat{i})$

Sol. **A**

$\vec{r}_g = \frac{a}{2}\hat{i} + \frac{a}{2}\hat{k}$

$\vec{r}_H = \frac{a}{2}\hat{j} + \frac{a}{2}\hat{k}$

$\vec{r}_H - \vec{r}_g = \frac{a}{2}(\hat{j}-\hat{i})$

3. A string of length 1 m and mass 5 g is fixed at both ends. The tension in the string is 8.0 N. The string is set into vibration using an external vibrator of frequency 100 Hz. The separation between successive nodes on the string is close to -

(A) 20.0 cm (B) 16.6 cm (C) 10.0 cm (D) 33.3 cm

Sol. **A**

Velocity of wave on string

$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{8}{5}\times 1000}$ = 40 m/s

Now, wavelength of wave $\lambda = \frac{v}{n} = \frac{40}{100}$ m

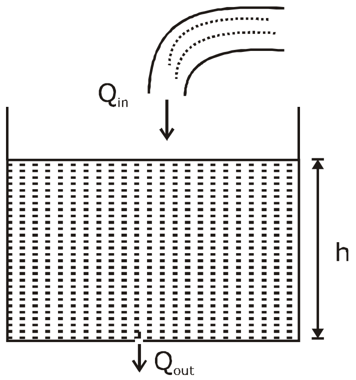
Separation b/w successive nodes, $\frac{\lambda}{2} = \frac{20}{100}$ m

= 20 cm

4. Water flows into a large tank with flat bottom at the rate of 10^{-4} m^3 s^{-1}. Water is also leaking out of a hole of area 1 cm^2 at its bottom. If the height of the water in the tank remains steady, then this height is -

(A) 2.9 cm (B) 5.1 cm (C) 4 cm (D) 1.7 cm

Sol. **B**



Since height of water column is constant therefore, water inflow rate (Q_{in})
= water outflow rate

$Q_{in} = 10^{-4}\ m^3s^{-1}$

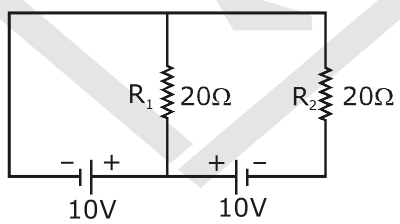
$Q_{out} = Au = 10^{-4} \times \sqrt{2gh}$

$10^{-4} = 10^{-4}\sqrt{20 \times h}$

$h = \frac{1}{20}$ m

h = 5 cm

5. In the given circuit the cells have zero internal resistance. The currents (in Amperes) passing through resistance R_1 and R_2 respectively, are -



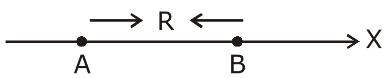
(A) 0.5, 0 (B) 2,2 (C) 0, 1 (D) 1, 2

Sol. **A**

$i_1 = \frac{10}{20} = 0.5$ A

$i_2 = 0$

6. Two electric dipoles, A,B with respective dipole moments $\vec{d}_A = -4qa\hat{i}$ and $\vec{d}_B = -2qa\hat{i}$ are placed on the x-axis with a separation R, as shown in the figrue.



The distance from A at which both of them produce the same potential is -

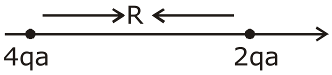
(A) $\frac{R}{\sqrt{2}+1}$ (B) $\frac{R}{\sqrt{2}-1}$ (C) $\frac{\sqrt{2}R}{\sqrt{2}-1}$ (D) $\frac{\sqrt{2}R}{\sqrt{2}+1}$

Sol. C

$$V = \frac{4qa}{(R+x)^2} = \frac{2qa}{(x^2)}$$

$$\sqrt{2}x = R + x$$

$$x = \frac{R}{\sqrt{2}-1}$$



$$\text{dist} = \frac{R}{\sqrt{2}-1} + R = \frac{\sqrt{2}R}{\sqrt{2}-1}$$

7. An insulating thin rod of length ℓ has a x linear charge density $\rho(x) = \rho_0 \frac{x}{\ell}$ on it. The rod is rotated about an axis passing through the origin (x = 0) and perpendicular to the rod. If the rod makes n rotations per second, then the time averaged magnetic moment of the rod is -

(A) $\pi n \rho \ell^3$ (B) $\frac{\pi}{3} n \rho \ell^3$ (C) $\frac{\pi}{4} n \rho \ell^3$ (D) $n \rho \ell^3$

sol. C

$\because M = NIA$

$dq = \lambda dx \ \& \ A = \pi x^2$

$$\int dm = \int (x) \frac{\rho_0 x}{\ell} dx . \pi x^2$$

$$M = \frac{n\rho_0\pi}{\ell} . \int_0^\ell x^3 . dx = \frac{n\rho_0\pi}{\ell} . \left[\frac{L^4}{4}\right]$$

$$M = \frac{n\rho_0\pi\ell^3}{4} \text{ or } \frac{\pi}{4} n\rho\ell^3$$

8. A plano convex lens of refractive index μ_1 and focal length f_1 is kept in contact with another plano concave lens of refarctive index μ_2 and focal length f_2. If the radius of curvature of their spherical faces is R each and $f_1 = 2f_2$, then μ_1 and μ_2 are related as -

(A) $\mu_1 + \mu_2 = 3$ (B) $3\mu_2 - 2\mu_1 = 1$ (C) $2\mu_2 - \mu_1 = 1$ (D) $2\mu_1 - \mu_2 = 1$

Sol. D

$$\frac{1}{2f_2} = \frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right)$$

$$\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right)$$

$$\frac{(\mu_1 - 1)}{R} = \frac{(\mu_2 - 1)}{2R}$$

$$2\mu_1 - \mu_2 = 1$$

9. A TV transmission tower has a height of 140 m and the height of the receiving antenna is 40 m. What is the maximum distance upto which signals can be broadcasted from this tower in LOS (Line of Sight) mode ? (Given : radius of earth = 6.4×10^6 m).

(A) 65 km (B) 40 km (C) 48 km (D) 80 km

Sol. **A**

Maximum distance upto which signal can be broadcasted is

$d_{max} = \sqrt{2gh_T} + \sqrt{2gh_R}$

where h_T and h_R are heights of transmitter tower and height of receiver respectively.

Putting all values -

$d_{max} = \sqrt{2 \times 6.4 \times 10^6}\left[\sqrt{140} + \sqrt{40}\right]$

on solving, d_{max} = 65 km

10. In an electron microscope, the resolution that can be achieved is of the order of the wavelength of electrons used. To resolve a width of 7.5 × 10^{-12} m, the minimum electron energy required is clsoe to -

(A) 25 keV (B) 500 keV (C) 100 keV (D) 1 keV

Sol. **A**

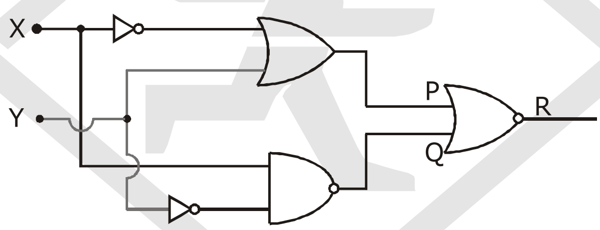
$\lambda = \frac{h}{p}$ $[\lambda = 7.5 \times 10^{-12}]$

$P = \frac{h}{\lambda}$

$KE = \frac{P^2}{2m} = \frac{(h/\lambda)^2}{2m} = \frac{\left\{\frac{6.6 \times 10^{-34}}{7.5 \times 10^{-12}}\right\}^2}{2 \times 9.1 \times 10^{-31}}$ J

KE = 25 Kev

11. To get output '1' at R, for the given logic gate circuit the input values must be -



(A) X = 0, Y = 1 (B) X = 1, Y = 0 (C) X = 0, Y = 0 (D) X = 1, Y = 1

Sol. **B**

$P = \overline{x} + y$

$Q = \overline{\overline{y}.x} = y + \overline{x}$

$O/P = \overline{P + Q}$

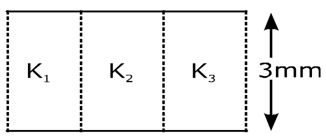
To make O/P

P + Q must be 'O'

So, y = 0

x = 1

12. A parallel plate capacitor is of area 6 cm^2 and a separation 3mm. The gap is filled with three dielectric materials of equal thickness (see figure) with dielectric constants K_1 = 10, K_2 = 12 and K_3 = 14. The dielectric constant of material which when fully inserted in above capacitor, gives same capacitance would be:



(A) 4 (B) 36 (C) 14 (D) 12

Sol. **D**

Let dielectric constant of material used be K.

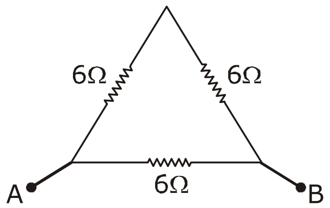
$$\therefore \frac{10 \in_0 A/3}{d} + \frac{12 \in_0 A/3}{d} + \frac{14 \in_0 A/3}{d} = \frac{K \in_0 A}{d}$$

$\Rightarrow K = 12$

13. A uniform metallic wire has a resistance of 18 Ω and is bent into an equilateral triangle. Then, the resistance between any two vertices of the triangle is :

(A) 4 Ω (B) 8 Ω (C) 12 Ω (D) 2 Ω

Sol. **A**



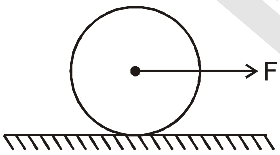
R_{eq} between any two vertex will be

$$\frac{1}{R_{eq}} = \frac{1}{12} + \frac{1}{6} \Rightarrow R_{eq} = 4\Omega$$

14. A homogeneous solid cylindrical roller of radius R and mass M is pulled on a cricket pitch by a horizontal force. Assuming rolling without slipping, angular acceleration of the cylinder is :

(A) $\frac{F}{2mR}$ (B) $\frac{F}{3mR}$ (C) $\frac{2F}{3mR}$ (D) $\frac{3F}{2mR}$

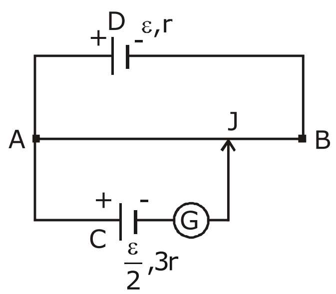
Sol. **C**



$$FR = \frac{3}{2} MR^2\alpha$$

$$\alpha = \frac{2F}{3MR}$$

15. A potentiometer wire AB having length L and resistance 12 r. is joined to a cell D of emf. ε and internal resistance r. A cell C having emf ε/2 and internal resistance 3r is connected. The length AJ at which the galvanometer as shown in fig. shows no deflection is.



(A) $\frac{13}{24}L$ (B) $\frac{5}{12}L$ (C) $\frac{11}{12}L$ (D) $\frac{11}{24}L$

Sol. **A**

$i = \frac{\varepsilon}{13r}$

$i\left(\frac{x}{L}12r\right) = \frac{\varepsilon}{2}$

$\frac{\varepsilon}{13r}\left[\frac{x}{L}.12r\right] = \frac{\varepsilon}{2} \Rightarrow x = \frac{13L}{24}$

16. A solid metal cube of edge length 2 cm is moving in a positive y-direction at a constant speed of 6 m/s. There is a uniform magnetic field of 0.1 T in the positive z-direction. The potential difference between the two faces of the cube perpendicular to the x-axis, is :

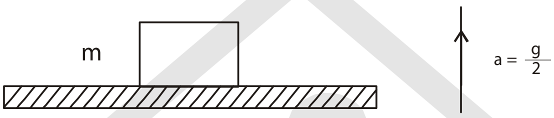
(A) 6 mV (B) 12 mV (C) 1 mV (D) 2 mV

Sol. **B**

Potential difference between two forces perpendicular to x-axis will be

$\ell.\ (\vec{V}\times\vec{B}) = 12$ mV

17 A block of mass m is kept on a platform which starts from rest with constant acceleration g/2 upward, as shown in fig. Work done by normal reaction on block in time t is :



(A) $\frac{m\ g^2t^2}{8}$ (B) $\frac{3mg^2t^2}{8}$ (C) $-\frac{mg^2t^2}{8}$ (D) 0

Sol. **B**

$N - mg = \frac{mg}{2} \Rightarrow N = \frac{3mg}{2}$

Now, work done $W = \vec{N}\vec{S} = \left(\frac{3mg}{2}\right)\left(\frac{1}{2}gt^2\right)$

$\Rightarrow W = \frac{3mg^2t^2}{4}$

18 A satellite is moving with a constant speed υ in circular orbit around the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of ejection, the kinetic energy of the object is :

(A) $2m\upsilon^2$ (B) $\frac{3}{2}m\upsilon^2$ (C) $m\upsilon^2$ (D) $\frac{1}{2}m\upsilon^2$

Sol. **C**

At height r from centre of earth. orbital velocity $= \sqrt{\frac{GM}{r}}$

$\therefore$ By energy conservation

KE of 'm' $+ \left(-\frac{GMm}{r}\right) = 0 + 0$

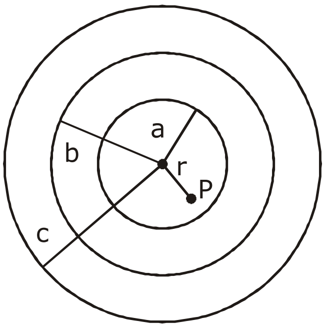
(At infinity, PE = KE = 0)

$\Rightarrow$ KE of 'm' $= \frac{GMm}{r} = \left(\sqrt{\frac{GM}{r}}\right)^2 m = mv^2$

19 A charge Q is distributed over three concentric spherical shells of radii a, b, c (a < b < c) such that their surface charge densities are equal to one another. The total potential at a point at distance r from their common centre, where r < a, would be :

(A) $\frac{Q}{4\pi\varepsilon_0(a+b+c)}$ (B) $\frac{Q(a+b+c)}{4\pi\varepsilon_0(a^2+b^2+c^2)}$ (C) $\frac{Q}{12\pi\varepsilon_0}\frac{ab+bc+ca}{abc}$ (D) $\frac{Q(a^2+b^2+c^2)}{4\pi\varepsilon_0(a^3+b^3+c^3)}$

Sol. B



Potential at point P, V = $\frac{kQ_a}{a} + \frac{kQ_b}{a} + \frac{kQ_c}{a}$

$\because Q_a : Q_b : Q_c :: a^2 : b^2 : c^2$

[since $\sigma_a = \sigma_b = \sigma_c$]

$\therefore Q_a = \left[\frac{a^2}{a^2+b^2+c^2}\right] Q$

$Q_b = \left[\frac{b^2}{a^2+b^2+c^2}\right] Q$

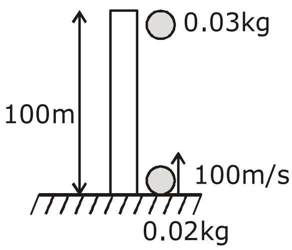
$Q_c = \left[\frac{c^2}{a^2+b^2+c^2}\right] Q$

$V = \frac{Q}{4\pi\epsilon_0}\left[\frac{(a+b+c)}{a^2+b^2+c^2}\right]$

20 A piece of wood of mass 0.03 kg is dropped from the top of a 100 m height building. At the same time, a bullet of a mass 0.02 kg is fired vertically upward, with a velocity 100 ms^{-1}, from the ground. The bullet gets embadded in the wood. Then the maximum height to which the combined system reaches above the top of the building before falling below is : (g = 10ms^{-2})

(A) 30 m (B) 10 m (C) 40 m (D) 20 m

Sol. C



Time taken for the particle to collide,

$t = \frac{d}{V_{rel}} = \frac{100}{100} = 1$ sec

Speed of wood just before collision = gt = 10 m/s

& speed of bullet just before collision v=u-gt

= 100 – 10 = 90 m/s

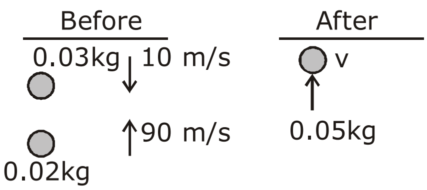
Now, conservation of linear momentum just before and after the collision -

–(0.02) (1) + (0.02) (90) = (0.05)v

$\Rightarrow$ 150 = 5 v

$\Rightarrow$ v = 30 m/s

Max. height reached by body h = $\frac{v^2}{2g}$



h = $\frac{30 \times 30}{2 \times 10}$ = 45 m

$\therefore$ Height above tower = 40 m

21. The density of a material in SI units is 128 kg m^{-3}. In certain units in which the unit of length is 25 cm and the unit of mass is 50 g, the numerical value of density of the material is :

(A) 40 (B) 410 (C) 16 (D) 640

Sol. **A**

$\frac{128\text{kg}}{\text{m}^3} = \frac{125(50\text{g})(20)}{(25\text{cm})^3(4)^3}$

= $\frac{128}{64}$ (20) units

= 40 units

22. A magnet of total magnetic moment $10^{-2}\hat{i}$ A-m^2 is placed in a time varying magnetic field, $B\hat{i}(\cos\omega t)$ where B=1 Tesla and $\omega = 0.125$ rad/s. The work done for reversing the direction of the magnetic moment at t = 1 second is :

(A) 0.01 J (B) 0.028 J (C) 0.007 J (D) 0.014 J

Sol. **D**

Work done, W = $(\Delta\vec{\mu}).\vec{B}$

= 2 × 10^{-2} × 1 cos (0.125)

= 0.02 J $\simeq$ 0.014 J (due to most close option available.)

23. In a Young's double slit experiment with slit separation 0.1 mm, one observes a bright fringe at angle $\frac{1}{40}$ rad by using light of wavelength λ_1. When the light of wavelength λ_2 is used a bright fringe is seen at the same angle in the same set up. Given that λ_1 and λ_2 are in visible range (380 nm to 740 nm), their values are :

(A) 625nm, 500 nm (B) 380 nm, 500 nm (C) 400 nm, 500 nm (D) 380 nm, 525 nm

Sol. **A**

Path difference = d sin $\theta \approx d\theta$

= 0.1 × $\frac{1}{40}$ = 2500 nm

or bright fringe, path difference must be integral multiple of λ.

$\therefore$ 2500 = nλ_1 = mλ_2

$\therefore$ λ_1 = 625, λ_2 = 500 (from m = 5)

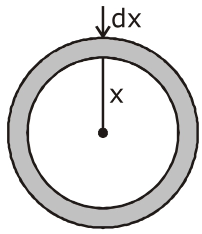
(for n = 4)

24. To mop-clean a floor, a cleaning machine presses a circular mop of radius R vertically down with a total force F and rotates it with a constant angular speed about its axis. If the force F is distributed uniformly over the mop and if coefficient of friction between the mop and the floor is μ, the torque, applied by the machine on the mop is :

(A) $\mu FR/3$ (B) $\mu FR/6$ (C) $\mu FR/2$ (D) $\frac{2}{3}\mu FR$

Sol. **D**

Consider a strip of radius x & thickness dx, Torque due to friction on this strip.



$$\int d\tau = \int_0^R \frac{x\mu F.2\pi x dx}{\pi R^2}$$

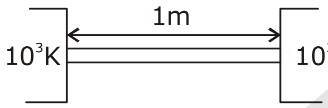
$$\tau = \frac{2\mu F}{R^2} \cdot \frac{R^3}{3}$$

$$\tau = \frac{2\mu FR}{3}$$

25. A heat source at T = 10^3 K is connected to another heat reservoir at T = 10^2 K by a copper slab which is 1 m thick, Given that the thermal conductivity of copper is 0.1 $WK^{-1}m^{-1}$, the energy flux through it in the steady state is :

(A) 65 Wm^{-2} (B) 200 Wm^{-2} (C) 90 Wm^{-2} (D) 120 Wm^{-2}

Sol. **C**



$$\left(\frac{dQ}{dt}\right) = \frac{kA\Delta T}{\ell}$$

$$\Rightarrow \frac{1}{A}\left(\frac{dQ}{dt}\right) = \frac{(0.1)(900)}{1} = 90\ W/m^2$$

26. If the magnetic field of a plane electromagnetic wave is given by (The speed of light = 3×10^8 m/s)

$B = 100 \times 10^{-6} \sin\left[2\pi \times 2 \times 10^{15}\left(t - \frac{x}{C}\right)\right]$ Then the maximum electric field associated with it is :

(A) 6×10^4 N/C (B) 4.5×10^4 N/C (C) 4×10^4 N/C (D) 3×10^4 N/C

Sol. **D**

$E_0 = B_0 \times C$

$= 100 \times 10^{-6} \times 3 \times 10^8$

$= 3 \times 10^4$ N/C

27. Two guns A and B can fire bullets at speeds 1 km/s and 2 km/s respectively. From a point on a horizontal ground, they are fired in all possible directions. The ratio of maximum areas covered by the bullets fired by the two guns, on the ground is :

(A) 1 : 8 (B) 1 : 16 (C) 1 : 4 (D) 1 : 2

Sol. **D**

$$R = \frac{u^2 \sin 2\theta}{g}$$

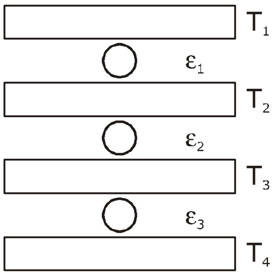
$A = \pi R^2$

$A \propto R^2$

$A \propto u^4$

$$\frac{A_1}{A_2} = \frac{u_1^4}{u_2^4} = \left[\frac{1}{2}\right]^4 = \frac{1}{16}$$

28. Three Carnot engines operate in series between a heat source at a temperature T_1 and a heat sink at temperature T_4 (see figure) There are two other reservoirs at temperature T_2 and T_3, as shown, with $T_1>T_2>T_3>T_4$. The three engines are equally efficient if :



(A) $T_2 = (T_1T_4)^{1/2};\ T_3 = (T_1^2T_4)^{1/3}$
(B) $T_2 = (T_1^2T_4)^{1/3};\ T_3 = (T_1T_4^2)^{1/3}$
(C) $T_2 = (T_1^3T_4)^{1/4};\ T_3 = (T_1T_4^3)^{1/4}$
(D) $T_2 = (T_1T_4^2)^{1/3};\ T_3 = (T_1^2T_4)^{1/3}$

Sol. **B**

$= 1 - \frac{T_2}{T_1} = 1 - \frac{T_3}{T_2} = 1 - \frac{T_4}{T_3}$

$\Rightarrow \frac{T_2}{T_1} = \frac{T_3}{T_4} = \frac{T_4}{T_3}$

$\Rightarrow T_2 = \sqrt{T_1T_3} = \sqrt{T_1\sqrt{T_2T_4}}$

$T_3 = \sqrt{T_2T_4}$

$T_2^{3/4} = \sqrt{T_1^{1/2}T_4^{1/4}}$

$T_2 = T_1^{2/3}\,T_4^{1/3}$

29. A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the speed of the train is reduced to 17 m/s, the frequency registered is f_2. If speed of sound is 340 m/s, then the ratio f_1/f_2 is :

(A) 21/20 (B) 19/18 (C) 18/17 (D) 20/19

Sol. **B**

$f_{app} = f_0\left[\frac{v_2 \pm v_0}{v_2 \mp v_s}\right]$

$f_1 = f_0\left[\frac{340}{340-34}\right]$

$f_2 = f_0\left[\frac{340}{340-17}\right]$

$\frac{f_1}{f_2} = \frac{340-17}{340-34} = \frac{323}{306} \Rightarrow \frac{f_1}{f_2} = \frac{19}{18}$

30. A 2 W carbon resistor is color coded with green black, red and brown respectively. The maximum current which can be passed through this resistor is :

(A) 20 mA (B) 0.4 mA (C) 63 mA (D) 100 mA

Sol. **A**

$P = i^2R$

$\therefore$ for i_{max}, R must be minimum

from color coding $R = 50 \times 10^2\Omega$

$\therefore i_{max} = 20$ mA