JEE MAIN JAN'19

QUESTION WITH SOLUTION

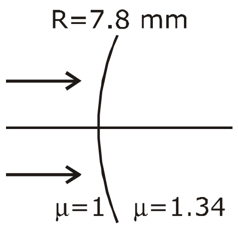
DATE : 10-01-2019 _ EVENING

[PHYSICS]

1. The eye can be regarded as a single refracting surface. The radius of curvature of this surface is equal to that of cornea (7.8 mm). This surface separates two media of refractive indices 1 and 1.34. Calculate the distance from the refracting surface at which a parallel beam of light will come to focus.

(A) 3.1 cm (B) 1 cm (C) 4.0 cm (D) 2 cm

Sol. A



$$\frac{1.34}{V} - \frac{1}{\infty} = \frac{1.34 - 1}{7.8}$$

$\therefore$ V = 30.7 mm

2. The modulation frequency of an AM radio station is 250 kHz, which is 10% of the carrier wave. If another AM station approaches you for license what broadcast frequency will you allot ?

(A) 2250 kHz (B) 2900 KHz (C) 2750 kHz (D) 2000 kHz

Sol. D

$$f_{carrier} = \frac{250}{0.1} = 2500KHZ$$

$\therefore$ Range of signal = 2250 KHz to 2750 KHz

Now check all options : for 2000 KHZ

f_{mod} = 200 KHz

$\therefore$ Range = 1800 KHZ to 2200 KHZ

3. A particle executes simple harmonic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity is SI units is equal to that of its acceleration. Then, its periodic time in second is -

(A) $\frac{3}{8}\pi$ (B) $\frac{7}{3}\pi$ (C) $\frac{4\pi}{3}$ (D) $\frac{8\pi}{3}$

Sol. D

$v = \omega\sqrt{A^2 - x^2}$ .....(1)

$a = -\omega^2 x$ .....(2)

$|V| = |a|$ .....(3)

$$\omega\sqrt{A^2 - x^2} = \omega^2 x$$

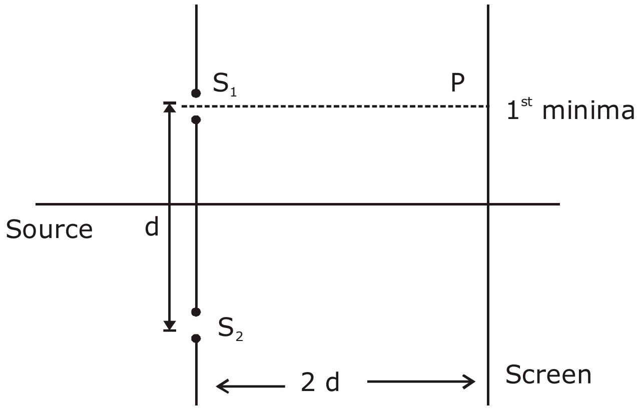
$$A^2 - x^2 = \omega^2 x^2$$

$$5^2 - 4^2 = \omega^2 (4^2)$$

$$\Rightarrow 3 = \omega \times 4$$

T = $2\pi/\omega$

4. Consider a Young's double slit experiment as shown in figure. what should be the slit separation d in terms of wavelength λ such that the first minima occurs directly in front of the slit (S_1) ?



(A) $\frac{\lambda}{(5-\sqrt{2})}$ (B) $\frac{\lambda}{2(5-\sqrt{2})}$ (C) $\frac{\lambda}{2(\sqrt{5}-2)}$ (D) $\frac{\lambda}{(\sqrt{5}-2)}$

Sol. **C**

$$\sqrt{5}d - 2d = \frac{\lambda}{2}$$

5. A cylindrical plastic bottle of negligible mass is filled with 310 ml of water and left floating in a pond with still water. If pressed downward slightly and released, it starts performing simple harmonic motion at angular frequency ω. if the radius of the bottle is 2.5 m then ω is close to : (density of water = 10^3 kg/m^3)

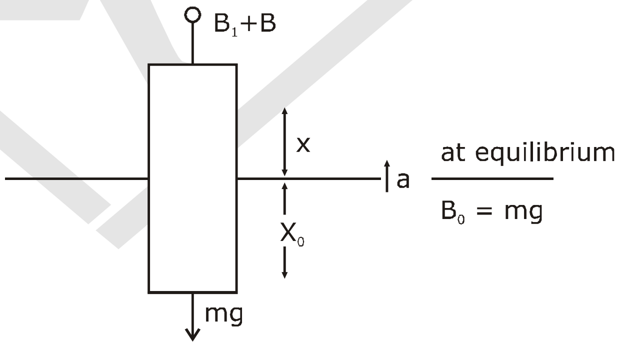
(A) 2.50 rad s^{-1} (B) 1.25 rad s^{-1} (C) 3.75 rad s^{-1} (D) 5.00 rad s^{-1}

Sol. **Bonus**

Extra Boyant force = δAxg

$B_0 + B \times mg = ma$

B = ma



$$a = \left(\frac{\delta Ag}{m}\right)^x$$

$$w^2 = \frac{\delta Ag}{m}$$

$$w = \sqrt{\frac{10^3 \times \pi(2.5)^2 \times 10^{-4} \times 10}{310 \times 10^{-6} \times 10^3}}$$

$$= \sqrt{63.30} = 7.95$$

6. Two kg of a monoatomic gas is at a pressure of 4×10^4 N/m^2. The density of the gas is 8 kg/m^3. What is the order of energy of the gas due to its thermal motion ?

(A) 10^3 J (B) 10^4 J (C) 10^6 J (D) 10^5 J

Sol. **B**

Thermal energy of N molecule $= N\left(\frac{3}{2}kT\right)$

$$= \frac{N}{N_A}\frac{3}{2}RT = \frac{3}{2}(nRT)$$

$=\frac{3}{2}PV = \frac{3}{2}P\left(\frac{m}{8}\right)$

$= \frac{3}{2} \times 4 \times 10^4 \times \frac{2}{8} = 1.5\times10^4$

7. A hoop and a solid cylinder of same mass and radius are made of a permanent magnetic material with their magnetic moment parallel to their respective axes. But the magnetic moment of hoop is twice of solid cylinder. They are placed in a uniform magnetic field in such a manner that their magnetic moments make a small angle with the field. If the oscillation periods of hoop and cylinder are T_h and T_c respectively, then :

(A) $T_h = 1.5\,T_C$ (B) $T_h = T_C$

(C) $T_h = 0.5\,T_c$ (D) $T_h = 2\,T_c$

Sol. **B**

$T = 2\pi\sqrt{\frac{I}{\mu B}}$

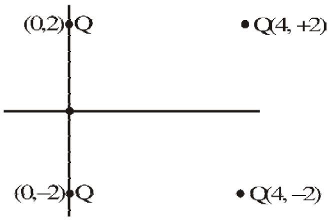
$T_h = 2\pi\sqrt{\frac{mR^2}{(2\mu)B}}$

$T_C = 2\pi\sqrt{\frac{1/2mR^2}{\mu B}}$

8. Four equal point charges Q each are placed in the xy plane at (0, 2), (4, 2), (4, –2) and (0, –2). The work required to put a fifth charge Q at the origin of the coordinate system will be :

(A) $\frac{Q^2}{4\pi\varepsilon_0}$ (B) $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{3}}\right)$ (C) $\frac{Q^2}{2\sqrt{2}\,\pi\varepsilon_0}$ (D) $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{5}}\right)$

Sol. **D**



Potential at origin = $\frac{KQ}{2}+\frac{KQ}{2}+\frac{KQ}{\sqrt{20}}+\frac{KQ}{\sqrt{20}}$

(Potential at $\infty = 0$)

$= KQ\left(1+\frac{1}{\sqrt{5}}\right)$

$\therefore$ Work required to put a fifth charge Q at origin is equal to $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{5}}\right)$

9. The self induced emf of a coil is 25 volts. When the current in it is changed at uniform rate from 10 A to 25 A in 1 s, the change in the energy of the inductance is :

(A) 740 J (B) 637.5 J (C) 437.5 J (D) 540 J

Sol. **C**

$$L\frac{di}{dt} = 25$$

$$L \times \frac{15}{1} = 25$$

$$L = \frac{5}{3}H$$

$$\Delta E = \frac{1}{2} \times \frac{5}{3} \times (25^2 - 10^2) = \frac{5}{6} \times 525 = 437.5J$$

10. A parallel plate capacitor having capacitance 12 pF is charged by a battery to a potential difference of 10 V between its plates. The charging battery is now disconnected and a porcelain slab of dielectric constant 6.5 is slipped between the plates. The work done by the capacitor on the slab is :

(A) 560 pJ (B) 692 pJ (C) 508 pJ (D) 600 pJ

Sol. **C**

Initial energy of capacitor

$$U_i = \frac{1}{2}\frac{v^2}{c}$$

$$= \frac{1}{2} \times \frac{120 \times 120}{12} = 600J$$

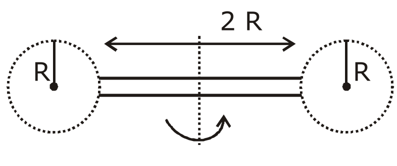
Since battery is disconnected so charge remain same.
final charge of capacitor,

$$U_r = \frac{1}{2}\frac{v^2}{c}$$

$$= \frac{1}{2} \times \frac{120 \times 120}{12 \times 6.5} = 92$$

$W + U_f = U_i$
$W = 508$ J

11. Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length 2R and mass M (see figure) The moment of inertia of the system about the axis passing perpendicularly thorugh the centre of the rod is :



(A) $\frac{137}{15}MR^2$ (B) $\frac{152}{15}MR^2$ (C) $\frac{209}{15}MR^2$ (D) $\frac{17}{15}MR^2$

Sol. **A**

For ball

using parallel axis theorem.

$$I_{ball} = \frac{2}{5}MR^2 + M(2R)^2 = \frac{22}{5}MR^2$$

2 Balls so $\frac{44}{5}MR^2$

Irod = for rod $\frac{M(2R)^2}{R} = \frac{MR^2}{3}$

$I_{system} = I_{Ball} + _{rod}$

$$= \frac{44}{5}MR^2 + \frac{MR^2}{3} = \frac{137}{15}MR^2$$

12. The diameter and height of a cylinder are measured by a meter scale to be 12.6 ± 0.1 cm and 34.2 ± 0.1 cm, respectively. What will be the value of its volume in appropriate significant figures ?

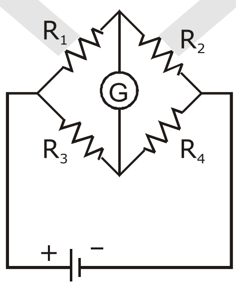
(A) 4300 ± 80 cm^3 (B) 4260 ± 80 cm^3 (C) 4264 ± 81 cm^3 (D) 4264.4 ± 81.0 cm^3

Sol. **B**

$$\frac{\Delta V}{V} = 2\frac{\Delta d}{d} + \frac{\Delta h}{h} = 2\left(\frac{0.1}{12.6}\right) + \frac{0.1}{34.2}$$

$$V = 12.6 \times \frac{\pi}{4} \times 314.2$$

13. The Wheatstone bridge shown in fig. here gets balanced when the carbon resistor used as R_1 has the colour code (orange, Red, Brown). The resistors R_2 and R_4 are 80 Ω and 40 Ω, respectively. Assuming that the colour code for the carbon resistors given their accurate values, the colour code for the carbon resistor, used as R_3, would be :



(A) Grey, Black, Brown (B) Brown, Blue, Brown

(C) Red, Green, Brown (D) Brown, Blue, Black

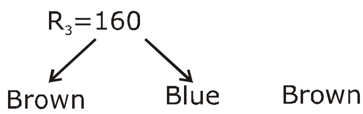
Sol. **B**

$R_1 = 32 \times 10 = 320$

for wheat stone bridge

$$\Rightarrow \frac{R_1}{R_3} = \frac{R_2}{R_4}$$

$$\frac{320}{R_3} = \frac{80}{40}$$

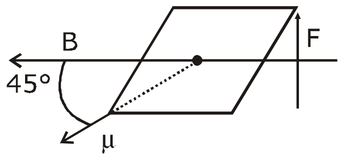


14. At some location on earth the horizontal component of earth's magnetic field is 18×10^{-6} T. At this location, magnetic needle of length 0.12 m and pole strength 1.8 Am is suspended from its mid-point using a thread, it makes 45° angle with horizontal in equilibrium. To keep this needle horizontal, the vertical force that should be applied at one of its ends is :

(A) 6.5×10^{-5} N (B) 1.8×10^{-5} N

(C) 3.6×10^{-5} N (D) 1.3×10^{-5} N

Sol. **A**



$\mu B \sin 45^\circ = F \frac{\ell}{2} \sin 45^\circ$

$F = 2g\mu B$

15. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. The magnitude of $(\vec{A}+\vec{B})$ is 'n' times the magnitude of $(\vec{A}-\vec{B})$. The angle between $\vec{A}$ and $\vec{B}$ is :

(A) $\cos^{-1}\left[\frac{n-1}{n+1}\right]$ (B) $\sin^{-1}\left[\frac{n-1}{n+1}\right]$ (C) $\sin^{-1}\left[\frac{n^2-1}{n^2+1}\right]$ (D) $\cos^{-1}\left[\frac{n^2-1}{n^2+1}\right]$

Sol. **D**

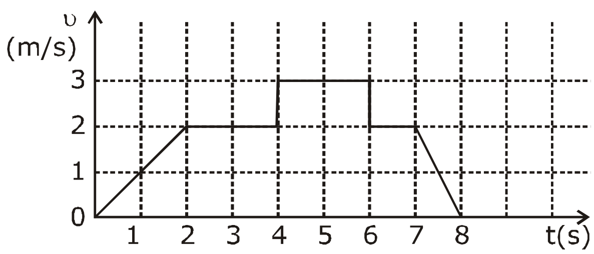
$\left|\vec{A}+\vec{B}\right| = 2a\cos\theta/2$ ....(1)

$\left|\vec{A}-\vec{B}\right| = 2a\cos\frac{(\pi-\theta)}{2} = 2a\sin\theta/2$ ....(2)

$\Rightarrow n\left(2a\cos\frac{\theta}{2}\right) = 2a\frac{\sin\theta}{2}$

$\Rightarrow \tan\frac{\theta}{2} = n$

16. A particle starts from the origin at time t = 0 and moves along the positive x - axis. The graph of velocity with respect to time is shown in figure. What is the position of the particle at time t = 5 s ?



(A) 10 m (B) 3 m (C) 9 m (D) 6 m

Sol. **C**

S = Area under graph

$\frac{1}{2} \times 2 \times 2 + 2 \times 2 + 3 \times 1 = 9\text{m}$

17. A current of 2 mA was passed through an unknown resistor which dissipated a power of 4.4 W. Dissipated power when an ideal power supply of 11 V is connected across it is :

(A) 11×10^{-5} W (B) 11×10^{-4} W (D) 11×10^{-3} W (D) 11×10^{5} W

Sol. **A**

$P = I^2R$

$4.4 = 4 \times 10^{-6}$ R

$R = 1.1 \times 10^6\ \Omega$

$P' = \frac{11^2}{R} = \frac{11^2}{1.1} \times 10^{-6} = 11 \times 10^{-5}\text{W}$

18. A particle which is experiencing a force, given by $\vec{F} = 3\vec{i} - 12\vec{j}$, undergoes a displacement $\vec{d} = 4\,\vec{i}$.

If the particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy at the end of the displacement ?

(A) 10 J (B) 12 J (C) 15 J (D) 9 J

Sol. **C**

Work done $= \vec{F}.\vec{d}$

$= 12\text{J}$

work energy theorem

$w_{net} = \Delta K.E.$

$12 = K_f - 3$

$K_f = 15\text{J}$

19. A closed organ pipe has a fundamental frequency of 1.5 kHz. The number of overtones that can be distinctly heard by a person with this organ pipe will be : (Assume that the highest frequency a person can hear is 20,000 Hz)

(A) 5 (B) 7 (C) 4 (D) 6

Sol. **B**

For closed organ pipe, resonate frequency is odd multiple of fundamental frequency.

$\therefore$ $(2n + 1)\, f_0 \leq 20{,}000$

(f_0 is fundamental frequency = 1.5 KHz)

$\therefore$ n = 6

$\therefore$ Total number of overtone that can be heared is 7. (0 to 6).

20. Two forces P and Q, of magnitude 2F and 3F, respectively, are at angle θ with each other. If the force Q is doubled, then their resultant also gets doubled. Then, the angle θ is :

(A) 60° (B) 90° (C) 120° (D) 30°

Sol. **C**

$4F^2 + 9F^2 + 12\, F^2 \cos\theta = R^2$

$4F^2 + 36F^2 + 24\, F^2 \cos\theta = 4R^2$

$4F^2 + 36F^2 + 24\, F^2 \cos\theta$

$=4(13F^2 + 12F^2\cos\theta) = 52\, F^2 + 48F^2\cos\theta$

$\cos\theta = -\frac{12F^2}{24F^2} = -\frac{1}{2}$

21. A metal plate of area 1×10^{-4} m^2 is illuminated by a radiation fo intensity 16 mW/m^2. The work function of the metal is 5 eV. The energy of the incident photons is 10 eV and only 10 % of it produces photo electrons. The number of emitted photo electrons per second and their maximum energy, respectively, will be :

[1 eV = 1.6×10^{-19} J]

(A) 10^{11} and 5 eV (B) 10^{10} and 5 eV

(C) 10^{14} and 10 eV (D) 10^{12} and 5 eV

Sol. **A**

$$I = \frac{nE}{At}$$

$$16 \times 10^{-3} = \left(\frac{n}{t}\right)_{photon} \frac{10 \times 1.6 \times 10^{-19}}{10^{-4}} = 10^{12}$$

22. Two stars of masses 3×10^{31} kg each and at distance 2×10^{11} m rotate in a plane about their common centre of mass O. A meteorite passes through O moving perpendicular to the star's rotation plane. In order to escape from the gravitational field of this double star, the minimum speed that meteorite should have at O is :

(Take Gravitational constant)

G = 6.67×10^{-11} Nm^2 Kg^{-2})

(A) 3.8×10^4 m/s (B) 2.8×10^5 m/s

(C) 1.4×10^5 m/s (D) 2.4×10^4 m/s

Sol. **B**

By energy convervation between 0 & ∞.

$$-\frac{GMm}{r} + \frac{-GMm}{r} + \frac{1}{2}mV^2 = 0 + 0$$

[M is mass of star m is mass of meteroite]

$$\Rightarrow V = \sqrt{\frac{4GM}{r}} = 2.8 \times 10^5 m/s$$

23. Consider the nuclear fission $Ne^{20} \rightarrow 2He^4 + C^{12}$

Given that the binding energy / nucleon of Ne^{20}, He^4 and C^{12} are, respectively, 8.03 MeV, 7.07 MeV and 7.86 MeV, identify the correct statement :

(A) energy of 12.4 MeV will be supplied

(B) energy of 3.6 MeV will be released

(C) energy of 11.9 MeV has to be supplied

(D) 8.3 MeV energy will be released

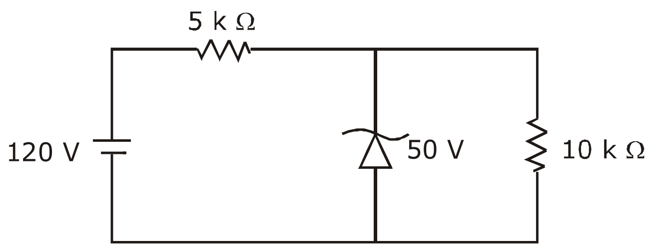
Sol. **C**

Ne^{20} $\rightarrow 2He^4 + C^{12}$

8.03×20 $2 \times 7.07 \times 4 + 7.86 \times 12$

$\therefore$ $E_B = (BE)_{react} - (BE)_{product} = 9.72$ MeV

24. For the circuit shown below, the current through the Zener diode is :

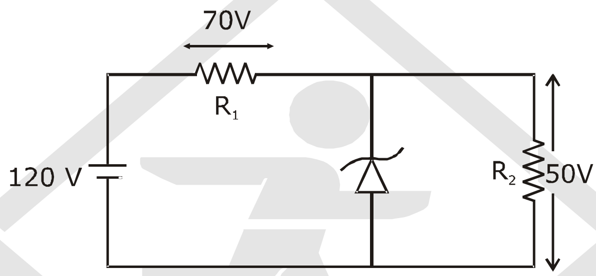


(A) 9 mA (B) 14 mA (C) Zero (D) 5 mA

Sol. **A**

Assuming zener diode doesnot undergo breakdown, current in circuit $= \frac{120}{15000} = 8\text{mA}$

$\therefore$ Voltage drop across diode = 80 V > 50 V.
The diode undergo breakdown.

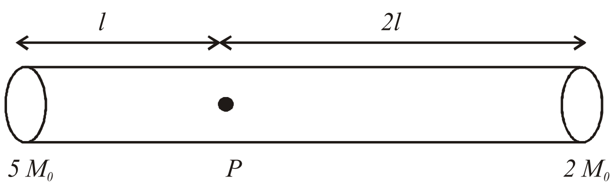


Current is $R_1 = \frac{70}{5000} = 14\text{mA}$

Current is $R_2 = \frac{50}{10000} = 5\text{mA}$

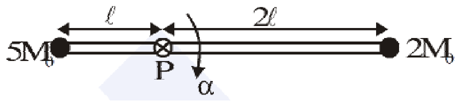
$\therefore$ Current through diode = 9mA

25. A rigid massless rod of length $3l$ has two masses attached at each end as shown in the figure. The rod is pivoted at point P on the horizontal axis (see figure). When released from initial horizontal position, its instantaneous angular acceleration will be :



(A) $\frac{g}{2\ell}$ (B) $\frac{7g}{3\ell}$ (C) $\frac{g}{3\ell}$ (D) $\frac{g}{13\ell}$

Sol. **D**



Applying torque equation about point P.

$2M_0 (2l) - 5 M_0 gl = I\alpha$

$I = 2 M_0 (2l)^2 + 5M_0 l^2 = 13 M_0 l^2 d$

$\therefore \quad \alpha = -\dfrac{M_0 g\ell}{13M_0\ell^2} \Rightarrow \alpha = -\dfrac{g}{13\ell}$

$\therefore \quad \alpha = \dfrac{g}{13\ell}$ anticlockwise

26. The electric field of a plane polarized electromagnetic wave in free space at time t = 0 is given by an expression

$\vec{E}(x, y) = 10\hat{j}\cos[(6x + 8z)]$

The magnetic field $\vec{B}(x, z, t)$ is given by : (c is the velocity of light)

(A) $\dfrac{1}{c}(6\hat{k} + 8\hat{i})\cos[(6x + 8z - 10ct)]$

(B) $\dfrac{1}{c}(6\hat{k} - 8\hat{i})\cos[(6x + 8z - 10ct)]$

(C) $\dfrac{1}{c}(6\hat{k} + 8\hat{i})\cos[(6x - 8z + 10ct)]$

(D) $\dfrac{1}{c}(6\hat{k} - 8\hat{i})\cos[(6x + 8z + 10ct)]$

Sol. **B**

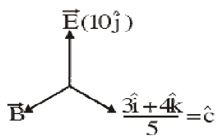
$\vec{E} = 10\hat{j}\cos\left[\left(6\hat{i} + 8\hat{k}\right).\left(x\hat{i} + z\hat{k}\right)\right]$

$= 10\hat{j}\cos[\vec{K}.\vec{r}]$

$\therefore \quad \hat{K} = 6\hat{i} + 8\hat{k};$ direction of waves travel.

i.e., direction of 'c'

$\therefore$ Direction of $\hat{B}$ will be along



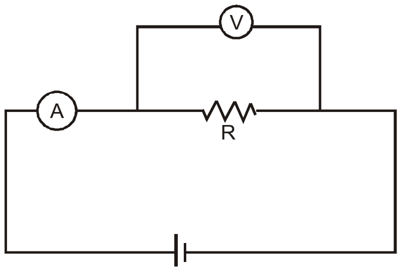
$\vec{C} \times \hat{E} = \dfrac{-4\hat{i} + 3\hat{k}}{5}$

Mag. of $\vec{B}$ will be along $\vec{C} \times \vec{E} = \dfrac{-4\hat{i} + 3\hat{k}}{5}$

Mag. of $\vec{B} = \dfrac{E}{C} = \dfrac{10}{C}$

$\therefore \vec{B} = \dfrac{10}{C}\left(\dfrac{-4\hat{i} + 3\hat{k}}{5}\right) = \dfrac{\left(-8\hat{i} + 6\hat{k}\right)}{C}$

27. The actual value of resistance R, shown in the figure is 30 Ω. This is measured in an experiment as shwon using the standard formula $R = \frac{V}{I}$, wehre V and I are the readings of the voltmeter and ammeter, respectively. If the measured value of R is 5 % less, then the internal resistance of the voltmeter is :



(A) 600 Ω (B) 350 Ω (C) 570 Ω (D) 35 Ω

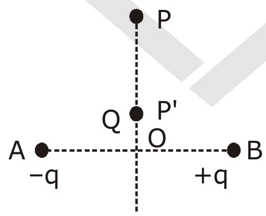
Sol. **C**

$$0.95R = \frac{RR_v}{R + R_v}$$

$0.95 \times 30 = 0.05\ R_v$

$R_V = 19 \times 30 = 570\ \Omega$

28. Charges – q and + q located at A and B, respectively, constitute an electric dipole. Distance AB = 2a, O is the mid point of the dipole and OP is perpendicular to AB. A charge Q is placed at P where OP = y and y >> 2a. The charge Q experiences an electrostatic force F. If Q is now moved along the equatorial line to P' such that OP' = $\left(\frac{y}{3}\right)$, the force on Q will be close to : $\left(\frac{y}{3} >> 2a\right)$



(A) $\frac{F}{3}$ (B) 27 F (C) 9F (D) 3 F

Sol. **B**

Electric field of equitorial plane of dipole

$$= -\frac{K\vec{P}}{r^3}$$

$$\therefore \text{At P, } F = -\frac{K\vec{P}}{r^3}Q$$

$$\text{At } P^1,\ F^1 = -\frac{K\vec{P}Q}{(r/3)^3} = 27F$$

29. An unknown metal of mass 192 g heated to a temperature of 100° C was immersed into a brass calorimeter of mass 128 g containing 240 g of water at a temperature of 8.4 °C. Calculate the specific heat of the unknown metal if water temperature stablizes at 21.5 °C. (Specific heat of brass is 394 J kg^{-1} K^{-1})

(A) 458 J Kg^{-1} K^{-1}

(B) 1232 J Kg^{-1} K^{-1}

(C) 916 J kg^{-1} K^{-1}

(D) 654 J kg^{-1} K^{-1}

Sol. **C**

$192 \times S \times (100 - 21.5)$

$= 128 \times 394 \times (21.5 - 8.4) + 240 \times 4200 \times (21.5 - 8.4)$

$\Rightarrow \quad S = 916$

30. Half mole of an ideal monoatomic gas is heated at constant pressure of 1 atm from 20°C to 90°C. Work done by gas is close to : (Gas constant R = 8.31 J / Mole - K)

(A) 291 J (B) 581 J (C) 146 J (D) 73 J

Sol. **A**

$WD = P\Delta V = nR\Delta T = \frac{1}{2} \times 8.31 \times 70$