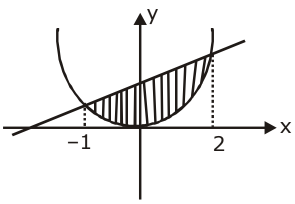

# QUESTION WITH SOLUTION

DATE : 11-01-2019 _ MORNING

**1.** The area (in sq. units) of the region bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$ is :

(A) $\frac{5}{4}$ (B) $\frac{7}{8}$ (C) $\frac{3}{4}$ (D) $\frac{9}{8}$

**Sol.** **D**




$x^2 = 4y$ ...(1)

$x = 4y - 2$

Solve (1) & (2)

$x^2 = 4\left(\frac{x+2}{4}\right)$

$x^2 = x + 2$

$x^2 - x - 2 = 0$

$x = -1, x = 2$

Bounded area is

$$A = \int_{-1}^{2}\left(\frac{x+2}{4} - \frac{x^2}{4}\right)dx$$

$$A = \frac{1}{4}\int_{-1}^{2}\left(x + 2 - x^2\right)dx \Rightarrow A = \frac{1}{4}\left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}$$

$$A = \frac{1}{4}\left\{\left(\frac{4}{2} + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)\right\}$$

$$A = \frac{1}{4}\left\{\frac{10}{3} + \frac{7}{6}\right\} \Rightarrow A = \frac{27}{24}$$

$$A = \frac{9}{8} \text{ sq. units}$$

**2.** If the system of linear equations

$2x + 2y + 3z = a$

$3x - y + 5z = b$

$x - 3y + 2z = c$

Where a,b,c are non - zero real numbers, has more than one solution, then :

(A) $b - c - a = 0$ (B) $a + b + c = 0$ (C) $b - c + a = 0$ (D) $b + c - a = 0$

**Sol.** **A**

Given equation

$2x + 2y + 3z = a$

$3x - y + 5z = b$

$x - 3y + 2z = c$

for more than one solution

$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\Delta = \begin{vmatrix} 2 & 2 & 3 \\ 3 & -1 & 5 \\ 1 & -3 & 2 \end{vmatrix} \Rightarrow 2(-2 + 15) - 2(6 - 5) + 3(-9 + 1)$$

$\Rightarrow$ 26 – 2 – 24 = 0
$\Delta = 0$

$$\Delta_1 = \begin{vmatrix} a & 2 & 3 \\ b & -1 & 5 \\ c & -3 & 2 \end{vmatrix} = 0$$

$\Rightarrow a(-2 + 15) - 2(2b - 5c) + 3(-3b + c) = 0$
$13a - 13b + 13c = 0$
$a - b + c = 0$

Also $\Delta_2 = 0 \Rightarrow \begin{vmatrix} 2 & a & 3 \\ 3 & b & 5 \\ 1 & c & 2 \end{vmatrix} = 0$

$\Rightarrow 2(2b - 5c) - a(6 - 5) + 3(3c - b) = 0$
$b - c - a = 0$
$\therefore$ from $\Delta_1 = \Delta_2 = \Delta_3 = 0$
$a - b + c = 0$
or $b - c - a = 0$

**3.** The sum of an infinite geometric series with positive terms is 3 and the sum of the cubes of its terms is $\frac{27}{19}$. Then the common ratio of this series is :

(A) $\frac{2}{9}$ (B) $\frac{2}{3}$ (C) $\frac{1}{3}$ (D) $\frac{4}{9}$

**Sol.** **B**
$S_\infty = 3$
let first term = a

$S_\infty = \frac{a}{1-r}$, $|r| < 1$

$3 = \frac{a}{1-r}$
$a = 3(1 - r)$ ....(1)
also given

sum of cubes = $\frac{27}{19}$

$\frac{a^3}{1-r^3} = \frac{27}{19}$
$19a^3 = 27(1 - r^3)$ ....(2)
Solve equation (1) & (2)
$19[3(1 - r)]^3 = 27(1 - r^3)$
$19 \times 27(1 - r)^3 = 27(1 - r)(1 + r + r^2)$
$19(1 - r)^2 = (1 + r + r^2)$
$19 + 19r^2 - 38r - 1 - r - r^2 = 0$
$18r^2 - 39r + 18 = 0$
$6r^2 - 13r + 6 = 0$
$6r^2 - 9r - 4r + 6 = 0$
$3r(2r - 3) - 2(2r - 3) = 0$

$r = \frac{3}{2}$ or $r = \frac{2}{3}$

But $|r| < 1 \therefore r = \frac{2}{3}$

**4.** Let $A = \begin{pmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{pmatrix}$. If $AA^T = I_3$, then |p| is :

(A) $\frac{1}{\sqrt{5}}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{\sqrt{6}}$

**Sol.** **C**

$A = \begin{bmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{bmatrix}$

$A.A^T = I_3$

$\begin{bmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{bmatrix}\begin{bmatrix} 0 & p & p \\ 2q & q & -q \\ r & -r & r \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 4q^2 + r^2 & 2q^2 - r^2 & -2q^2 + r^2 \\ 2q^2 - r^2 & p^2 + q^2 + r^2 & p^2 - q^2 - r^2 \\ -2q^2 + r^2 & p^2 - q^2 - r^2 & p^2 + q^2 + r^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Now, $4q^2 + r^2 = 1$
$2q^2 - r^2 = 0$,
$p^2 - q^2 - r^2 = 0$
$p^2 + q^2 + r^2 = 1$
Solving

$4q^2 + r^2 = 1$
$2q^2 - r^2 = 0$
$6q^2 = 1$

$q^2 = \frac{1}{6}$

$q = \frac{1}{\sqrt{6}}$

Soving
$r^2 = 2q^2$

$r^2 = \frac{1}{3} \Rightarrow r = \frac{1}{\sqrt{3}}$

$\therefore p^2 = q^2 + r^2$

$p^2 = \frac{1}{6} + \frac{1}{3}$

$p^2 = \frac{1}{2}$

$|P| = \frac{1}{\sqrt{2}}$

**5.** If y(x) is the solution of the differential equation $\frac{dy}{dx}+\left(\frac{2x+1}{x}\right)y = e^{-2x}$, x > 0, where $y(1) = \frac{1}{2}e^{-2}$, then :

(A) y(x) is decreasing in $\left(\frac{1}{2},1\right)$
(B) $y(\log_e 2) = \frac{\log_e 2}{4}$
(C) $y(\log_e 2) = \log_e 4$
(D) y(x) is decreaing in (0,1)

**Sol.** **A**

$\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y = e^{-2x}, x > 0$

it is linear differential equation.

I.F. $= e^{\int\left(2+\frac{1}{x}\right)dx} = e^{(2x+\ell nx)} = e^{2x}.\ e^{\ln x} = x.e^{2x}$

$\therefore$ I.F. $= x.e^{2x}$

$x.e^{2x}\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y.\,x.e^{2x} = x.e^{2x}.\ e^{-2x}$

$\int d\left(y.x.e^{2x}\right) = \int x\,dx$

$y.x.e^{2x} = \frac{x^2}{2}+C$

Now given $y(1) = \frac{1}{2}.e^{-2}$

$\therefore \frac{1}{2}.e^{-2}.(1).e^{2} = \frac{1}{2}+C \Rightarrow C = 0$

$\therefore y.x.e^{2x} = \frac{x^2}{2} \Rightarrow y = \frac{x.e^{-2x}}{2}$

$\frac{dy}{dx} = \frac{e^{-2x}}{2}+\frac{x.e^{-2x}(-2)}{2} = e^{-2x}\left(\frac{1}{2}-x\right)$

$\therefore \frac{dy}{dx} = e^{-2x}\left(\frac{1}{2}-x\right)$

$\therefore$ when $x > \frac{1}{2}, \frac{dy}{dx} < 0$

$\therefore$ y(x) is decreasing in $\left(\frac{1}{2},1\right)$

**6.** Let $f(x) = \begin{cases} -1, & -2 \le x < 0 \\ x^2-1, & 0 \le x \le 2 \end{cases}$ and g(x) =|f(x)| + f(|x|). Then, in the interval (–2,2), g is :

(A) not differentiable at two points
(B) differentiable at all points
(C) not differentiable at one point
(D) not continuous

**Sol.** **C**

$|f(x)| = \begin{cases} 1, & -2 \le x < 0 \\ 1-x^2, & 0 \le x < 1 \\ x^2-1, & 1 \le x \le 2 \end{cases}$

and $f(|x|) = x^2 - 1,\ x \in [-2,2]$

Hence $g(x) = \begin{cases} x^2, & x \in [-2,0) \\ 0, & x \in [0,1) \\ 2(x^2-1), & x \in [1,2] \end{cases}$

It is not differentiable at x = 1

**7.** In a triangle, the sum of lengths of two sides is x and the product of the lengths of the same two sides is y . If $x^2 - c^2 = y$, where c is the length of the third side of the triangle, then the circumradius of the triangle is :

(A) $\frac{c}{\sqrt{3}}$ (B) $\frac{y}{\sqrt{3}}$ (C) $\frac{c}{3}$ (D) $\frac{3}{2}y$

**Sol.** **A**

In $\triangle ABC$

$a + b = x$ & $ab = y$

$x^2 - c^2 = y$

$(a + b)^2 - c^2 = ab$

$a^2 + b^2 + 2ab - c^2 = ab$

$a^2 = b^2 - c^2 = -ab$

$\frac{a^2+b^2-c^2}{2ab} = -1/2 \therefore \cos C = -\frac{1}{2} \therefore \angle C = 120$

$\therefore R = \frac{C}{2\sin C}$

$\therefore R = \frac{C}{2.\sin 120^\circ} = \frac{c}{2.\left(\frac{\sqrt{3}}{2}\right)}$

$\therefore r = \frac{C}{\sqrt{3}}$

**8.** The direction ratios of normal to the plane through the points (0,–1,0) and (0,0,1) and making an angle $\frac{\pi}{4}$ with the plane y – z + 5 = 0 are :

(A) $2,\sqrt{2},-\sqrt{2}$ (B) 2, – 1, 1 (C) $\sqrt{2},1,-1$ (D) $2\sqrt{3},1,-1$

**Sol.** **A, C**

A(0,–1,0)

B(0,0,1)

Points A & B lies in the plane

$\therefore \overline{AB}$ also lies m plane

$\overline{AB} = 0\hat{i} + \hat{j} + \hat{k}$

another plane $P_2$ is y – z + 5 = 0

$\therefore \vec{n}_2 = 0\hat{i} + \hat{j} - \hat{k}$

Let plane is ax + by + cz + d = 0

$\vec{n} = a\hat{i} + b\hat{j} + c\hat{k}$

this $\vec{n} \perp \overline{AB}$

$\therefore \vec{n}.\overline{AB} = 0$

a(0) + b(1) + c(1) = 0

b + c = 0
b = – c

angle b/w planes is $\frac{\pi}{4}$

$\therefore \cos\frac{\pi}{4} = \left|\frac{\vec{n}.\vec{n}_2}{|\vec{n}||\vec{n}_2|}\right|$

$\frac{1}{\sqrt{2}} = \left|\frac{b-c}{\sqrt{a^2+b^2+c^2}\sqrt{2}}\right| \Rightarrow 1 = \left|\frac{2b}{\sqrt{a^2+2b^2}}\right| \Rightarrow 4b^2 = a^2 + 2b^2$

$a^2 = 2b^2$

$a = \pm\sqrt{2}\,b$

or $a = \pm\sqrt{2}\,c$

& b = – c

$\therefore$ Direction ratios are

$\left(\sqrt{2},-1,1\right)$ or $\left(\sqrt{2},1,-1\right)$

**9.** The value of r for which ${}^{20}C_r\,{}^{20}C_0 + {}^{20}C_{r-1}\,{}^{20}C_1 + {}^{20}C_{r-2}\,{}^{20}C_2 + \ldots. + {}^{20}C_0\,{}^{20}C_r$ is maximum, is :

(A) 15 (B) 20 (C) 11 (D) 10

**Sol.** **B**

${}^{20}C_r\,{}^{20}C_0 + {}^{20}C_{r-1}\,{}^{20}C_1 + {}^{20}C_{r-2}\,{}^{20}C_2 + \ldots. + {}^{20}C_0\,{}^{20}C_r$

$(1 + x)^{20}(1 + x)^{20}$

sum is ${}^{40}C_r$

maximum when r = 20

**10.** Two integers are selected at random from the set {1,2,....,11}.Given that the sum of selected numbers is even, the conditional probability that both the numbers are even is :

(A) $\frac{3}{5}$ (B) $\frac{7}{10}$ (C) $\frac{2}{5}$ (D) $\frac{1}{2}$

**Sol.** **C**

either both even or both odd

required probability = $\frac{{}^5C_2}{{}^5C_2 + {}^6C_2}$

$= \frac{10}{10+15} = \frac{10}{25} = \frac{2}{5}$

**11.** Let $\left(-2-\frac{1}{3}i\right)^3 = \frac{x+iy}{27}\left(i=\sqrt{-1}\right)$, where x and y are real numbers, then y – x equals :

(A) –85 (B) –91 (C) 91 (D) 85

**Sol.** **C**

$\left(-2-\frac{1}{3}i\right)^3 = \frac{x+iy}{27}$

$\left(\frac{-6-i}{3}\right)^3 = \frac{x+iy}{27}$

$-\frac{(6+i)^3}{27} = \frac{(x+iy)}{27}$

$-(216 + 108i + 18i^2 + i^3) = (x + iy)$

$-(216 + 108i - 18 - i) = (x + iy)$

$-(198 + 107i) = x + iy$

$x = -198, y = -107$

$\therefore y - x = -107 + 198 = 91$

**12.** Two circles with equal radii are intersecting at the points (0,1) and (0,–1).The tangent at the point (0,1) to one of the circles passes through the centre of the other circle. Then the distance between the centres of these circles is :

(A) 1 (B) $\sqrt{2}$ (C) 2 (D) $2\sqrt{2}$

**Sol.** **C**

In $\Delta C_1PC_2$

$r^2 + r^2 = (C_1C_2)^2$

$(C_1C_2)^2 = 2r^2$

$C_1C_2 = \sqrt{2}\,r$

In $\Delta C_1MP$

$r^2 = 1 + (C_1M)^2$

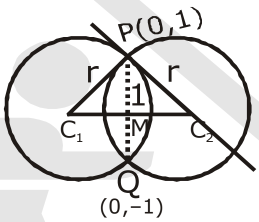
$r^2 = 1 + \left(\frac{\sqrt{2}\,r}{2}\right)^2$

$r^2 = 1 + \frac{r^2}{2}$

$2r^2 - r^2 - 2 = 0$

$r = \sqrt{2}$

$\therefore C_1C_2 = 2$




**13.** If $x\log_e(\log_e x) - x^2 + y^2 = 4 (y > 0)$, then $\frac{dy}{dx}$ at x = e is equal to :

(A) $\frac{e}{\sqrt{4+e^2}}$ (B) $\frac{(1+2e)}{\sqrt{4+e^2}}$ (C) $\frac{(1+2e)}{2\sqrt{4+e^2}}$ (D) $\frac{(2e-1)}{2\sqrt{4+e^2}}$

**Sol.** **D**

$x\log_e(\log_e x) - x^2 + y^2 = 4, (y > 0).$ ...(1)

$\log_e(\log_e x) + x.\frac{1}{\log_e x}.\frac{1}{x} - 2x + 2y.\frac{dy}{dx} = 0$

put x = e

$\log_e(\log_e e) + e.\frac{1}{\log_e e}.\frac{1}{e} - 2e + 2y.\frac{dy}{dx} = 0$

$\log_e(1) + 1 - 2e + 2y.\frac{dy}{dx} = 0$ ...(2)

from equation (1) at x = e

$e\log_e(\log_e e) - e^2 + y^2 = 4$

$y^2 = 4 + e^2$

$y = \sqrt{4+e^2}$

put $y = \sqrt{4+e^2}$ in equation (2)

$\therefore\ 1 - 2e + 2\sqrt{4+e^2}.\dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx} = \dfrac{2e-1}{2\sqrt{4+e^2}}$

**14.** If q is false and $p \wedge q \leftrightarrow r$ is true, then which one of the following statements is a tautology ?

(A) $p \vee r$ (B) $(p \wedge r) \to (p \vee r)$ (C) $(p \vee r) \to (p \wedge r)$ (D) $p \wedge r$

**Sol.** **B**

q : F

$(p \wedge q) \leftrightarrow r : T$

**Case I**

$p \wedge q : T$ and r : T

It is not possible when q :F

**Case II**

$p \wedge q : F$ and r : F

P : T or F q : F, r : F

**1.** $p \vee r$

$T \vee F : T$

$f \vee f : f$

**2.** $(p \wedge r) \to (p \vee r)$

$T \wedge f \to T \vee F$

$F \to T : T$

$F \wedge F \to F \vee F$

$F \to F : T$

**3.** $(p \vee r) \to (p \wedge r)$

$T \wedge f \to (T \wedge F)$

$T \to F : F$

$F \vee F \to F \wedge F$

$F \to F : T$

**(4)** $p \wedge r$

$T \wedge F : F$

$T \wedge F : F$

**15.** Equation of a common tangent to the parabola $y^2 = 4x$ and the hyperbola xy = 2 is :

(A) x + 2y + 4 = 0 (B) x – 2y + 4 = 0 (C) x + y + 1 = 0 (D) 4x + 2y + 1 = 0

**Sol.** **A**

$y^2 = 4x$ & xy = 2.

for parabola $y^2 = 4x$

let tangent is $y = mx + \dfrac{1}{m}$ ...(1)

it also touches hyperbola xy = 2 ...(2)

$\therefore$ solve (1) & (2) & apply D = 0

$x\left(mx+\frac{1}{m}\right) = 2$

$m^2x^2 - 2m + x = 0 \Rightarrow D = 0$

$(1)^2 - 4(m^2)(-2m) = 0$

$8m^3 = -1,\ m^3 = -\frac{1}{8}$

$m = -\frac{1}{2}$

$\therefore$ common tangent is

$y = -\frac{1}{2}x + \frac{1}{(-1/2)}$

$y = -\frac{1}{2}x - 2$

$x + 2y + 4 = 0$

**16.** Let $a_1, a_2, \ldots, a_{10}$ be a GP. If $\frac{a_3}{a_1} = 25$, then $\frac{a_9}{a_5}$ equals :

(A) $5^4$ (B) $5^3$ (C) $2(5^2)$ (D) $4(5^2)$

**Sol. A**

$a_1, a_2, \ldots\ldots, a_{10} \to GP$

$a, ar, ar^2, \ldots\ldots\ldots, ar^9 \to GP$

$\frac{a_3}{a_1} = 25 \Rightarrow \frac{ar^2}{a} = 25$

$r = \pm 5$

$\frac{a_9}{a_5} = \frac{ar^8}{ar^4} \Rightarrow r^4$

$\therefore r^4 = (25)^2 = 5^4$

**17.** The outcome of each of 30 itmes was observed ; 10 items gave an outcome $\frac{1}{2} - d$, 10 items gave outcome $\frac{1}{2}$ each and the remaining 10 items gave outcome $\frac{1}{2}+d$ each. If the variance of this outcome data is $\frac{4}{3}$ then $|d|$ equals :

(A) $\sqrt{2}$ (B) $\frac{2}{3}$ (C) $\frac{\sqrt{5}}{2}$ (D) 2

**Sol. A**

variance is independent of origin shift data by $\frac{1}{2}$.

$\sum \frac{x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 =$

$\frac{10d^2 + 10\times(0)^2 + 10d^2}{30} - (0)^2 = \frac{4}{3}$

$d^2 = 2 \Rightarrow |d| = \sqrt{2}$

**18.** If one real root of the quadratic equation $81x^2 + kx + 256 = 0$ is cube of the other root, then a value of k is :

(A) 144 (B) −81 (C) 100 (D) −300

**Sol.** **D**

$81x^2 + kx + 256 = 0$

roots are $\alpha$ & $\alpha^3$

$$\alpha + \alpha^3 = -\frac{k}{81}$$

$$\alpha^4 = \frac{256}{81}$$

$$\alpha = \pm\frac{4}{3}$$

$$\therefore \alpha + \alpha^3 = -\frac{k}{81}$$

$$\frac{4}{3} + \frac{64}{27} = -\frac{k}{81}$$

$\therefore k = -300$

**19.** If $\int \frac{\sqrt{1-x^2}}{x^4}dx = A(x)\left(\sqrt{1-x^2}\right)^m + C$, for a suitable chosen integer m and a function A(x), where C is a constant of integration, then $(A(x))^m$ equals :

(A) $\frac{1}{9x^4}$ (B) $\frac{1}{27x^6}$ (C) $\frac{-1}{27x^9}$ (D) $\frac{-1}{3x^3}$

**Sol.** **C**

$$\int \frac{\sqrt{1-x^2}}{x^4}dx = A(x)\left(\sqrt{1-x^2}\right)^m + C$$

$$\int \frac{x\sqrt{\frac{1}{x^2}-1}}{x^4}dx = A(x)\left(\sqrt{1-x^2}\right)^m + C$$

Put $\left(\frac{1}{x^2}-1\right) = t \Rightarrow \frac{dt}{dx} = \frac{-2}{x^3}$

$$\therefore -\frac{1}{2}\int \sqrt{t}\,dt \Rightarrow \frac{-t^{3/2}}{3} + C$$

$$\Rightarrow \frac{\left(\sqrt{1-x^2}\right)^3}{-3x^2} + C$$

$\therefore A(x) = -\frac{1}{3x^3}$ & $m = 3$

$$A((x))^3 \Rightarrow \left(-\frac{1}{3x^3}\right)^3$$

$$= \frac{-1}{27x^9}$$

**20.** The maximum value of the function $f(x) = 3x^3 - 18x^2 + 27x - 40$ on the set $S = \{x \in R : x^2 + 30 \le 11x\}$ is :

(A) 122 (B) −122 (C) −222 (D) 222

**Sol.** **A**

$f(x) = 3x^3 - 18x^2 + 27x - 40$

$f'(x) = 9x^2 - 36x + 27$

$f'(x) = 9(x^2 - 4x + 3)$

$f'(x) = 9(x - 1)(x - 3)$

Now $S = \{x \in R, x^2 + 30 - 11x \le 0\}$

$= \{x \in R, x \in [5,6]\}$

$\therefore$ where $x \in [5,6]$, $f'(x)$ is positive

$\therefore$ $f(x)$ is increasing in $[5,6]$

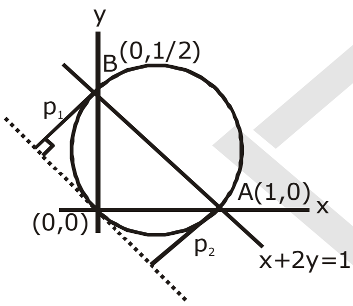
$\therefore$ max. value, $f(6) = 122$

**21.** The straight line $x + 2y = 1$ meets the coordinate axes at A and B. A circle is drawn through A,B and the origin. Then the sum of perpendicular distances from A and B on the tangent to the circle at the origin is :

(A) $\frac{\sqrt{5}}{4}$ (B) $4\sqrt{5}$ (C) $\frac{\sqrt{5}}{2}$ (D) $2\sqrt{5}$

**Sol.** **C**

$x + 2y = 1$




equation of circle

$(x - 1)(x - 0) + (y - 0)(y - 1/2) = 0$

$x^2 + y^2 - x - \frac{y}{2} = 0$

Tangent at (0,0) is

From T = 0

$0 + 0 - \left(\frac{x+0}{2}\right) - \frac{1}{2}\left(\frac{y+0}{2}\right) = 0 \Rightarrow 2x + y = 0$

$p_1 + p_2 = \left|\frac{0 + \frac{1}{2}}{\sqrt{5}}\right| + \left|\frac{2+0}{\sqrt{5}}\right| = \frac{1}{2\sqrt{5}} + \frac{2}{\sqrt{5}}$

$p_1 + p_2 = \frac{5}{2\sqrt{5}} \Rightarrow p_1 + p_2 = \frac{\sqrt{5}}{2}$

**22.** The plane containing the line $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z-1}{3}$ and also contiaining its projection on the plane $2x + 3y - z = 5$, contains which one of the following points ?

(A) (−2,2,2) (B) (0, − 2, 2) (C) (2,0,−2) (D) (2,2,0)

**Sol.** **C**

line. $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z-1}{3}$ & $P_1 \equiv 2x + 3y - z = 5$

$\bar{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ $\therefore$ $\bar{n}_1 = 2\hat{i} + 3\hat{j} - \hat{k}$

normal vector of required plane is $\perp$ to $\bar{b}$ & $\bar{n}_1$

$\therefore$ $\vec{n} = \bar{b} \times \bar{n}_1$

$\bar{n} = -8\hat{i} + 8\hat{j} + 8\hat{k}$

$\therefore$ D.R.'s of $\bar{n}$ of required plane are – 1,1,1

$\therefore$ equation of required plane is

$-1(x - 3) + 1(y + 2) + 1(z - 1) = 0$

$-x + y + z + 4 = 0$

$x - y - z - 4 = 0$

it is the required plane

Now check options

**23.** Let [x] denote the greatest integer less than or equal to x. then :

$$\lim_{x\to 0}\frac{\tan(\pi\sin^2 x)+\left(|x|-\sin(x[x])\right)^2}{x^2}$$

(A) does not exist (B) equals 0 (C) equals $\pi + 1$ (D) equals $\pi$

**Sol.** **A**

RHL

$$\lim_{x\to 0^+}\frac{\tan(\pi\sin^2 x)+\left(|x|-\sin(x[x])\right)^2}{x^2}$$

where $x \to 0^+$, $[x] = 0$

$$\therefore \lim_{x\to 0^+}\frac{\tan(\pi\sin^2 x)+x^2}{x^2}$$

$$\lim_{x\to 0^+}\left(\frac{\tan(\pi\sin^2 x)}{(\pi\sin^2 x)}\times\frac{(\pi\sin^2 x)}{x^2}\right)+1$$

$\therefore$ RHL = $\pi + 1$

LHL

$$\lim_{x\to 0^-}\frac{\tan(\pi\sin^2 x)+\left(|x|-\sin(x[x])\right)^2}{x^2}$$

as $x \to 0^+$ , $[x] = -1$

$$\therefore \lim_{x\to 0^-}\frac{\tan(\pi\sin^2 x)+(-x+\sin x)^2}{x^2}$$

$$\lim_{x\to 0^-}\left(\frac{\tan(\pi\sin^2 x)}{(\pi\sin^2 x)}\times\frac{(\pi\sin^2 x)}{x^2}\right)+\left(\frac{\sin x}{x}\right)^2-1$$

$\therefore$ LHS = $\pi$

$\therefore$ RHL $\neq$ LHL

$\therefore$ Limit does not exist

**24.** The value of the integral $\int_{-2}^{2} \frac{\sin^2 x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} dx$ (where [x] denotes the greatest integer less than or equal to x) is :

(A) sin 4 (B) 4 (C) 4 – sin4 (D) 0

**Sol.** **D**

$$I = \int_{-2}^{2} \frac{\sin^2 x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} dx$$

$$I = \int_{0}^{2} \left( \frac{\sin^2 x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} + \frac{\sin^2(-x)}{\left[-\frac{x}{\pi}\right] + \frac{1}{2}} \right) dx$$

$$\left( \left[\frac{x}{\pi}\right] + \left[\frac{-x}{\pi}\right] = -1 \text{as } x \neq n\pi \right)$$

$$I = \int_{0}^{2} \left( \frac{\sin^2 x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} + \frac{\sin^2 x}{-1 - \left[\frac{x}{\pi}\right] + \frac{1}{2}} \right) dx \quad = 0$$

**25.** A square is inscribed in the circle $x^2 + y^2 - 6x + 8y - 103 = 0$ with its sides parallel to the coordinates axes. Then the distance of the vertex of this square which is nearest to the origin is :

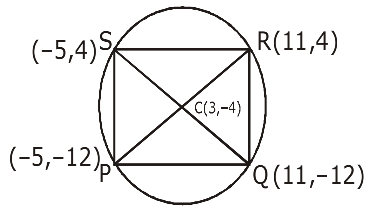
(A) 6 (B) $\sqrt{41}$ (C) $\sqrt{137}$ (D) 13

**Sol.** **B**

$x^2 + y^2 - 6x + 8y - 103 = 0$

center (3, –4), r = $\sqrt{9+16+103}$ = $\sqrt{128}$ = $8\sqrt{2}$

CP = CR = CQ = CS = $8\sqrt{2}$




$$R = \left(3 + 8\sqrt{2}.\frac{1}{\sqrt{2}}, -4 + 8\sqrt{2}.\frac{1}{\sqrt{2}}\right)$$

$R \equiv (11, 4)$ $\therefore$ OR = $\sqrt{137}$

$$p \equiv \left(3 - 8\sqrt{2}.\frac{1}{\sqrt{2}}, -4 - 8\sqrt{2}.\frac{1}{\sqrt{2}}\right)$$

$P \equiv (-5, -12)$ $\therefore$ OP = 13

$\therefore$ Q $\equiv (11, -12)$ & S $\equiv (-5, 4)$

$\therefore$ OQ = $\sqrt{265}$ $\therefore$ OS = $\sqrt{41}$

$\therefore$ Minimum distance from origin is $\sqrt{41}$

**26.** Let f : R → R be defined by $f(x) = \frac{x}{1+x^2}$, $x \in R$. Then the range of f is

(A) (–1,1) –{0} (B) $R - \left[-\frac{1}{2}, \frac{1}{2}\right]$ (C) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (D) R – [–1,1]

**Sol.** **C**

f(0) = 0 & f(x) is odd.

Further, If x > 0 then

$f(x) = \dfrac{1}{x + \frac{1}{X}} \in \left(0, \frac{1}{2}\right]$

Hence, $f(x) \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

**27.** Let $\vec{a} = \hat{i} + 2\hat{j} + 4\hat{k}$ , $\vec{b} = \hat{i} + \lambda\hat{j} + 4\hat{k}$ and $\vec{c} = 2\hat{i} + 4\hat{j} + (\lambda^2 - 1)\hat{k}$ be coplaner vectors. Then the non - zero vector $\vec{a} \times \vec{c}$ is :

(A) $-10\hat{i} + 5\hat{j}$ (B) $-14\hat{i} - 5\hat{j}$ (C) $-14\hat{i} + 5\hat{j}$ (D) $-10\hat{i} - 5\hat{j}$

**Sol. A**

$\vec{a}, \vec{b}, \vec{c}$ are coplanar

$\therefore \left[\vec{a}\ \vec{b}\ \vec{c}\right]$

$$\begin{vmatrix} 1 & 2 & 4 \\ 1 & \lambda & 4 \\ 2 & 4 & \lambda^2 - 1 \end{vmatrix} = 0$$

$\lambda(\lambda^2 - 1) - 16 - 2(\lambda^2 - 1 - 8) + 4(4 - 2\lambda) = 0$

$\lambda^3 - \lambda - 16 - 2\lambda^2 + 18 + 16 - 8\lambda$

$\lambda^3 - 2\lambda^2 - 9\lambda + 18 = 0$

$l^2(\lambda - 2) - 9(\lambda - 2) = 0$

$(\lambda - 2)(\lambda^2 - 9) = 0$

$\lambda = 2$ , $\lambda = \pm 3$

Now $\vec{a} = \hat{i} + 2\hat{j} + 4\hat{k}$

$\vec{b} = \hat{i} + \lambda\hat{j} + 4\hat{k}$

$\vec{c} = 2\hat{i} + 4\hat{j} + (\lambda^2 - 1)\hat{k}$

when $\lambda = \pm 3$, $\vec{a} \,||\, \vec{c}$ $\therefore \lambda \neq \pm 3$

$\therefore \lambda = 2$

$\vec{c} = 2\hat{i} + 4\hat{j} + 3\hat{k}$

$$\therefore \vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 4 \\ 2 & 4 & 3 \end{vmatrix}$$

$\vec{a} \times \vec{c} = -10\hat{i} + 5\hat{j}$

**28.** If tangents are drawn to the ellipse $x^2 + 2y^2 = 2$ at all points on the ellipse other than its four vertices then the mid points of the tangents intercepted between the coordinates axes lie on the curve.

(A) $\dfrac{1}{2x^2} + \dfrac{1}{4y^2} = 1$ (B) $\dfrac{x^2}{2} + \dfrac{y^2}{4} = 1$ (C) $\dfrac{x^2}{4} + \dfrac{y^2}{2} = 1$ (D) $\dfrac{1}{4x^2} + \dfrac{1}{2y^2} = 1$

**Sol. A**

Equation of generel tangent on ellipse

$\frac{x}{a\sec\theta}+\frac{y}{b\,\text{cosec}\theta}=1$

a = $\sqrt{2}$, b = 1

$\Rightarrow \frac{x}{\sqrt{2}\sec\theta}+\frac{y}{\text{cosec}\theta}=1$

let the midpoint be (h,k)

h = $\frac{\sqrt{2}\sec\theta}{2} \Rightarrow \cos\theta = \frac{1}{\sqrt{2}h}$

and k = $\frac{\text{cosec}\theta}{2} \Rightarrow \sin\theta = \frac{1}{2k}$

$\because \sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \frac{1}{2h^2}+\frac{1}{4k^2} = 1$

$\Rightarrow \frac{1}{2x^2}+\frac{1}{4y^2} = 1$

**29.** The sum of the real values of x for which the middle term in the binomial expansion of $\left(\frac{x^3}{3}+\frac{3}{x}\right)^8$ equals 5670 is :

(A) 0 (B) 8 (C) 6 (D) 4

**Sol.** **A**

$T_5 = {}^8C_4 \frac{x^{12}}{81}\times\frac{81}{x^4} = 5670$

$\Rightarrow 70x^8 = 5670$

$\Rightarrow x = \pm\sqrt{3}$

**30.** Let $f_k(x) = \frac{1}{k}\left(\sin^k x + \cos^k x\right)$ for k = 1,2,3..... Then for all $x \in R$, the value of $f_4(x) - f_6(x)$ is equal to :

(A) $\frac{1}{4}$ (B) $\frac{1}{12}$ (C) $\frac{-1}{12}$ (D) $\frac{5}{12}$

**Sol.** **B**

$f_4(x) - f_6(x)$

$= \frac{1}{4}\left(\sin^2 x + \cos^4 x\right) - \frac{1}{6}\left(\sin^6 x + \cos^6 x\right)$

$= \frac{1}{4}\left(1 - \frac{1}{2}\sin^2 2x\right) - \frac{1}{6}\left(1 - \frac{3}{4}\sin^2 2x\right) = \frac{1}{12}$