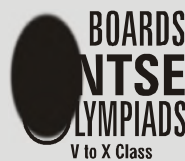


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QUESTION WITH SOLUTION
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[MATHEMATICS] 12-01-2019_Morning

1. For $x > 1$, If $(2x)^{2y} = 4e^{2x-2y}$, then $(1 + \log_e 2x)^2 \frac{dy}{dx}$ is equal to :

- (A) $\frac{x \log_e 2x + \log_e 2}{x}$ (B) $\frac{x \log_e 2x - \log_e 2}{x}$ (C) $\log_e 2x$ (D) $x \log_e 2x$

Sol. B

$$2y \ln 2x = \ln 4 + 2x - 2y$$

$$2y (1 + \ln 2x) = \ln 4 + 2x$$

$$y = \frac{x + \ln 2}{(1 + \ln 2x)} \Rightarrow \frac{dy}{dx} = \frac{(1 + \ln 2x) - (x + \ln 2) \cdot \frac{1}{x}}{(1 + \ln 2x)^2}$$

$$y'(1 + \ln 2x)^2 = \left[\frac{x \ln 2x - \ln 2}{x} \right]$$

2. If λ be the ratio of the roots of the quadratic equation in x , $3m^2x^2 + m(m-4)x + 2 = 0$, then the least value of m for which $\lambda + \frac{1}{\lambda} = 1$, is

- (A) $4 - 2\sqrt{3}$ (B) $4 - 3\sqrt{2}$ (C) $-2 + \sqrt{2}$ (D) $2 - \sqrt{3}$

Sol. B

$$\lambda = \frac{\alpha}{\beta} \Rightarrow \lambda + \frac{1}{\lambda} = 1 \text{ (given)}$$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = 1$$

$$\Rightarrow \frac{\alpha^2 + \beta^2}{\alpha\beta} = 1 \Rightarrow \frac{(\alpha + \beta)^2 - 4\alpha\beta}{\alpha\beta} = 1$$

$$\Rightarrow \frac{(\alpha + \beta)^2}{\alpha\beta} = 5 \Rightarrow (\alpha + \beta)^2 = 5\alpha\beta$$

$$\Rightarrow \left(\frac{m(m-4)}{3m^2} \right)^2 = \frac{5 \cdot 2}{3m^2} \Rightarrow m = 4 \pm \sqrt{18}, 4 \pm 3\sqrt{2}$$

3. Consider three boxes, each containing 10 balls labelled 1,2,...,10. Suppose one ball is randomly drawn from each of the boxes. Denote by n_i , the label of the ball drawn from the i^{th} box, ($i=1,2,3$). Then, the number of ways in which the balls can be chosen such that $n_1 < n_2 < n_3$ is :

- (A) 164 (B) 82 (C) 240 (D) 120

Sol. D

Chose any 3 balls it will always be $n_1 < n_2 < n_3$

$$\Rightarrow \text{No of ways} = {}^{10}C_3 = \frac{10 \cdot 9 \cdot 8}{1 \cdot 2 \cdot 3} = 120$$

4. The sum of the distinct real values of μ , for which the vectors, $\mu\hat{i} + \hat{j} + \hat{k}$, $\hat{i} + \mu\hat{j} + \hat{k}$, $\hat{i} + \hat{j} + \mu\hat{k}$ are co-planar, is :

- (A) 0 (B) 1 (C) 2 (D) -1

Sol. D

$$\begin{vmatrix} \mu & 1 & 1 \\ 1 & \mu & 1 \\ 1 & 1 & \mu \end{vmatrix} = 0$$

$$\begin{aligned}\mu(\mu^2 - 1) - 1(\mu - 1) + 1(1 - \mu) &= 0 \\ \mu^3 - 3\mu + 2 &= 0 \Rightarrow \mu^3 - 1 - 3\mu + 3 = 0 \\ \mu &= 1 \text{ \& } \mu^2 + \mu - 2 = 0 \\ \mu &= 1, \mu = -2 \text{ sum} = 1 + (-2) = -1\end{aligned}$$

5. Let $S = \{1, 2, 3, \dots, 100\}$. The number of non - empty subsets A of S such that the product of elements in A is even is :
 (A) $2^{50}(2^{50} - 1)$ (B) $2^{100} - 1$ (C) $2^{50} - 1$ (D) $2^{50} + 1$

Sol. **A**

$$\begin{aligned}S &= \{1, 2, 3, \dots, 100\} \\ &= \text{Total Non empty subsets} - (\text{Subsets with prod.} = \text{odd}) \\ &= 2^{100} - 1 - \{2^{50} - 1\} \rightarrow \text{exactly have} \\ &= 2^{100} - 2^{50} = 2^{50}(2^{50} - 1)\end{aligned}$$

6. The integral $\int \cos(\log_e x) dx$ is equal to : (where C is a constant of integration)

$$\begin{aligned}(\text{A}) \frac{x}{2} [\sin(\log_e x) - \cos(\log_e x)] + C & \quad (\text{B}) x [\cos(\log_e x) - \sin(\log_e x)] + C \\ (\text{C}) x [\cos(\log_e x) + \sin(\log_e x)] + C & \quad (\text{D}) \frac{x}{2} [\cos(\log_e x) + \sin(\log_e x)] + C\end{aligned}$$

Sol. **D**

$$\int \cos(\log_e x) dx$$

$$I = x \cos(\ln x) + \int \frac{x}{x} \sin(\ln x) dx$$

$$I = x \cos(\ln x) + [x \sin(\ln x) - \int \cos(\ln x) dx]$$

$$I = \frac{x}{2} \cos(\ln x) + \sin(\ln x) + C$$

7. Considering only the principal values of inverse functions, the set $A =$

$$A = \left\{ x \geq 0 : \tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4} \right\}$$

- (A) contains two elements (B) contains more than two elements
 (C) is a singleton (D) is an empty set

Sol. **C**

$$\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$$

$$\tan^{-1} \left(\frac{2x + 3x}{1 - 6x^2} \right) = \frac{\pi}{4}$$

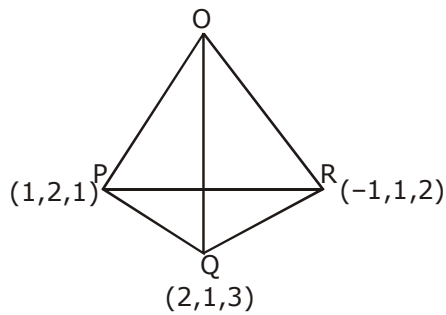
$$\Rightarrow 6x^2 + 5x - 1 = 0 \Rightarrow x = \frac{1}{6}, x = -1$$

\therefore No. of element = one

8. A tetrahedron has vertices $P(1, 2, 1)$, $Q(2, 1, 3)$, $R(-1, 1, 2)$ and $O(0, 0, 0)$. The angle between the faces OPQ and PQR is :

$$(\text{A}) \cos^{-1} \left(\frac{17}{31} \right) \quad (\text{B}) \cos^{-1} \left(\frac{7}{31} \right) \quad (\text{C}) \cos^{-1} \left(\frac{9}{35} \right) \quad (\text{D}) \cos^{-1} \left(\frac{19}{35} \right)$$

Sol. **D**



$$\text{Vector } \perp^r \text{ to face OPQ} = \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 5\hat{i} - \hat{j} - 3\hat{k}$$

$$\text{Vectore } \perp^r \text{ to face ABC} = \begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 1 & -1 & 2 \end{vmatrix} = \hat{i} - 5\hat{j} - 2\hat{k}$$

$$\text{Angle between faces} = \cos\theta = \frac{|5+5+9|}{\sqrt{35}\sqrt{35}} = \frac{19}{35}$$

$$\theta = \cos^{-1}\left(\frac{19}{35}\right)$$

9. Let $y = y(x)$ be the solution of the differential equation, $x \frac{dy}{dx} + y = x \log_e x$, ($x > 1$). If $2y(2) = \log_e 4 - 1$, then $y(e)$ is equal to :

- (A) $\frac{e}{4}$ (B) $-\frac{e}{2}$ (C) $-\frac{e^2}{2}$ (D) $\frac{e^2}{4}$

Sol. **A**

$$\frac{dy}{dx} = \frac{y}{x} = \ln x$$

$$e^{\int \frac{1}{x} dx} = x$$

$$xy = \int x \ln x + C$$

$$\ln x \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2}$$

$$xy = \frac{x}{2} \ln x - \frac{x^2}{4} + C$$

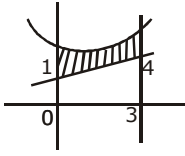
for $2y(2) = 2 \ln 2 - 1$, $c = 0$

$$y = \frac{x}{2} \ln x - \frac{x}{4}$$

$$y(e) = \frac{e}{4}$$

10. The area (in sq. units) of the region bounded by the parabola, $y = x^2 + 2$ and the lines, $y = x + 1$, $x = 0$ and $x = 3$, is :

- (A) $\frac{21}{2}$ (B) $\frac{15}{2}$ (C) $\frac{17}{4}$ (D) $\frac{15}{4}$

Sol. B

$$\begin{aligned} \text{Req area} &= \int_0^3 (x^2 + 2) dx - \frac{1}{2} \times 5 \times 3 \\ &= 9 + 6 - \frac{15}{2} \\ &= \frac{15}{2} \end{aligned}$$

11. Let $S_k = \frac{1+2+3+\dots+k}{k}$. If $S_1^2 + S_2^2 + \dots + S_{10}^2 = \frac{5}{12} \cdot A$, then A is equal to :

- (A) 156 (B) 283 (C) 303 (D) 301

Sol. C

$$S_k = \frac{k+1}{2}$$

$$\sum S_k^2 = \frac{5}{12} A$$

$$\sum_{k=1}^{10} \left(\frac{k+1}{2} \right)^2 = \frac{2^2 + 3^2 + \dots + 11^2}{4} = \frac{5}{12} A$$

$$\frac{11 \times 12 \times 23}{6} - 1 = \frac{5}{3} A$$

$$505 = \frac{5}{3} A$$

$$\Rightarrow A = 303$$

12. If the straight line, $2x - 3y + 17 = 0$ is perpendicular to the line passing through the points $(7,17)$ and $(15,\beta)$, then β equals :

- (A) $-\frac{35}{3}$ (B) $\frac{35}{3}$ (C) 5 (D) -5

Sol. C

$$\left(\frac{17-\beta}{7-15} \right) \frac{2}{3} = -1 \quad \therefore \beta = 5$$

13. An ordered pair (α, β) for which the system of linear equations

$$(1 + \alpha)x + \beta y + z = 2$$

$$\alpha x + (1 + \beta)y + z = 3$$

$$\alpha x + \beta y + 2z = 2$$

has a unique solution, is :

- (A) $(1, -3)$ (B) $(-3, 1)$ (C) $(-4, 2)$ (D) $(2, 4)$

Sol. D

For unique solution

$$D \neq 0 \quad \begin{vmatrix} 1+\alpha & \beta & 1 \\ \alpha & 1+\beta & 1 \\ \alpha & \beta & 2 \end{vmatrix} \neq 0$$

$$R_1 \rightarrow R_1 - R_2, R_2 \rightarrow R_2 - R_3$$

$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \alpha & \beta & 2 \end{vmatrix} \neq 0$$

$$\therefore \alpha + \beta \neq -2$$

- 14.** If a variable line, $3x + 4y - \lambda = 0$ is such that the two circles $x^2 + y^2 - 2x - 2y + 1 = 0$ and $x^2 + y^2 - 18x - 2y + 78 = 0$ are on its opposite sides, then the set of all values of λ is the interval :
 (A) (23,31) (B) [12,21] (C) (2,17) (D) [13,23]

Sol. B

Center of circle are opposite side of the line

$$(3 + 4 - \lambda)(27 + 4 - \lambda) < 0$$

$$(\lambda - 7)(\lambda - 31) < 0$$

$$\lambda \in (7, 31)$$

Distance from s_1

$$\left| \frac{3 + 4 - \lambda}{5} \right| \geq 1 \Rightarrow \lambda \in (-\infty, 2] \cup [12, \infty)$$

distance from s_2 .

$$\left| \frac{27 + 4 - \lambda}{5} \right| \geq 2$$

$$\lambda \in [-\infty, 21] \cup [41, \infty)$$

- 15.** The maximum value of $3\cos\theta + 5\sin\left(\theta - \frac{\pi}{6}\right)$ for any real value of θ is :

(A) $\sqrt{31}$ (B) $\frac{\sqrt{79}}{2}$ (C) $\sqrt{34}$ (D) $\sqrt{19}$

Sol. D

$$y = 3\cos\theta + 5\sin\left(\theta - \frac{\pi}{6}\right)$$

$$y = 3\cos\theta + 5\left(\sin\theta \frac{\sqrt{3}}{2} - \cos\theta \frac{1}{2}\right)$$

$$\frac{5\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta$$

$$y_{\max} = \sqrt{\frac{75}{4} + \frac{1}{4}} = \sqrt{19}$$

- 16.** In a random experiment, a fair die is rolled until two fours are obtained in succession. The probability that the experiment will end in the fifth throw of the die is equal to :

(A) $\frac{175}{6^5}$ (B) $\frac{225}{6^5}$ (C) $\frac{200}{6^5}$ (D) $\frac{150}{6^5}$

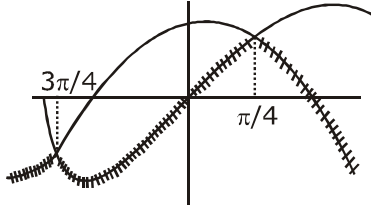
Sol. A

$$\frac{1}{6^2} \left(\frac{5^3}{6^3} + \frac{{}^2C_1 \cdot 5^2}{6} \right) = \frac{175}{6^5}$$

17. Let S be the set of all points in $(-\pi, \pi)$ at which the function, $f(x) = \min\{\sin x, \cos x\}$ is not differentiable. Then S is a subset of which of the following ?

(A) $\left\{-\frac{\pi}{2}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{2}\right\}$ (B) $\left\{-\frac{3\pi}{4}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{4}\right\}$ (C) $\left\{-\frac{\pi}{4}, 0, \frac{\pi}{4}\right\}$ (D) $\left\{-\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{4}\right\}$

Sol.



$$S \in \left\{-\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{4}\right\}$$

18. Let C_1 and C_2 be the centres of the circles $x^2 + y^2 - 2x - 2y - 2 = 0$ and $x^2 + y^2 - 6x - 6y + 14 = 0$ respectively. If P and Q are the points of intersection of these circles, then the area (in sq. units) of the quadrilateral PC_1QC_2 is :

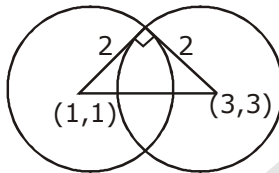
(A) 8

(B) 4

(C) 6

(D) 9

Sol. B



$$\text{Area} = 2 \times \frac{1}{2} \cdot 4 = 4$$

19. The maximum area (in sq. units) of a rectangle having its base on the x -axis and its other two vertices on the parabola, $y = 12 - x^2$ such that the rectangle lies inside the parabola, is :

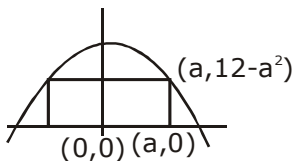
(A) 32

(B) 36

(C) $18\sqrt{3}$

(D) $20\sqrt{2}$

Sol. A



$$f(a) = 2a(12 - a^2)$$

$$f'(a) = 2(12 - 3a^2)$$

$$\text{maximum at } a = 2$$

$$\text{maximum area} = f(2) = 32$$

20. If the vertices of a hyperbola be at $(-2,0)$ and $(2,0)$ and one of its foci be at $(-3,0)$, then which one of the following points does not lie on this hyperbola ?

(A) $(6, 5\sqrt{2})$

(B) $(2\sqrt{6}, 5)$

(C) $(-6, 2\sqrt{10})$

(D) $(4, \sqrt{15})$

Sol. A

$$ae = 3$$

$$e = \frac{3}{2}$$

$$b^2 = 4\left(\frac{9}{4} - 1\right)$$

$$b^2 = 5$$

$$\frac{x^2}{4} - \frac{y^2}{5} = 1$$

21. If $\frac{z-\alpha}{z+\alpha}$ ($a \in \mathbb{R}$) is a purely imaginary number and $|z| = 2$, then a value of α is :

- (A) $\frac{1}{2}$ (B) $\sqrt{2}$ (C) 2 (D) 1

Sol. B

$$\frac{z-a}{z+a} + \frac{\bar{z}-\alpha}{\bar{z}+\alpha} = 0$$

$$z\bar{z} + z\alpha - \alpha\bar{z} - \alpha^2 + z\bar{z} - \bar{z}\alpha - \alpha^2 = 0$$

$$|z|^2 = \alpha^2$$

$$\therefore \alpha = \pm 2$$

22. $\lim_{x \rightarrow \pi/4} \frac{\cot^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$ is :

- (A) $8\sqrt{2}$ (B) 8 (C) $4\sqrt{2}$ (D) 4

Sol. B

$$\lim_{x \rightarrow \pi/4} \frac{\cot^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$$

$$\Rightarrow \lim_{x \rightarrow \pi/4} \frac{(1 - \tan^4 x)}{\cos\left(x + \frac{\pi}{4}\right)}$$

$$\Rightarrow 2 \lim_{x \rightarrow \pi/4} \frac{(1 - \tan^2 x)}{\cos\left(x + \frac{\pi}{4}\right)}$$

$$4\sqrt{2} \lim_{x \rightarrow \pi/4} (\cos x + \sin x) = 8$$

23. If the sum of deviations of 50 observations from 30 is 50, then the mean of these observations is :

- (A) 51 (B) 50 (C) 30 (D) 31

Sol. D

$$\sum_{i=1}^{50} (x_i - 30) = 50$$

$$\sum x_i = 50 \times 30 + 50$$

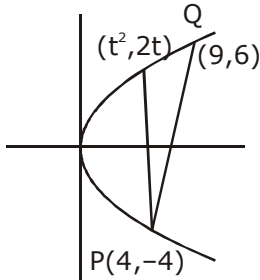
$$\text{Mean} = \bar{x} = \frac{\sum x_i}{N} = \frac{50 \times 30 + 50}{50}$$

$$= 30 + 1 = 31$$

24. Let $P(4, -4)$ and $Q(9,6)$ be two points on the parabola, $y^2 = 4x$ and let X be any point on the arc PQ of this parabola, where O is the vertex of this parabola, such that the area of ΔPXQ is maximum then this maximum area (in sq. units) is :

- (A) $\frac{625}{4}$ (B) $\frac{75}{2}$ (C) $\frac{125}{4}$ (D) $\frac{125}{2}$

Sol. C



$$y^2 = 4x$$

$$2yy' = 4 \Rightarrow y' = \frac{2}{y} = \frac{2}{2t} = \frac{1}{t}$$

max. Area will happen when tangent at $(t^2, 2t)$

$$||^r \text{ to } m_{PQ} = \frac{6+4}{9-4} = 2$$

$$\frac{1}{t} = 2 \Rightarrow t = \frac{1}{2} \therefore \text{pt}(t^2, 2t) = \left(\frac{1}{4}, 1\right)$$

$$\text{area} = \frac{125}{4} \text{ sq. units}$$

25. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$ and $Q = [q_{ij}]$ be two 3×3 matrices such that $Q - P^5 = I_3$. Then $\frac{q_{21} + q_{31}}{q_{32}}$ is equal

to :

- (A) 135 (B) 9 (C) 10 (D) 15

Sol. C

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix} \& P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 3+3 & 1 & 0 \\ 9+9+9 & 3+3 & 1 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 1 & 0 & 0 \\ 3+3+3 & 1 & 0 \\ 6.9 & 3+3+3 & 1 \end{bmatrix}$$

$$P^n = \begin{bmatrix} 1 & 0 & 0 \\ 3n & 1 & 0 \\ \frac{n(n+1)}{2} \cdot 3^2 & 3n & 1 \end{bmatrix} \Rightarrow P^5 = \begin{bmatrix} 1 & 0 & 0 \\ 5.3 & 1 & 0 \\ 15.9 & 5.3 & 1 \end{bmatrix}$$

$$Q = P^5 + I_3 = \begin{bmatrix} 2 & 0 & 0 \\ 15 & 2 & 0 \\ 135 & 15 & 2 \end{bmatrix}$$

$$\frac{q_{21} + q_{31}}{q_{32}} = \frac{15 + 135}{15} = 10$$

26. The product of three consecutive terms of a G.P. is 512. If 4 is added to each of the first and the second of these terms, the three terms now form an A.P. Then the sum of the original three terms of the given G.P. is

(A) 28 (B) 36 (C) 32 (D) 24

Sol. A

Let terms are $\frac{a}{r}, a, ar \rightarrow$ G.P

$$\therefore a^3 = 512 \Rightarrow a = 8$$

$$\frac{8}{r} + 4, 12, 5r \rightarrow \text{A.P.}$$

$$24 = \frac{8}{r} + 5 + 8r$$

$$r = 2, r = \frac{1}{2}$$

$$r = 2 (4, 8, 16)$$

$$r = \frac{1}{2} (16, 8, 4)$$

$$\text{Sum} = 28$$

27. The perpendicular distance from the origin to the plane containing the two lines,

$$\frac{x+2}{3} = \frac{y-2}{5} = \frac{z+5}{7} \text{ and } \frac{x-1}{1} = \frac{y-4}{4} = \frac{z+4}{7} \text{ is :}$$

(A) 11 (B) $11\sqrt{6}$ (C) $11/\sqrt{6}$ (D) $6\sqrt{11}$

Sol. C

$$\begin{vmatrix} i & j & k \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix}$$

$$\hat{i}(35 - 28) - \hat{j}(21 - 7) + \hat{k}(12 - 5)$$

$$7\hat{i} - 14\hat{j} + 7\hat{k} \Rightarrow \hat{i} - 2\hat{j} + \hat{k}$$

$$1(x + 2) - 2(y - 2) + 1(z + 15) = 0$$

$$x - 2y + z + 11 = 0$$

$$\frac{11}{\sqrt{4+1+1}} = \frac{11}{\sqrt{6}}$$

28. The Boolean expression $((p \wedge q) \vee (p \vee \sim q)) \wedge (\sim p \wedge \sim q)$ is equivalent to :

(A) $p \wedge (\sim q)$ (B) $p \wedge q$ (C) $p \vee (\sim q)$ (D) $(\sim p) \wedge (\sim q)$

Sol. C

29. A ratio of the 5th term from the beginning to the 5th term from the end in the binomial expansion

of $\left(2^{1/3} + \frac{1}{2(3)^{1/3}}\right)^{10}$ is

- (A) $1 : 4(16)^{1/3}$ (B) $1 : 2(6)^{1/3}$ (C) $2(36)^{1/3} : 1$ (D) $4(36)^{1/3} : 1$

Sol. D

$$\frac{T_5}{T_5^1} = \frac{{}^{10}C_4 (2^{1/3})^{10-4} \left(\frac{1}{2(3)^{1/3}}\right)^4}{{}^{10}C_4 \left(\frac{1}{2(3)^{1/3}}\right)^{10-4} (2^{1/3})^4} = 4 \cdot (36)^{1/3} : 1$$

30. Let f and g be continuous functions on $[0, a]$ such that $f(x) = f(a - x)$ and $g(x) + g(a - x) = 4$, then

$\int_0^a f(x)g(x)dx$ is equal to :

- (A) $-3\int_0^a f(x)dx$ (B) $4\int_0^a f(x)dx$ (C) $\int_0^a f(x)dx$ (D) $2\int_0^a f(x)dx$

Sol. D

$$I = \int_0^a f(x)g(x)dx$$

$$I = \int_0^a f(a-x)g(a-x)dx$$

$$I = \int_0^a f(x)(4 - g(x))dx$$

$$I = 4\int_0^a f(x)dx - I$$

$$\Rightarrow I = 2\int_0^a f(x)dx$$