JEE MAIN

QUESTION PAPER WITH SOLUTION

MATHEMATIC

26^{th} June 2022 | Shift - 2

MOTION®

JEE (Main+Advanced) | NEET | NTSE | Olympiads | Boards

SECTION - A

1. Let f : R → R be defined as f(x) = x – 1 and g : R – {1 , - 1) → R, be defined as $g(x) = \frac{x^2}{x^2-1}$ Then the function fog is

(A) one – one but not onto
(B) onto but not one - one
(C) both one – one and onto
(D) neither one – one nor onto

Sol. **(D)**

$f(g(x)) = \frac{x^2}{x^2-1} - 1 = \frac{1}{x^2-1}$

since fog is even function

⇒ many – one function

let $y = \frac{1}{x^2-1}$

$\Rightarrow x^2 - 1 = \frac{1}{y} \Rightarrow x^2 = \frac{1+y}{y}$

$\Rightarrow \frac{1+y}{y} \geq 0$

$\Rightarrow y \in (-\infty, -1] \cup (0, \infty)$

∵ Range ≠ co-domain

⇒ Into - function

2. If the system of equations
αx + y + z = 5, x + 2y + 3z = 4, x + 3y + 5z = β . has infinitely many solutions, then the ordered pair (α, β) is equal to :

(A) (1, -3)
(B) (-1, 3)
(C) (1, 3)
(D) (-1, -3)

Sol. **(C)**

For infinitely many solutions

$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$\Delta = 0$

$\Rightarrow \begin{vmatrix} \alpha & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{vmatrix} = 0$

⇒ α (10 – 9) – 1(5 – 3) + 1(3 – 2) = 0 ⇒ α = 1

$\Delta_1 = 0$

$\begin{vmatrix} 5 & 1 & 1 \\ 4 & 2 & 3 \\ \beta & 3 & 5 \end{vmatrix} = 0$

⇒ 5 – 1(20 – 3β) + (12 – 2β) =0

⇒ β = 3

3. If $A = \sum_{n=1}^{\infty} \frac{1}{(3+(-1)^n)^n}$ and $B = \sum_{n=1}^{\infty} \frac{(-1)^n}{(3+(-1)^n)^n}$, then $\frac{A}{B}$ is equal to :

(A) $\frac{11}{9}$
(B) 1
(C) $\frac{-11}{9}$
(D) $\frac{-11}{3}$

Sol. **(C)**

$A = \sum_{n=1}^{\infty} \frac{1}{(3+(-1)^n)^n}$

$= \frac{1}{2} + \frac{1}{4^2} + \frac{1}{2^3} + \frac{1}{4^4} \ldots\ldots$

$= \left(\frac{\frac{1}{2}}{1-\frac{1}{4}}\right) + \frac{\frac{1}{16}}{1-\frac{1}{16}}$

$= \frac{2}{3} + \frac{1}{15} = \frac{11}{15}$

$B = \sum_{n=1}^{\infty} \frac{(-1)^n}{(3+(-1)^n)^n}$

$= \frac{-1}{2} + \frac{1}{4^2} - \frac{1}{2^3} + \frac{1}{4^4}$........

$= -\left(\frac{\frac{1}{2}}{1-\frac{1}{4}}\right) + \frac{\frac{1}{6}}{1-\frac{1}{16}}$

$= \frac{-2}{3} + \frac{1}{15} = \frac{-9}{15}$

$\frac{A}{B} = \frac{-11}{9}$

4. $\lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^4}$ is equal to

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{6}$ (D) $\frac{1}{12}$

Sol. **(C)**

$\lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^4}$

$= \lim_{x\to 0} \frac{2\sin\left(\frac{x+\sin x}{2}\right)\sin\left(\frac{x-\sin x}{2}\right)}{x^4}$

$= \lim_{x\to 0} \frac{\left(\frac{x+\sin x}{2}\right)\left(\frac{x-\sin x}{2}\right).2\sin\left(\frac{x+\sin x}{2}\right)\sin\left(\frac{x-\sin x}{2}\right)}{x^4 \times \left(\frac{x+\sin x}{2}\right)\left(\frac{x-\sin x}{2}\right)}$

$= \lim_{x\to 0} \frac{x^2 - \sin^2 x}{2x^4}$

$= \lim_{x\to 0} \frac{x^2 - \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots\right)^2}{2x^4}$

$= \lim_{x\to 0} \frac{x^2 - (x^2 - \frac{2x^4}{6} + \ldots)}{2x^4}$

$= \lim_{x\to 0} \frac{\left(\frac{x^4}{3}\right)}{2x^4} = \frac{1}{6}$

5. Let f(x) = min {1, 1 + x sinx}, $0 \le x \le 2\pi$, if m is the number of point, where f is not differentiable and n is the number of point, where f is not continuous, then the ordered pair (m, n) is equal to

(A) (2, 0) (B) (1, 0) (C) (1, 1) (D) (2, 1)

Sol. **(B)**

f(x) = min{1 , 1 + x sinx}

$f(x) = \begin{cases} 1; & 0 \le x \le \pi \\ 1 + x\sin x; & \pi < x \le 2\pi \end{cases}$

at x = 0

$f(0) = \lim_{x\to 0^+} f(x) = 1$

at x = π

$f(\pi) = \lim_{x\to \pi^-} f(x) = \lim_{x\to \pi^+} f(x) = 1$

at x = 2π

$f(2\pi) = \lim_{x\to 2\pi^-} f(x) = 1$

function is continuous everywhere

differentiability

at x = π

$f'(x) = \begin{cases} 0\,;\, 0 \le x \le \pi \\ x\cos x + \sin x\,;\; x < x \le 2\pi \end{cases}$

$f'(\pi) = \begin{cases} 0\,; 0 \le x \le \pi \\ -\pi; \pi < x \le 2\pi \end{cases}$

L.H.D ≠ R.H.D

f(x) is not differentiable at x = π

6. Consider a cuboid of sides 2x, 4x and 5x and a closed hemisphere of radius r. if the sum of their surface areas is a constant k, then the ratio x : r, for which the sum of their volumes is maximum, is

(A) 2 : 5 (B) 19 : 45 (C) 3 : 8 (D) 19 : 15

Sol. **(B)**

Total surface area = $76x^2 + 3\pi r^2 = k$

$\Rightarrow r = \left(\frac{k-76x^2}{3\pi}\right)^{\frac{1}{2}}$

total volume (v) = $40x^3 + \frac{2}{3}\pi r^3 = 40x^3 + \frac{2}{3}\pi\left(\frac{k-76x^2}{3\pi}\right)^{3/2}$

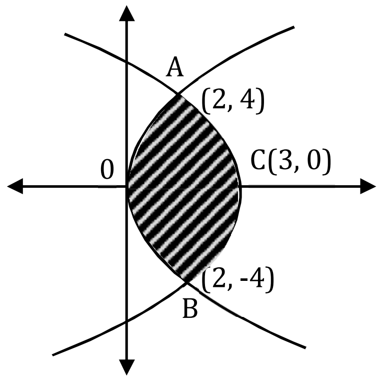
$\frac{dv}{dx} = 120x^2 + \frac{2\pi}{3}\left(\frac{3}{2}\right)\left(\frac{k-76x^2}{3\pi}\right)^{\frac{1}{2}}\left(\frac{-152x}{3\pi}\right)$

Put $\frac{dv}{dx} = 0 \Rightarrow \frac{x}{r} = \frac{19}{45}$

7. The area of the region bounded by $y^2 = 8x$ and $y^2 = 16(3 - x)$ is equal to :

(A) $\frac{32}{3}$ (B) $\frac{40}{3}$ (C) 16 (D) 19

Sol. **(C)**



Given curves

$y^2 = 16(3 - x)$ and $y^2 = 8x$

$8x = 16(3 - x)$

$\Rightarrow x = 6 - 2x$

$\Rightarrow x = 2$

$y = \pm 4$

Area bounded between curves

A = 2(area OAC)

$= 2\int_0^4 [(3 - \frac{y^2}{16}) - \frac{y^2}{8}]dy = 2\int_0^4 \left(3 - \frac{3y^2}{16}\right)dy$

$= 2\left[3y - \frac{y^3}{16}\right]_0^4$

$= 2[12 - 4] = 16$

8. If $\int \frac{1}{x}\sqrt{\frac{1-x}{1+x}}dx = g(x) + c$, g(1) = 0 then $g\left(\frac{1}{2}\right)$ is equal to :

(A) $\log_e\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right) + \frac{\pi}{3}$ (B) $\log_e\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) + \frac{\pi}{3}$ (C) $\log_e\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) - \frac{\pi}{3}$ (D) $\frac{1}{2}\log_e\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right) - \frac{\pi}{6}$

Sol. **(A)**

Let $I = \int \frac{1-x}{x\sqrt{1-x^2}}dx$

$= \int \frac{1}{x\sqrt{1-x^2}}dx - \int \frac{1}{\sqrt{1-x^2}}dx$

Put $x = \frac{1}{t}$

$dx = -\frac{1}{t^2}$

$= \int \frac{\frac{-1}{t^2}}{\frac{1}{t}\sqrt{1-\frac{1}{t^2}}}dt - \sin^{-1}(x) + C_1$

$= \int \frac{-dt}{\sqrt{t^2-1}}$

$= -\ln\left|t + \sqrt{t^2-1}\right|$

$= -\ln\left|\frac{1}{x} + \sqrt{\frac{1}{x^2} - 1}\right| - \sin^{-1}(x) + C_1$

$= -\ln\left|\frac{1}{x} + \sqrt{\frac{1}{x^2} - 1}\right| - \left(\frac{\pi}{2} - \cos^{-1}x\right) + C_1$

$= -\ln\left|\frac{1}{x} + \sqrt{\frac{1}{x^2} - 1}\right| + \cos^{-1}(x) - \frac{\pi}{2} + C_1$

$g(x) = \cos^{-1}(x) - \ell n\left|\frac{1}{x} + \sqrt{\frac{1}{x^2} - 1}\right|$

$g(1) = \cos^{-1}(1) - \ell n|1| = 0$

$g\left(\frac{1}{2}\right) = \cos^{-1}\left(\frac{1}{2}\right) - \ell n|2 + \sqrt{3}|$

$= \frac{\pi}{3} - \ell n\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$

$= \frac{\pi}{3} + \ell n\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$

9. If y = y(x) is the solutions of the differential equation $\frac{dy}{dx} + 2y = xe^x$, y(1) = 0 then the local maximum value of the function z(x) = x^2y(x) – e^x, x ∈ R is :

(A) 1 - e (B) 0 (C) 1/2 (D) $\frac{4}{e} - e$

Sol. **(D)**

$\frac{dy}{dx} + \frac{2y}{x} = e^x$

Linear Differential Equation

$I.F. = e^{\int \frac{2}{x}dx} = x^2$

$yx^2 = \int e^x x^2 dx$

$yx^2 = x^2e^x - 2xe^x + 2e^x + c$

Put y(1) = 0

$= e - 2e + 2e + c \Rightarrow c = -e$

$z(x) = x^2 e^x - 2xe^x + 2e^x - e - e^x$

$= x^2 e^x - 2xe^x + e^x - e$

$z'(x) = x^2 e^x + 2xe^x - 2e^x - 2xe^x + e^x$

$= x^2 e^x - e^x = 0$

$\Rightarrow x = \pm 1$

$z''(x) = 2xe^x + xe^x - e^x$

$z''(1) = 2e + e - e = 2e > 0$

$z''(-1) = \frac{-2}{e} - \frac{1}{e} - \frac{1}{e} < 0$

Local maximum at x = – 1

$z(-1) = \frac{1}{e} + \frac{2}{e} + \frac{1}{e} - e = \frac{4}{e} - e$

10. If the solution of the differential equation $\frac{dy}{dx} + e^x(x^2-2)y = (x^2-2x)(x^2-2)e^{2x}$ satisfies y(0) = 0 then the value of y(2) is ....

(A) – 1 (B) 1 (C) 0 (D) e

Sol. **(C)**

$\frac{dy}{dx} + e^x(x^2-2)y = (x^2-2x)(x^2-2)e^{2x}$

Linear D. E.

$I.F. = e^{\int e^x(x^2-2)dx}$

$= e^{(x^2-2)e^x - e^x(2x) + 2e^x}$

$\Rightarrow ye^{e^x(x^2-2x)} = \int e^{e^x(x^2-2x)(x^2-2x)(x^2-2)e^x.e^x}dx$

Put $e^x(x^2 - 2x) = t$

$[e^x (2x - 2) + e^x(x^2 - 2x)]dx = dt$

$e^x (x^2 - 2)dx = dt$

$ye^{e^x(x^2-2x)} = \int e^t.t\,dt$

$= te^t - e^t + C$

$= e^{e^x(x^2-2x)}[e^x(x^2-2x) - 1] + C$

Put y(0) = 0

0 = 1[–1] +C = c = 1

put x = 2

y = –1 + 1 = 0

11. If m is the slope of a common tangent to the curves $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and $x^2 + y^2 = 12$, then $12m^2$ is equal to :

(A) 6 (B) 9 (C) 10 (D) 12

Sol. **(B)**

tangent line to circle $x^2 + y^2 = 12$

$y = mx \pm \sqrt{12 + 12m^2}$ ... (i)

tangent line to ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

$y = mx \pm \sqrt{16m^2 + 9}$ .... (ii)

equation (i) and (ii) are identical

$mx \pm \sqrt{12 + 12m^2} = mx \pm \sqrt{16m^2 + 9}$

$\Rightarrow 12 + 12m^2 = 16m^2 + 9$

$\Rightarrow m^2 = \frac{3}{4}$

$12m^2 = \frac{3}{4} \times 12 = 9$

12. The locus of the mid point of the line segment joining the point (4, 3) and the points on the ellipse $x^2 + 2y^2 = 4$ is an ellipse with eccentricity :

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2\sqrt{2}}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{2}$

Sol. **(C)**

Let point on ellipse $Q(2\cos\theta, \sqrt{2}\sin\theta)$

given point P(4, 3)

mid point of P and Q

$(h, k) = \left(\frac{2\cos\theta + 4}{2}, \frac{\sqrt{2}\sin\theta + 3}{2}\right)$

$\cos\theta = \frac{2h-4}{2}, sin\theta = \frac{2k-3}{\sqrt{2}}$

squaring and adding

$(h-2)^2 + \left(\frac{2k-3}{\sqrt{2}}\right)^2 = 1$

$\frac{(x-2)^2}{1} + \frac{\left(y-\frac{3}{2}\right)^2}{\frac{1}{2}} = 1$

$e^2 = 1 - \frac{1}{2} = \frac{1}{2}$

$e = \frac{1}{\sqrt{2}}$

13. The normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{9} = 1$ at the point z$(8, 3\sqrt{3})$ on it passes through the point

(A) $(15, -2\sqrt{3})$ (B) $(9, 2\sqrt{3})$ (C) $(-1, 9\sqrt{3})$ (D) $(-1, 6\sqrt{3})$

Sol. **(C)**

$\frac{x^2}{a^2} - \frac{y^2}{9} = 1$, point $(8, 3\sqrt{3})$ will satisfy given equation

$\frac{64}{a^2} - \frac{27}{9} = 1$

$\Rightarrow a^2 = 16 \Rightarrow a = 4$

$\frac{x^2}{16} - \frac{y^2}{9} = 1$

Equation of normal

$\frac{x-x_1}{\frac{x_1}{a^2}} = \frac{y-y_1}{-\frac{y_1}{b^2}}$

Put $(x_1, y_1) = (8, 3\sqrt{3})$

$\Rightarrow \frac{x-8}{\left(\frac{8}{16}\right)} = \frac{y-3\sqrt{3}}{-\left(\frac{3\sqrt{3}}{9}\right)}$

$\Rightarrow 2(x-8) = -\sqrt{3}(4-3\sqrt{3})$

$\Rightarrow 2x + \sqrt{3}y - 25 = 0$

$(-1, 9\sqrt{3})$ satisfies equation.

14. If the plane $2x + y - 5z = 0$ is rotated about its line of intersection with the plane $3x - y + 4z - 7 = 0$ by an angle of $\frac{\pi}{2}$, then the plane after the rotation passes through the point :

(A) (2, - 2, 0) (B) (-2, 2, 0) (C) (1, 0 , 2) (D) (- 1, 0 , - 2)

Sol. **(C)**

Equation of family of planes passing through line of intersection of two planes $2x + y - 5z = 0$ and $3x - y + 4z - 7 = 0$

is $(3x - y + 4z - 7) + \lambda(2x + y - 5z) = 0$

$(3 + 2\lambda)x + (-1 + \lambda)y + (4 - 5\lambda)z - 7 = 0$ ... (i)

is perpendicular to plane

$2x + y - 5z = 9$

$(3 + 2\lambda).2 + (-1 + \lambda).1 + (4 - 5\lambda)(-5) = 0$

$\Rightarrow 6 + 4\lambda - 1 + \lambda - 20 + 25\lambda = 0$

$\Rightarrow 30\lambda - 15 = 0$

$\Rightarrow \lambda = \frac{15}{30} = \frac{1}{2}$

Put λ in equation (i)

$\left(3 + 2 \times \frac{1}{2}\right)x + \left(-1 + \frac{1}{2}\right)y + \left(4 - \frac{5}{2}\right)z - 7 = 0$

$\Rightarrow 4x - \frac{y}{2} + \frac{3}{2}z - 7 = 0$, $\Rightarrow 8x - y + 3z - 14 = 0$ $\Rightarrow$ Point (1, 0, 2) satisfy equation.

15. If the lines $\vec{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda(3\hat{j} - \hat{k})$ and $\vec{r} = (\alpha\hat{i} - \hat{j}) + \mu(2\hat{i} - 3\hat{k})$ are co-planer, the the distance of the plane containing these two lines from the point (α , 0 , 0) is :

(A) $\frac{2}{9}$ (B) $\frac{2}{11}$ (C) $\frac{4}{11}$ (D) 2

Sol. **(B)**

$L_1 : \vec{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda(3\hat{i} - \hat{k})$

$L_2 : \vec{r} = (\alpha\hat{i} - \hat{j}) + \mu(2\hat{i} - 3\hat{k})$

Normal vector to both lines

$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & -1 \\ 2 & 0 & -3 \end{vmatrix}$

= $\hat{i}(-9) - \hat{j}(2) + \hat{k}(-6)$

= $\vec{n} = -9\hat{i} - 2\hat{j} - 6\hat{k}$

equation of plane with normal vector $\vec{n}$ and containing point (1, -1, 1)

$\vec{r}.\vec{n} = \vec{a}.\vec{n}$

⇒ -9x – 2y -6z = - 9 + 2 – 6

⇒ 9x + 2y + 6z – 13 = 0

point (α , -1, 0) on plane

9α -2 – 13 = 0 ⇒ $\alpha = \frac{5}{3}$

distance of plane from $\left(\frac{5}{3}, 0, 0\right)$

$d = \frac{\left|9\left(\frac{5}{3}\right) - 13\right|}{\sqrt{81+4+36}} = \frac{2}{11}$

16. Let $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}, \vec{b} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + \hat{k}$ be three given vectors, Let $\vec{v}$ be a vector in the plane of $\vec{a}$ and $\vec{b}$ whose projection on $\vec{c}$ is $\frac{2}{\sqrt{3}}$. if $\vec{v}.\hat{j} = 7$, then $\vec{v}.(\hat{i} + \hat{k})$ is equal to :

(A) 6 (B) 7 (C) 8 (D) 9

Sol. **(D)**

$\vec{v} = x\vec{a} + y\vec{b}$

$\vec{v} = (x + 2y)\hat{i} + (x - 3y)\hat{j} + (2x + y)\hat{k}$

$\frac{\vec{v}.\vec{c}}{|\vec{c}|} = \frac{2}{\sqrt{3}}$

$\Rightarrow \frac{(x+2y).1+(x-3y)(-1)+(2x+y).1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$

⇒ 2 = 2x + 6y

⇒ x + 3y = 1 .... (i)

$\vec{v}.\hat{j} = 7$ ⇒ x – 3y = 7 .... (ii)

(i) – (ii)

6y = - 6 ⇒ y = -1

and x = 4

$\vec{v} = 2\hat{i} + 7\hat{j} + 7\hat{k}$

$\vec{v}.(\hat{i} + \hat{k})$ = 2 + 7 = 9

17. The mean and standard deviation of 50 observations are 15 and 2 respectively. it was found that one incorrect observation was taken such that the sum of correct and incorrect observations is 70. If the correct mean is 16. Then the correct variance is equal to

(A) 10 (B) 36 (C) 43 (D) 60

Sol. **(C)**

$\bar{x} = \frac{\sum x_i}{50} = 15$

$\Rightarrow \sum x_i = 750$

$\frac{\sum x_i^2}{50} - (\bar{x})^2 = \sigma^2$

$\Rightarrow \frac{\sum x_i^2}{50} - (15)^2 = 4$

$\Rightarrow \sum x_i^2 = 11450$

New $\bar{x} = 16$

$\sum x_{i_{new}} = 16 \times 50 = 800$

let a be the incorrect observation
then correct observation = a + 50
a + (a + 50) = 70
⇒ a = 10
correct observation a + 50 = 10 + 50 = 60

New variance = $\frac{11450 - 10^2 + 60^2}{50} - (16)^2$

= 299 – 256 = 43

18. 16sin(20°) sin(40°) sin(80°) is equal to :

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) 3 (D) $4\sqrt{3}$

Sol. (B)

16sin20° sin40°sin80°

= 16 sin20°. sin(60° - 20°). sin(60° + 20°) $\left(\because \sin(\theta)\cdot\sin(60-\theta)\cdot\sin(60+\theta) = \frac{1}{4}\sin 3\theta\right.$

$= 16\left(\frac{1}{4}\sin 60°\right)$

$= 4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$

19. If the inverse trigonometric functions take principal values, then

$\cos^{-1}\left(\frac{3}{10}\cos\left(\tan^{-1}\left(\frac{4}{3}\right)\right) + \frac{2}{5}\sin\left(\tan^{-1}\left(\frac{4}{3}\right)\right)\right)$ is equal to :

(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

Sol. (C)

$\cos^{-1}\left(\frac{3}{10}\cos\left(\tan^{-1}\left(\frac{4}{3}\right)\right) + \frac{2}{5}\sin\left(\tan^{-1}\left(\frac{4}{3}\right)\right)\right)$

$= \cos^{-1}\left(\frac{3}{10}.\frac{3}{5} + \frac{2}{5}.\frac{4}{5}\right)$

$= \cos^{-1}\left(\frac{9}{50} + \frac{8}{25}\right) = \cos^{-1}\left(\frac{25}{50}\right)$

$= \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

20. Let r ∈ {p, q, ~p, ~q} be such that the logical statement $r \vee (\sim p) \Rightarrow (p \wedge q) \vee r$ is a tautology. Then r is equal to :

(A) p (B) q (C) ~p (D) ~q

Sol. (C)

(A)

p = r	q	~p	r ∨ (~p)	(p∧ q)	(p∧ q)∨ r	r∨ (~p)⇒ (p∧ q)∨ r
T	F	F	T	F	T	T
F	T	T	T	F	F	F
T	T	F	T	T	T	T
F	F	T	T	F	F	T

(B)

p	~p	r ∨ (~p)	q = r	(p∧ q)	(p∧ q)∨ r	r∨ (~p)⇒ (p∧ q)∨ r
T	F	T	T	T	T	T
F	T	T	T	F	T	T
T	F	F	F	F	F	T
F	T	T	F	F	F	F

(C)

p	q	r = ~p	r ∨ (~p)	(p∧ q)	(p∧ q)∨ r	r∨ (~p)⇒ (p∧ q)∨ r
T	T	F	F	T	T	T
F	T	T	T	F	T	T
T	F	F	F	F	F	T
F	F	T	T	F	T	T

(D)

~p	p	q	r ∨ (~p)	r = ~q	(p ∧ q)	(p∧ q)∨ r	r∨ (~p)⇒ (p∧ q)∨ r
F	T	T	F	F	T	T	T
F	T	F	T	T	F	T	T
T	F	T	T	F	F	F	F
T	F	F	T	T	F	T	T

21. Let f : R → R satisfy $f(x + y) = 2^x f(y) + 4^y f(x)$, ∀ x, y ∈ R. If f(2) = 3, then $14.\frac{f'(4)}{f'(2)}$ is equal to .....

Sol. **(248)**

$f(x + y) = 2^x f(y) + 4^y f(x)$ ... (1)

x ⇔ y

$f(y + x) = 2^y f(x) + 4^x f(y)$ ... (2)

(1) – (2)

$0 = f(x)(4^y - 2^y) + f(y)(2^x - 4^x)$

$\Rightarrow \frac{f(x)}{(2^x-4^x)} = \frac{f(y)}{(2^y-4^y)} = \lambda(say)$

$\Rightarrow f(x) = \lambda(2^x - 4^x), f(y) = \lambda(2^y - 4^y)$

$f'(x) = \lambda[2^x \ln^2 - 4^x \ln^4]$

$\frac{f'(4)}{f'(2)} = \frac{16\ln^2 - 256\ln^4}{4\ln^2 - 16\ln^4} = \frac{\ln^2[16-256\times2]}{\ln^2[4-32]} = \frac{496}{28} = \frac{248}{14} \Rightarrow$ Answer $= \frac{248}{14} \times 14 = 248$

22. Let P and q be two real numbers such that p + q = 3 and $p^4 + q^4 = 369$. then $\left(\frac{1}{p}+\frac{1}{q}\right)^{-2}$ is equal to .....

Sol. **(4)**

P + q = 3

⇒ $(p + q)^2 = 9$

⇒ $p^2 + q^2 + 2pq = 9$

given $p^4 + q^4 = 369$

$(p^2 + q^2)^2 - 2(pq)^2 = 369$

⇒ $(9 - 2pq)^2 - 2(pq)^2 = 369$

⇒ $81 - 36pq + 4p^2q^2 - 2p^2q^2 = 369$

⇒ $2(pq)^2 - 36(pq) - 288 = 0$

⇒ $(pq)^2 - 18(pq) - 144 = 0$

pq = 24, – 6

if pq = 24

$p^2 + q^2 = 9 - 2(pq) =$ –ve(not possible)

take pq = – 6

$\left(\frac{1}{p}+\frac{1}{q}\right)^{-2} = \left(\frac{pq}{p+q}\right)^2 = \left(\frac{-6}{3}\right)^2 = 4$

23. If $z^2 + z + 1 = 0$, $z \in C$, then $\left|\sum_{n=1}^{15}\left(z^n + (-1)^n\frac{1}{z^n}\right)^2\right|$ is equal to ......

Sol. **(2)**

$z^2 + z + 1 = 0 \Rightarrow z = \omega, \omega^2$

$\left|\sum_{n=1}^{15}\left(z^n + (-1)^n\frac{1}{z^n}\right)^2\right|$

$= \left|\sum_{n=1}^{15} z^{2n} + \sum_{n=1}^{15}\frac{1}{z^{2n}} + \sum_{n=1}^{15}(-1)^n\right|$

$= \left|\sum_{n=1}^{15} \omega^{2n} + \sum_{n=1}^{15}\frac{1}{\omega^{2n}} + \sum_{n=1}^{15}(-1)^n\right|$ $\left[\omega = \frac{1}{\omega^2}\right]$

$= \left|\sum_{n=1}^{15} \omega^{2n} + \sum_{n=1}^{15}\omega^n + \sum_{n=1}^{15}(-1)^n\right|$

$= \left|(\omega^2 + \omega^4 + \omega^6 \ldots \omega^{30}) + (\omega + \omega^2 + \omega^3 + \ldots \omega^{15}) + (-1 + 1 - 1 + 1 \ldots -1)\right|$

$= \left|\omega^2(1 + \omega^2 + \omega^3 + \ldots \omega^{15}) + \omega(1 + \omega + \omega^2 + \ldots + \omega^{14}) - 1\right|$

$= \left|\omega^2\left(\frac{1-\omega^{15}}{1-\omega}\right) + \omega\left(\frac{1-\omega^{14}}{1-\omega}\right) - 1\right|$

$= |\omega^2(0) + \omega(1 + \omega) - 1|$

$= |\omega^2 + \omega - 1| = |-1 - 1| = 2$

24. Let $X = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, $y = \alpha I + \beta X + \gamma X^2$ and $Z = \alpha^2 I - \alpha\beta X + (\beta^2 - \alpha\gamma)X^2, \alpha, \beta, \gamma \in R$,

If $Y^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{-2}{5} & \frac{1}{5} \\ 0 & \frac{1}{5} & \frac{-2}{5} \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$, then $(\alpha - \beta + \gamma)^2$ is equal to :

Sol. **(100)**

$$X=\begin{bmatrix}0&1&0\\0&0&1\\0&0&0\end{bmatrix}$$

$$X^2=\begin{bmatrix}0&0&1\\0&0&0\\0&0&0\end{bmatrix}$$

$$Y=\begin{bmatrix}\alpha&0&0\\0&\alpha&0\\0&0&\alpha\end{bmatrix}+\begin{bmatrix}0&\beta&0\\0&0&\beta\\0&0&0\end{bmatrix}+\begin{bmatrix}0&0&\gamma\\0&0&0\\0&0&0\end{bmatrix}$$

$$\Rightarrow Y=\begin{bmatrix}\alpha&\beta&\gamma\\0&\alpha&\beta\\0&0&\alpha\end{bmatrix}$$

$Y Y^{-1} = I$

$$\Rightarrow\begin{bmatrix}\alpha&\beta&\gamma\\0&\alpha&\beta\\0&0&\alpha\end{bmatrix}\begin{bmatrix}\frac{1}{5}&\frac{-2}{5}&\frac{1}{5}\\0&\frac{1}{5}&\frac{-2}{5}\\0&0&\frac{1}{5}\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$\begin{bmatrix}\frac{\alpha}{5}&\frac{-2\alpha+\beta}{5}&\frac{\alpha-2\beta+\gamma}{5}\\0&\frac{\alpha}{5}&\frac{-2\alpha+\beta}{5}\\0&0&\frac{\alpha}{5}\end{bmatrix}$$

⇒ Comparing every elements

$\alpha = 5, \beta = 10, \gamma = 15$

$(\alpha-\beta+\gamma)^2=(5-10+15)^2=100$

25. The total number of 3 - digit numbers, whose greatest common divisor with 36 is 2.

Sol. **(150)**

$36 = 2^2 \times 3^2$

to get GCD (n, 36) = 2

Power of 2 in n must be exactly 1

⇒ Number is divisible by 2 but not divisible by 4 and not divisible by 3 also.

Total 3 digit numbers

= (divisible by 2) – (divisible by 4)

–(divisible by 3) + (divisible by 12)

= 451 – 226 – 150 + 75

= 150

26. If $({}^{40}C_0)+({}^{41}C_1)+({}^{42}C_2)+....+({}^{60}C_{20})=\frac{m}{n}\ {}^{60}C_{20}$ m and n are coprime, then m + n is equal to

Sol. **(102)**

${}^{40}C_0 + {}^{41}C_1 + {}^{42}C_2 + ... + {}^{60}C_{20} = \frac{m}{n}({}^{60}C_{20})$

${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$

$= {}^{41}C_0 + {}^{41}C_1 + {}^{42}C_2 + ... + {}^{60}C_{20}$ $\quad (\because {}^{40}C_0 = {}^{41}C_0)$

$= {}^{42}C_1 + {}^{42}C_2 + ... + {}^{60}C_{20}$

$= {}^{60}C_{19} + {}^{60}C_{20} = {}^{61}C_{20} = \frac{m}{n}\ {}^{60}C_{20}$

$\Rightarrow \frac{61!}{20!41!} = \frac{m}{n}\left(\frac{60!}{20!40!}\right)$

$\Rightarrow \frac{61}{41} = \frac{m}{n}$

m + n = 102

27. If a_1 (> 0), a_2, a_3, a_4, a_5 are in a G.P., $a_2 + a_4 = 2a_3 + 1$ and $3a_2 + a_3 = 2a_4$, then $a_2 + a_4 + 2a_5$ is equal to .....

Sol. **(40)**

$a_2 + a_4 = 2a_3 + 1$

$\Rightarrow a_1r + a_1r^3 = 2a_1r^2 + 1$ ... (i) (r = common ratio)

and $3a_2 + a_3 = 2a_4$

$\Rightarrow 3a_1r + a_1 r^2 = 2a_1r^3$

$\Rightarrow 2r^2 - r - 3 = 0$

$(2r - 3)(r + 1)$

$\Rightarrow r = -1, 3/2$

for r = – 1

$-a_1, -a_1 = 2a_1 + 1$

$a_1 = \frac{-1}{4}(rejected)$

Hence $r = \frac{3}{2}$ put in equation (i)

$a_1\left(\frac{3}{2} + \frac{27}{8} - \frac{9}{2}\right) = 1$

$\Rightarrow a_1\left(\frac{12+27-36}{8}\right) = 1$

$\Rightarrow a_1 = \frac{8}{3}$

$a_2 + a_4 + 2a_5 = a_1r + a_1r^3 + 2a_1r^4$

$= \frac{3}{2}\left(\frac{8}{3}\right) + \frac{8}{3}\left(\frac{27}{8}\right) + 2\left(\frac{8}{3}\right)\left(\frac{81}{16}\right)$

= 4 + 9 + 27

= 40

28. The integral $\frac{24}{\pi}\int_0^{\sqrt{2}} \frac{(2-x^2)dx}{(2+x^2)\sqrt{4+x^4}}$ is equal to ......

Sol. **(3)**

$\frac{24}{\pi}\int_0^{\sqrt{2}} \frac{(2-x^2)dx}{(2+x^2)\sqrt{4+x^4}}$

let $I = \int_0^{\sqrt{2}} \frac{2-x^2}{(2+x^2)\sqrt{4+x^4}} dx$

$= \int_0^{\sqrt{2}} \frac{(2-x^2)}{x(\frac{2}{x}+x)x\sqrt{\frac{4}{x^2}+x^2}} dx$

$= \int_0^{\sqrt{2}} \frac{(\frac{2}{x^2}-1)}{(\frac{2}{x}+x)\sqrt{(x+\frac{2}{x})^2-2^2}} dx$

Put $x + \frac{2}{x} = t$

$\left(1 - \frac{2}{x^2}\right) dx = dt$

$I = -\int_{\infty}^{2\sqrt{2}} \frac{dt}{t.\sqrt{t^2-2^2}}$

$= \frac{1}{2}\left[\sec^{-1}\frac{t}{2}\right]_{2\sqrt{2}}^{\infty}$

$= \frac{1}{2}\left[\sec^{-1}(\infty) - \sec^{-1}(\sqrt{2})\right]$

$= \frac{1}{2}\left[\frac{\pi}{2} - \frac{\pi}{4}\right]$

$= \frac{\pi}{8}$

Answer = $\frac{24}{\pi} \times \frac{\pi}{8} = 3$

29. Let a line L_1 be tangent to the hyperbola $\frac{x^2}{16} - \frac{y^2}{4} = 1$ and Let L_2 be the line passing through the origin and perpendicular to L_1. if the locus of the point intersection of L_1 and L_2 is $(x^2 + y^2)^2 = \alpha x^2 + \beta y^2$, , then $\alpha + \beta$ is equal to ......

Sol. **(12)**

$\frac{x^2}{16} - \frac{y^2}{4} = 1$

equation of tangent to hyperbola

$y = mx \pm \sqrt{a^2m^2 - b^2}$

$\Rightarrow y = mx \pm \sqrt{16m^2 - 4}$ ... (i)

equation of line perpendicular to tangent line and passing through origin

$y = \frac{-x}{m}$

Put $m = \frac{-x}{y}$

to get locus of point of intersection

$y = \frac{-x^2}{y} \pm \sqrt{\frac{16x^2}{y^2} - 4}$

$\Rightarrow \left(y + \frac{x^2}{y}\right)^2 = \frac{16x^2 - 4y^2}{y^2}$

$\Rightarrow (x^2 + y^2)^2 = 16x^2 - 4y^2$

$(\alpha, \beta) = (16, -4)$

$\alpha + \beta = 12$

30. If the probability that a randomly chosen 6 – digit number formed by using digits 1 and 8 only is a multiple of 21 is p, then 96 p is equal to ...

Sol. **(33)**

6 digit numbers formed by digits 1 and 8 only

for a number to be multiple of 21 it must be divisible by both 3 and 7

To make divisible by '3'

sum must be divisible by 3

possible cases

(i) All digits are 1's → 1

(ii) All digits are 8's → 1

(iii) 3 1's and 3 8's → $\frac{6!}{3!3!}$ = 20

To make divisible by 7

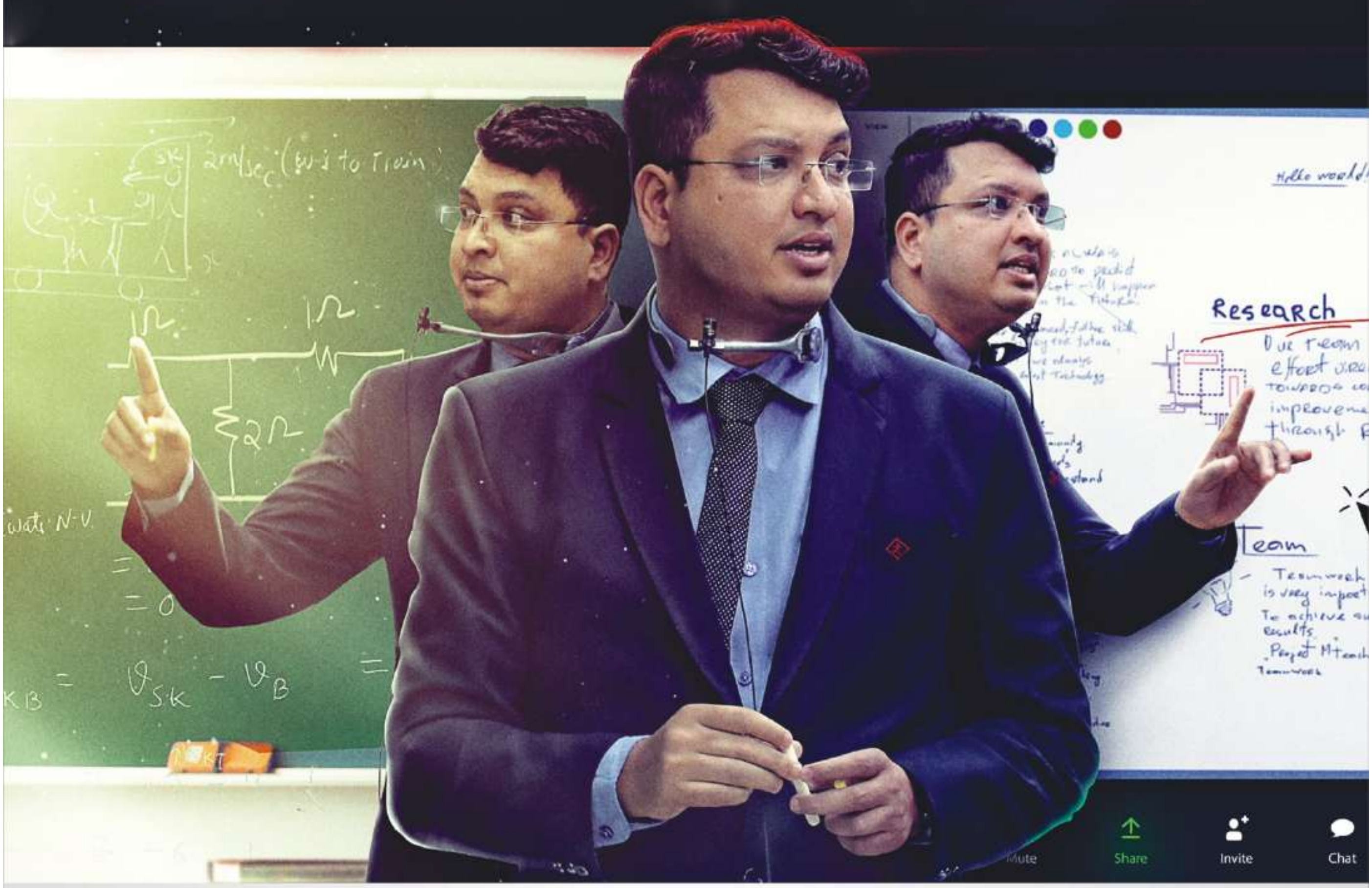
|2(last digit) – (remaining number)| = 7k , k ∈ Z

Numbers formed in all three cases are also divisible by 7

total possibilities = 2^6

total numbers divisible by 21 = 20 + 1 + 1 = 22

$P = \frac{22}{2^6} \Rightarrow 96P = \frac{22}{64} \times 96 = 33$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