JEE MAIN

QUESTION PAPER WITH SOLUTION

MATHEMATICS

25^{th} June 2022 | Shift - 1

Umeed Rank Ki Ho Ya Selection Ki,
JEET NISCHIT HAI!

MOST PROMISING RANKS
PRODUCED BY MOTION FACULTIES

NEET / AIIMS

- AIR-1 TO 10 — 25 TIMES
- AIR-11 TO 25 — 37 TIMES
- AIR-26 TO 50 — 43 TIMES
- AIR-51 TO 100 — 78 TIMES

JEE MAIN+ADVANCED

- AIR-1 TO 10 — 8 TIMES
- AIR-11 TO 25 — 6 TIMES
- AIR-26 TO 50 — 18 TIMES
- AIR-51 TO 100 — 30 TIMES

MOTION®
JEE I NEET I NTSE I BOARDS I OLYMPIADS

NATION'S BEST SELECTION
PERCENTAGE (%) RATIO

STUDENT QUALIFIED IN NEET

- 2021: 3276 / 3411 = 93.12%
- 2020: 2663 / 2843 = 93.66%
- 2019: 2041 / 2212 = 92.27%

STUDENT QUALIFIED IN JEE ADVANCED

- 2021: 1256 / 2994 = 41.95%
- 2020: 994 / 2538 = 39.16%
- 2019: 769 / 2105 = 36.53%

STUDENT QUALIFIED IN JEE MAIN

- 2021: 2994 / 4087 = 73.25%
- 2020: 2538 / 3554 = 71.44%
- 2019: 2288 / 3316 = 68.99%

NITIN VIJAY (NV Sir)
Founder & CEO

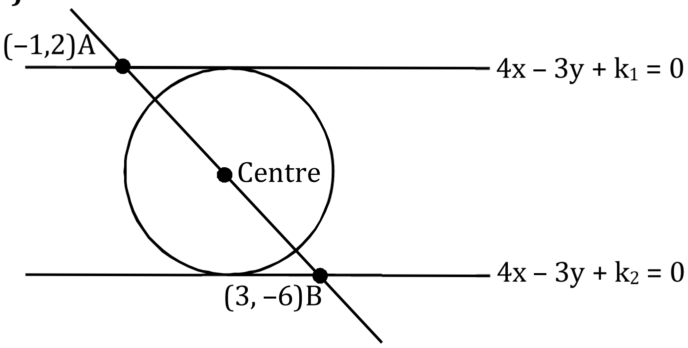
SECTION - A

1. Let a circle C touch the lines $L_1 : 4x - 3y + k_1 = 0$ and $L_2 : 4x - 3y + k_2 = 0$, $k_1, k_2 \in R$. If a line passing through the centre of the circle C intersects L_1 at (-1, 2) and L_2 at (3, -6), then the equation of the circle C is :

(A) $(x - 1)^2 + (y - 2)^2 = 4$ (B) $(x + 1)^2 + (y - 2)^2 = 4$

(C) $(x - 1)^2 + (y + 2)^2 = 16$ (D) $(x - 1)^2 + (y - 2)^2 = 16$

Sol. **(C)**



$L_1 : 4x - 3y + k_1 = 0$ (put A in L_1)

$-4 - 6 + k_1 = 0$

$k_1 = 10$

$L_2 : 4x - 3y + k_2 = 0$

Put B in L_2

$12 + 18 + k_2 = 0$

$K_2 = -30$

distance between L_1 and L_2 = $diameter = \left|\frac{40}{\sqrt{4^2+3^2}}\right| = 8$

$\therefore$ radius = 4

Centre is mid point of AB $\Rightarrow$ center is (1, - 2)

$\therefore$ circle is $(x - 1)^2 + (y + 2)^2 = 16$ Ans.

2. The value of $\int_0^{\pi} \frac{e^{\cos x}\sin x}{(1+\cos^2 x)(e^{\cos x}+e^{-\cos x})} dx =$

(A) $\frac{\pi^2}{4}$ (B) $\frac{\pi^2}{2}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

Sol. **(C)**

$I = \int_0^{\pi} \frac{e^{\cos x}\sin x\, dx}{(1+\cos^2 x)(e^{\cos x}+e^{-\cos x})}$ .... (A)

Replace $x \to \pi - x$

$I = \int_0^{\pi} \frac{e^{-\cos x}.\sin x\, dx}{(1+\cos^2 x)(e^{\cos x}+e^{-\cos x})}$ .... (B)

Add (A) and (B)

$2I = \int_0^{\pi} \frac{\sin x\, dx}{1+\cos^2 x}$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin x\ dx}{1+\cos^2 x}$

Put

cosx = t

– sinx dx = dt

$$I = -\int_1^0 \frac{dt}{1+t^2} = [\tan^{-1}t]_0^1 = \frac{\pi}{4} \text{ Ans.}$$

3. Let a, b and c be the length of sides of a triangle ABC such that $\frac{a+b}{7} = \frac{b+c}{8} = \frac{c+a}{9}$. If r and R are the radius of incircle and radius of circumcircle of the triangle ABC, respectively, then the value of R/r is equal to :

(A) $\frac{5}{2}$ (B) 2 (C) $\frac{3}{2}$ (D) 1

Sol. **(A)**

a + b = 7k

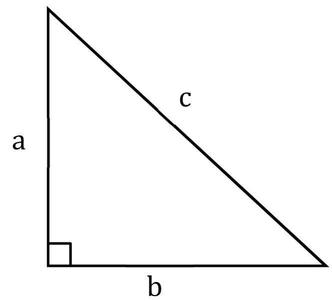
b + c = 8k

c + a = 9k

⇒ a + b + c = 12k

∴ a = 4k, b = 3k, c = 5k

ΔABC will be Right angled triangle



∴ 2R = c

$R = \frac{5k}{2}$

$r = \frac{\Delta}{S} = \frac{\left(\frac{1}{2}\times 4\times 3\right)k^2}{6k}$

r = k

$\Rightarrow \frac{R}{r} = \frac{5}{2}$

4. If f : N → R be a function such that f(x + y) = 2f(x)f(y) for natural numbers x and y. If f(1) = 2, then the value of α for which $\sum_{k=1}^{10} f(\alpha+k) = \frac{512}{3}(2^{20}-1)$

(A) 2 (B) 3 (C) 4 (D) 6

Sol. **(C)**

f(x + y) = 2f(x) · f(y) f(1) = 2

Put x = y = 1 f(2) = 2 · 2 · 2 = 2^3

x = 1, y = 2 f(3) = 2 · f(1) · f(2) = 2 · 2 · 2^3 = 2^5

$T_n = 2\{4^{n-1}\} = f(n)$

f(α + k) = 2. f(α). f(k)

$\sum_{k=1}^{10} f(\alpha+k) = 2f(\alpha)\sum_{k=1}^{10} f(k)$

$= 2f(\alpha)[2 + 2^3 + 2^5 + \ldots \text{upto 10 terms}]$ G.P.

$= 2f(\alpha)\left[\frac{2[4^{10}-1]}{4-1}\right]$

$= \frac{2}{3}f(\alpha)[2(2^{20}-1)] = \frac{512}{3}(2^{20}-1)$

⇒ 4f(α) = 512

⇒ f(α) = 128

$\Rightarrow 128 = 2 \cdot 4^{n-1}$

$\Rightarrow 64 = 4^{n-1} = 4^3$

⇒ n = 4

5. Let A be a 3 × 3 real matrix such that

$$A\begin{pmatrix}1\\1\\0\end{pmatrix}=\begin{pmatrix}1\\1\\0\end{pmatrix};A\begin{pmatrix}1\\0\\1\end{pmatrix}=\begin{pmatrix}-1\\0\\1\end{pmatrix}\, and\ A\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}1\\1\\2\end{pmatrix}$$

If $X = (x_1, x_2, x_3)^T$ and I is an identity matrix of order 3, then the system $(A - 2I)X = \begin{pmatrix}4\\1\\1\end{pmatrix}$ has:

(A) no solution
(B) infinitely many solutions
(C) unique solution
(D) exactly two solutions

Sol. **(B)**

$$A=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\a_{21} & a_{22} & a_{23}\\a_{31} & a_{32} & a_{33}\end{bmatrix}$$

$$A\begin{bmatrix}1\\1\\0\end{bmatrix}=\begin{bmatrix}a_{11}+a_{12}\\a_{21}+a_{22}\\a_{31}+a_{32}\end{bmatrix}=\begin{bmatrix}1\\1\\0\end{bmatrix}\Rightarrow\begin{cases}a_{11}+a_{12}=1\\a_{21}+a_{22}=1\\a_{31}+a_{32}=0\end{cases}$$

$$A\begin{bmatrix}1\\0\\1\end{bmatrix}=\begin{bmatrix}-1\\0\\1\end{bmatrix}\Rightarrow\begin{matrix}a_{11}+a_{13}=-1\\a_{21}+a_{23}=0\\a_{31}+a_{33}=1\end{matrix}$$

$$A\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}1\\1\\2\end{pmatrix}\Rightarrow\begin{matrix}a_{13}=1\\a_{23}=1\\a_{33}=2\end{matrix}$$

Solving all the equations

$a_{11} = -2, a_{12} = 3$

$a_{31} = -1, a_{32} = 1$

$a_{21} = -1, a_{22} = 2$

$$A=\begin{pmatrix}-2 & 3 & 1\\-1 & 2 & 1\\-1 & 1 & 2\end{pmatrix}$$

$$A-2I=\begin{pmatrix}-4 & 3 & 1\\-1 & 0 & 1\\-1 & 1 & 0\end{pmatrix}$$

$$(A-2I)\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}4\\1\\1\end{pmatrix}\Rightarrow\begin{matrix}-4x_1+3x_2+x_3=4\\-x_1+x_3=1\\-x_1+x_2=1\end{matrix}$$

$\Delta = -1(3) - 1(-4 + 1)$

$= -3 + 3 = 0$

$$\Delta_1=\begin{pmatrix}4 & 3 & 1\\1 & 0 & 1\\1 & 1 & 0\end{pmatrix}=1(3)-1\,(4-1)=0$$

$$\Delta_2=\begin{pmatrix}-4 & 4 & 1\\-1 & 1 & 1\\-1 & 1 & 0\end{pmatrix}=0$$

$$\Delta_3=\begin{pmatrix}-4 & 3 & 4\\-1 & 0 & 1\\-1 & 1 & 1\end{pmatrix}=0$$

∴ Infinite solutions Ans.

6. Let f : R → R be defined as $f(x) = x^3 + x - 5$. If g(x) is a function such that f(g(x)) = x, ∀ x ∈ R, then g'(63) is equal to .....

(A) $\frac{1}{49}$ (B) $\frac{3}{49}$ (C) $\frac{43}{49}$ (D) $\frac{91}{49}$

Sol. **(A)**

$f(x) = x^3 + x - 5 \quad \Rightarrow \quad f'(x) = 3x^2 + 1$

and f(4) = 63

f(g(x)) = x $\quad \therefore g(x) = f^{-1}(x)$

$g'(f(x)) = \frac{1}{f'(x)}$

$\Rightarrow g'(f(x)) = \frac{1}{3x^2+1}$

$\Rightarrow g'(63) = \frac{1}{49}$, for x = 4

7. Consider the following two propositions

$P_1 : \sim(p \to \sim q)$

$P_2 : (p \wedge \sim q) \wedge ((\sim p) \vee q)$

If the proposition $p \to ((\sim p) \vee q)$ is evaluated as FALSE, then :

(A) P_1 is TRUE and P_2 is FALSE (B) P_1 is FALSE and P_2 is TRUE

(C) Both P_1 and P_2 are FALSE (D) Both P_1 and P_2 are TRUE

Sol. **(C)**

$P_1 : \sim(p \to \sim q)$

$P_2 : (p \wedge \sim q) \wedge ((\sim p) \vee q)$

$p \to ((\sim p) \vee q)$ is false

∴ p is true and $(\sim p) \vee q$ false

∴ q is false

P_1 is false

P_2 is false

8. If $\frac{1}{2\cdot 3^{10}} + \frac{1}{2^2\cdot 3^9} + \ldots + \frac{1}{2^{10}\cdot 3} = \frac{k}{2^{10}\cdot 3^{10}}$, then the remainder when k is divided by 6 is :

(A) 1 (B) 2 (C) 3 (D) 5

Sol. **(D)**

$S = \frac{1}{2\cdot 3^{10}} + \frac{1}{2^2\cdot 3^9} + \ldots + \frac{1}{2^{10}\cdot 3}$ is a G.P.

First term = $\frac{1}{2\cdot 3^{10}}$

$r = \frac{3}{2}$, n = 10

$$s = \frac{1}{2\cdot 3^{10}}\left\{\frac{\left(\frac{3}{2}\right)^{10}-1}{\frac{3}{2}-1}\right\} = \frac{1}{3^{10}}\left\{\frac{3^{10}-2^{10}}{2^{10}}\right\}$$

$$= \frac{3^{10}-2^{10}}{2^{10}\cdot 3^{10}}$$

$\therefore k = 3^{10} - 2^{10}$

$3^{10} - 2^{10} = (3^5 - 2^5)(3^5 + 2^5) = 211 \times 275$

= (210 + 1) (270 + 5)

= (6λ + 1)(6μ + 5)

∴ remainder = 5 Ans.

9. Let f(x) be a polynomial function such that f(x) + f' (x) + f"(x) = x^5 + 64. Then, the value of $\lim_{x\to1}\frac{f(x)}{x-1}$ is equal to :

(A) – 15 (B) – 60 (C) 60 (D) 15

Sol. **(A)**

f(x) + f'(x) + f"(x) = x^5 + 64

f is polynomial of degree 5

f(x) = $x^5 + ax^4 + bx^3 + cx^2 + dx + e$

f'(x) = $5x^4 + 4ax^3 + 3bx^2 + 2cx + d$

f"(x) = $20x^3 + 12ax^2 + 6bx + 2c$

∴ a + 5 = 0 ⇒ a = – 5

b + 4a + 20 = 0 ⇒ b = 0

c + 3b + 12a = 0 ⇒ c = + 60

d = –120 and e = 64

f(x) = $x^5 - 5x^4 + 60x^2 - 120x + 64$

= (x – 1) ($x^4 - 4x^3 - 4x^2 + 56x - 64$)

∴ $\lim_{x\to1}\frac{f(x)}{x-1}$ = 1 – 4 – 4 + 56 – 64 = – 15 Ans.

10. Let E_1 and E_2 be two events such that the conditional probabilities $P(E_1|E_2) = \frac{1}{2}$, $P(E_2|E_1) = \frac{3}{4}$ and $(E_1 \cap E_2) = \frac{1}{8}$. Then :

(A) $P(E_1 \cap E_2) = P(E_1).P(E_2)$ (B) $P(E_1' \cap E_2') = P(E_1').P(E_2)$

(C) $P(E_1 \cap E_2') = P(E_1).P(E_2)$ (D) $P(E_1' \cap E_2) = P(E_1).P(E_2)$

Sol. **(C)**

$P(E_1/E_2) = 1/2$

$\Rightarrow \frac{P(E_1\cap E_2)}{P(E_2)} = \frac{1}{2}$

$\Rightarrow P(E_2) = \frac{\frac{1}{8}}{\frac{1}{2}} = \frac{1}{4}$

$P(E_2/E_1) = \frac{3}{4} \Rightarrow P(E_1) = \frac{\frac{1}{8}}{\frac{3}{4}} = \frac{1}{6}$

(A) $P(E_1).P(E_2) = \frac{1}{6}.\frac{1}{4} = \frac{1}{24}$ Wrong

(B) $P(\bar{E}_1 \cap \bar{E}_2) = P(\overline{E_1 \cup E_2}) = 1 - P_1(E_1 \cup E_2)$

$= 1 - \left\{\frac{1}{6} + \frac{1}{4} - \frac{1}{8}\right\}$

$= 1 - \left\{\frac{4+6-3}{24}\right\} = 1 - \frac{7}{24} = \frac{17}{24}$

$P(\bar{E}_1).P(E_2) = \frac{5}{6} \times \frac{1}{4} = \frac{5}{24}$

(C) $P(E_1 \cap \bar{E}_2) = P(E_1) - P(E_1 \cap E_2) = \frac{1}{6} - \frac{1}{8} = \frac{1}{24} = P(E_1) \cdot P(E_2)$ Correct

11. Let A = $\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$. If M and N are two matrices given by $M = \sum_{k=1}^{10} A^{2k}$ and $N = \sum_{k=1}^{10} A^{2k-1}$ then MN^2 is :

(A) a non – identity symmetric matrix

(B) a skew- symmetric matrix

(C) neither symmetric nor skew – symmetric matrix

(D) an identity matrix

Sol. **(A)**

$A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix} = -4I$

$A^3 = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 8 \\ -8 & 0 \end{bmatrix}$

$A^4 = \begin{bmatrix} 0 & 8 \\ -8 & 0 \end{bmatrix}\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}$

$\therefore M = \sum_{k=1}^{10} A^{2k} = A^2 + A^4 + \ldots A^{20}$

$= -4I + 16I - 2^6 I + \ldots$ upto 10 terms

$= -4I\{1 - 4 + 4^2 + \ldots$ upto 10 terms$\}$

$= -4I\left\{\frac{(-4)^{10}-1}{-4-1}\right\} = \frac{4I}{5}\{4^{10}-1\} = \lambda I$

$M = \lambda I, \lambda \in R$

$N = \sum_{k=1}^{10} A^{2k-1} = A + A^3 + \ldots$ upto 10 terms

$= A - 4A + 16A + \ldots$ upto 10 terms $\quad (\because A^2 = -4I)$

$= A\left\{\frac{(-4)^{10}-1}{-5}\right\} = -\frac{A}{5}\{4^{10}-1\} = \mu A$

$N = \mu A$

$MN^2 = (kA^2) = -4kI \Rightarrow$ symmetric matrix Ans.

12. Let $g : (0, \infty) \to R$ be a differentiable function such that

$\int \left(\frac{x(\cos x - \sin x)}{e^x+1} + \frac{g(x)(e^x+1-xe^x)}{(e^x+1)^2}\right)dx = \frac{xg(x)}{e^x+1} + c$, for all $x > 0$, where c is an arbitrary constant. Then :

(A) g is decreasing in $\left(0, \frac{\pi}{4}\right)$ (B) g' is increasing in $\left(0, \frac{\pi}{4}\right)$

(C) g + g' is increasing in $\left(0, \frac{\pi}{2}\right)$ (D) g – g' is increasing in $\left(0, \frac{\pi}{2}\right)$

Sol. **(D)**

$\frac{x(\cos x - \sin x)}{e^x+1} + \left\{\frac{g(x)((e^x+1)-xe^x)}{(e^x+1)^2}\right\} = \frac{(e^x+1)(g(x)+xg'(x)) - e^x x g(x)}{(e^x+1)^2}$

$\Rightarrow (e^x + 1)(x\cos x - x\sin x) + g(x)(e^x + 1) - xe^x g(x) = (e^x + 1)g(x) + (e^x + 1)xg'(x) - e^x . x\, g(x)$

$\Rightarrow (e^x + 1)(x\cos x - x\sin x) = (e^x + 1) . xg'(x)$

$\Rightarrow x(\cos x - \sin x) = x.g'(x)$

$\therefore g'(x) = \cos x - \sin x \Rightarrow g'(x) = \sqrt{2}\cos\left(x + \frac{\pi}{4}\right) \downarrow$ in $\left(0, \frac{\pi}{4}\right)$

$g(x) = \sin x + \cos x + \lambda \Rightarrow g(x) = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right) + \lambda \uparrow$ in $\left(0, \frac{\pi}{4}\right)$

$g(x) + g'(x) = 2\cos x + \lambda$ is decreasing in $\left(0, \frac{\pi}{2}\right)$

$g(x) - g'(x) = 2\sin x + \lambda$ is Increasing in $\left(0, \frac{\pi}{2}\right)$

13. Let $f : R \to R$ and $g : R \to R$ be two functions defined by $f(x) = \log_e(x^2 + 1) - e^{-x} + 1$ and $g(x) = \frac{1-2e^{2x}}{e^x}$. Then, for which of the following range of α, the inequality

$f\left(g\left(\frac{(\alpha-1)^2}{3}\right)\right) > f\left(g\left(\alpha - \frac{5}{3}\right)\right)$ holds ?

(A) (2, 3) (B) (-2, - 1) (C) (1, 2) (D) (-1, 1)

Sol. **(A)**

$f(x) = \log_e(x^2 + 1) - e^{-x} + 1$

$f'(x) = \frac{2x}{1+x^2} + e^{-x}$

$f'(x) > 0$

$\therefore f(x) \uparrow$ function

$g(x) = \frac{1-2e^{2x}}{e^x}$

$g'(x) = -e^{-x} - 2e^x$

$g'(x) = -\left\{2e^x + \frac{1}{e^x}\right\} < 0$

$g(x) \downarrow$ function

$f\left(g\left(\frac{(\alpha-1)^2}{3}\right)\right) > f\left(g\left(\alpha-\frac{5}{3}\right)\right)$

$\Rightarrow g\left(\frac{(\alpha-1)^2}{3}\right) > g\left(\alpha-\frac{5}{3}\right)$

$\Rightarrow \frac{(\alpha-1)^2}{3} < \alpha - \frac{5}{3}$

$\Rightarrow \alpha^2 + 1 - 2\alpha < 3\alpha - 5$

$\alpha^2 - 5\alpha + 6 < 0$

$\alpha \in (2, 3)$ Ans.

14. Let $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ where $a_i > 0$, i = 1, 2,3 be a vector which makes equal angles with the coordinate axes OX, OY and OZ. Also, let the projection of $\vec{a}$ on the vector $3\hat{\imath} + 4\hat{\jmath}$ be 7. Let $\vec{b}$ be a vector obtained by rotating $\vec{a}$ with 90°. If $\vec{a}, \vec{b}$ and x – axis are coplanar, then projection of a vector $\vec{b}$ on $3\hat{\imath} + 4\hat{\jmath}$ is equal to :

(A) $\sqrt{7}$ (B) $\sqrt{2}$ (C) 2 (D) 7

Sol. **(B)**

$\vec{a} = \lambda\hat{\imath} + \lambda\hat{\jmath} + \lambda\hat{k}$

also $\frac{\vec{a}.(3\hat{\imath}+4\hat{\jmath})}{5} = 7 \Rightarrow \frac{7\lambda}{5} = 7 \Rightarrow \lambda = 5$

$\therefore \vec{a} = 5\hat{\imath} + 5\hat{\jmath} + 5\hat{k}$

Let $\vec{b} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$\vec{a}\cdot\vec{b} = 0 \Rightarrow x + y + z = 0$ --- (1)

also $\begin{vmatrix} x & y & z \\ 5 & 5 & 5 \\ 1 & 0 & 0 \end{vmatrix} = 0$

$\Rightarrow -y(-5) + z(-5) = 0$

$\Rightarrow y = z$

From (1), $x = -2y$

$x^2 + y^2 + z^2 = 25 \times 3$

$4y^2 + y^2 + y^2 = 25 \times 3$

$6y^2 = 25 \times 3$

$y = \frac{5}{\sqrt{2}} = z \Rightarrow x = \frac{-10}{\sqrt{2}}$

So, $\left|\frac{\vec{b}\cdot(3\hat{\imath}+4\hat{\jmath})}{5}\right| = \left|\frac{3x+4y}{5}\right| = \left|\frac{\frac{-30}{\sqrt{2}}+\frac{20}{\sqrt{2}}}{5}\right| = \left|\frac{-10}{5\sqrt{2}}\right| = \left|-\sqrt{2}\right| = \sqrt{2}$ Ans.

15. Let y = y(x) be the solutions of the differential equation $(x+1)y' - y = e^{3x}(x+1)^2$, with $y(0) = \frac{1}{3}$. Then, the point $x = \frac{-4}{3}$ for the curve y = y(x) is :

(A) not a critical point (B) a point of local minima

(C) a point of local maxima (D) a point of inflection

Sol. **(B)**

$(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$

$\frac{dy}{dx} - \frac{y}{x+1} = e^{3x}(x+1)$

$\text{IF} = e^{-\int\frac{dx}{x+1}} = \frac{1}{x+1}$

$\therefore y \times \frac{1}{x+1} = \int e^{3x}dx$

$\frac{y}{x+1} = \frac{e^{3x}}{3} + C$

Given y(0) = 1/3

$\therefore \frac{1}{3} = \frac{1}{3} + C$

C = 0

$\therefore y = \frac{e^{3x}(x+1)}{3}$

$\frac{dy}{dx} = \frac{1}{3}\{e^{3x} + 3e^{3x}(x+1)\}$

$= \frac{e^{3x}}{3}\{3x+4\}$

$\frac{dy}{dx} < 0$ | $\frac{dy}{dx} > 0$

$\frac{-4}{3}$

So, x = $-\frac{4}{3}$ is a pt. of local minima. Ans.

16. If $y = m_1x + c_1$ and $y = m_2x + c_2$, $m_1 \neq m_2$ are two common tangents of circle $x^2 + y^2 = 2$ and parabola $y^2 = x$, then the value of $8|m_1m_2|$ is equal to:

(A) $3 + 4\sqrt{2}$ (B) $-5 + 6\sqrt{2}$ (C) $-4 + 3\sqrt{2}$ (D) $7 + 6\sqrt{2}$

Sol. **(C)**

$x^2 + y^2 = 2$;

tangent: $y = mx \pm \sqrt{2}\sqrt{1+m^2}$

$y^2 = x$

tangent : $y = mx + \frac{1}{4m}$

$\therefore \pm\sqrt{2}\sqrt{1+m^2} = \frac{1}{4m}$

$2 + 2m^2 = \frac{1}{16m^2}$

$32m^4 + 32m^2 - 1 = 0$

$m^4 + m^2 - \frac{1}{32} = 0$

$\Rightarrow \left(m^2 + \frac{1}{2}\right)^2 = \frac{9}{32}$

$m^2 + \frac{1}{2} = \pm\frac{3}{4\sqrt{2}}$

$m^2 = \frac{3}{4\sqrt{2}} - \frac{1}{2}$ $(\because m^2 > 0)$

$\therefore 8|m_1m_2| = 8\left(\frac{3-2\sqrt{2}}{4\sqrt{2}}\right)$

$= 3\sqrt{2} - 4$ Ans.

17. Let Q be the mirror image of the point P(1, 0, 1) with respect to the plane S : x + y + z = 5. If a line L passing through (1, -1, -1), parallel to the line PQ meets the plane S at R, then QR^2 is equal to :

(A) 2 (B) 5 (C) 7 (D) 11

Sol. **(B)**

Equation of L

$\frac{x-1}{1} = \frac{y+1}{1} = \frac{z+1}{1} = \lambda$

$x = \lambda + 1, y = \lambda - 1, z = \lambda - 1$

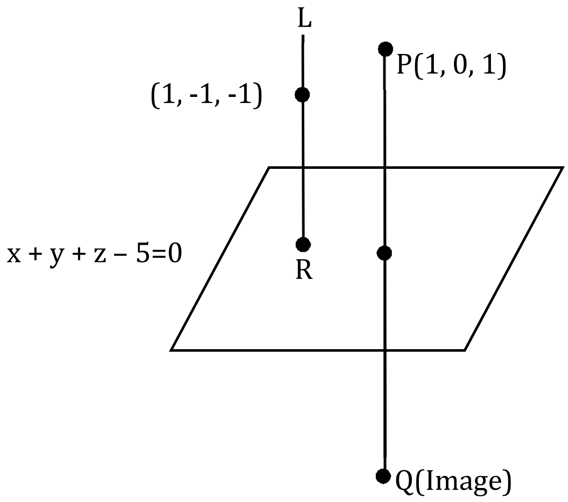
Putting in equation of plane

$\lambda + 1 + \lambda - 1 + \lambda - 1 = 5$

$3\lambda = 6$

$\lambda = 2$

$\therefore$ R(3, 1, 1)



$QR^2 = PR^2 = 4 + 1 + 0 = 5$ Ans.

18. If the solution curve y = y(x) of the differential equation $y^2dx + (x^2 - xy + y^2)dy = 0$ which passes through the point (1, 1) and intersects the line $y = \sqrt{3}x$ at the point $(\alpha, \sqrt{3}\alpha)$, then value of $\log_e(\sqrt{3}\alpha)$ is equal to:

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{12}$ (D) $\frac{\pi}{6}$

Sol. **(C)**

$y^2\,dx + (x^2 - xy + y^2)dy = 0$

$\frac{dy}{dx} = \frac{-y^2}{x^2-xy+y^2}$

Put y = tx

$\frac{dy}{dx} = t + x\frac{dt}{dx}$

$t + x\frac{dt}{dx} = \frac{-t^2x^2}{x^2-tx^2+t^2x^2}$

$t + x\frac{dt}{dx} = \frac{-t^2}{1-t+t^2}$

$\left(-\frac{1}{t} + \frac{1}{t^2+1}\right)dt = \frac{dx}{x}$

$-\log t + \tan^{-1}t = \ln x + c$

$-\log\left(\frac{y}{x}\right) + \tan^{-1}\left(\frac{y}{x}\right) = \ln x + C$

Putting x = 1, y = 1 we get C = $\frac{\pi}{4}$

Put $y = \sqrt{3}x$

$-\log(\sqrt{3}) + \tan^{-1}(\sqrt{3}) = \ln x + \frac{\pi}{4}$

$\therefore \ln(\sqrt{3}x) = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$ Ans.

19. Let x = 2t, $y = \frac{t^2}{3}$ be a conic. Let S be the focus and B be the point on the axis of the conic such that $SA \perp BA$, where A is any point on the conic. If k is the ordinate of the centroid of the ΔSAB, then $\lim_{t\to 1} k$ is equal to :

(A) $\frac{17}{18}$ (B) $\frac{19}{18}$ (C) $\frac{11}{18}$ (D) $\frac{13}{18}$

Sol. **(D)**

x = 2t, $y = \frac{t^2}{3}$

$y = \frac{x^2}{12} \Rightarrow x^2 = 12y$

$m_{AS} = \frac{\frac{t^2}{3}-3}{2t}$

$m_{AB} = \frac{2t}{3-\frac{t^2}{3}}$

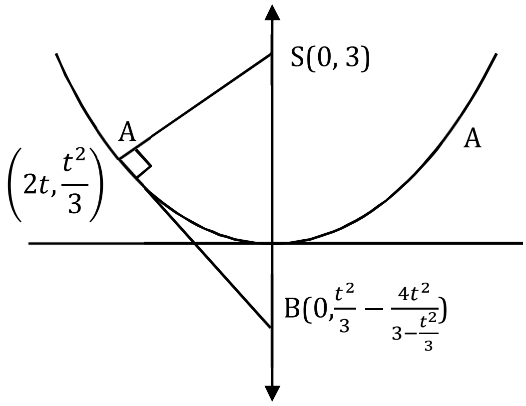
Equation of AB : $y - \frac{t^2}{3} = \frac{2t}{3-\frac{t^2}{3}}(x - 2t)$

For B , put x = 0

$\therefore y = \frac{t^2}{3} - \frac{4t^2}{3-\frac{t^2}{3}}$

$\therefore k = \frac{3+\frac{t^2}{3}+\frac{t^2}{3}-\frac{4t^2}{3-\frac{t^2}{3}}}{3}$

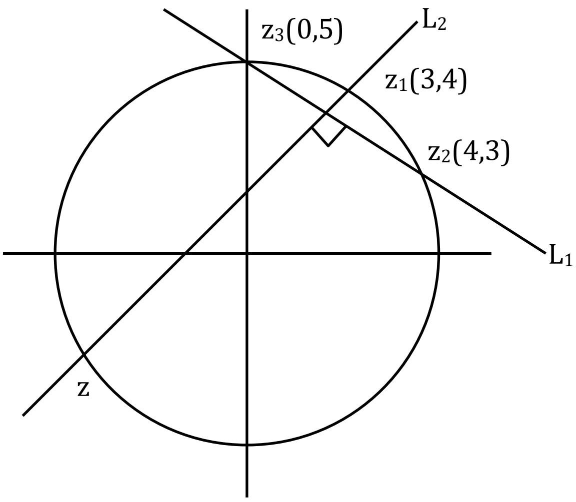
$\lim_{t\to1} k = \frac{3+\frac{2}{3}-\frac{3}{2}}{3} = \frac{13}{18}$



20. Let a circle C in complex plane pass through the point $z_1 = 3 + 4i$, $z_2 = 4 + 3i$ and $z_3 = 5i$. If $z(\neq z_1)$ is a point on C such that the line through z and z_1 is perpendicular to the line through z_2 and z_3, then arg(z) is equal to :

(A) $\tan^{-1}\left(\frac{2}{\sqrt{5}}\right) - \pi$ (B) $\tan^{-1}\left(\frac{24}{7}\right) - \pi$ (C) $\tan^{-1}(3) - \pi$ (D) $\tan^{-1}\left(\frac{3}{4}\right) - \pi$

Sol. **(B)**



Slope of line $L_1 = \frac{2}{-4} = \frac{-1}{2}$

Slope of line $L_2 = 2$

So, equation of line L_2 : $y - 4 = 2(x - 3)$

$y = 2x - 2 = 2(x - 1)$

Equation of circle : $x^2 + y^2 = 25$

$x^2 + 4(x - 1)^2 = 25$

$x^2 + 4(x^2 - 2x + 1) = 25$

$5x^2 - 8x - 21 = 0$

$5x^2 - 15x + 7x - 21 = 0$

$(5x + 7)(x - 3) = 0$
$x = 3$ gives point z_1
So, for point z, $x = \frac{-7}{5}$
$y = 2\left(-\frac{7}{5} - 1\right) = 2\left(-\frac{12}{5}\right) = \frac{-24}{5}$
$z = \frac{-7}{5} - \frac{24}{5}i$, 3rd quadrant
so, $\arg z = -\pi + \tan^{-1}\left(\frac{\frac{24}{5}}{\frac{7}{5}}\right) = -\pi + \tan^{-1}\left(\frac{24}{7}\right)$ Ans.

21. Let C_r denote the binomial coefficient of x^r in the expansion of $(1 + x)^{10}$. If for , $\beta \in R$, $C_1 + 3 \cdot 2C_2 + 5 \cdot 3C_3$ + ... upto 10 terms = $\frac{\alpha \times 2^{11}}{2^\beta - 1}\left(C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \ldots \text{upto 10 terms}\right)$ then the value of $\alpha + \beta$ is equal to ....

Sol. **(286)**
$C_1 + 3.2C_2 + 5.3C_3 + \ldots\ldots$ upto 10 terms
$T_r = (2r - 1)\, r.\, C_r = 2r^2\, C_r - r.\, C_r$
$S_n = 2\sum r^2 C_r - \sum r\, C_r$
$T_r = 2(r^2 - r + r).\, C_r - rCr$
$T_r = (2r(r - 1) + 2r)\, C_r - rC_r$
$T_r = 2r(r - 1)C_r + rC_r$
$T_r = 2\, n(n-1)\, {}^{n-2}C_{r-2} + n\, {}^{n-1}C_{r-1}$
$S_n = 2n(n-1).\, 2^8 + n.\, 2^9$
$= n\, .\, 2^9\, \{(n - 1) + 1\} = n^2\, .\, 2^9$

RHS
$C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \ldots + \frac{C_{10}}{11}$
$\int (1 + x)^{10} dx = C_0 x + \frac{C_1 x^2}{2} + \frac{C_2 x^3}{3} + \ldots + \frac{C_{10} x^{11}}{11} + k$
$\frac{(1+x)^{11}}{11} = C_0 x + \frac{C_1 x^2}{2} + \ldots + \frac{C_{10} x^{11}}{11} + k$
Putting x = 0, we get k = $\frac{1}{11}$
$x = 1 \Rightarrow \frac{2^{11}}{11} - \frac{1}{11} = C_0 + \frac{C_1}{2} + \ldots.. + \frac{C_0}{11}$
$\therefore 100.2^9 = \frac{2^{11}-1}{11} \frac{(\alpha.2^{11})}{2^\beta - 1}$
$2^2.5^2.2^9 = \frac{2^{11}-1}{11} \frac{(\alpha.2^{11})}{2^\beta - 1}$
$\therefore \frac{2^\beta - 1}{\alpha} = \frac{2^{11}-1}{25 \times 11}$
$\alpha = 275$
$\beta = 11$
$\therefore \alpha + \beta = 286$.

Infinite solutions are possible.

22. The number of 3 – digit odd numbers, whose sum of digits is a multiple of 7, is ......

Sol. **63**
a + b + c = 7, 14, 21
Case I : If a + b + c = 7
c = 1 a + b = 6 ⇒ 6 cases
c = 3 a + b = 4 ⇒ 4 cases
c = 5 a + b = 2 ⇒ 2 case

Case 2 : If a + b + c = 14

c = 1 a + b = 13 ⇒ 6 cases
c = 3 a + b = 11 ⇒ 8 cases
c = 5 a + b = 9 ⇒ 9 cases
c = 7 a + b = 7 ⇒ 7 cases
c = 4 a + b = 5 ⇒ 5 cases

Case 3 : If a + b + c = 21 ;

c = 3 a + b = 18 ⇒ 1 case
c = 5 a + b = 16 ⇒ 3 cases
c = 7 a + b = 14 ⇒ 5 cases
c = 9 a + b = 12 ⇒ 7 cases

∴ 63 numbers

23. Let θ be the angle between the vectors $\vec{a}$ and $\vec{b}$, where $|\vec{a}| = 4, |\vec{b}| = 3$ and $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$ Then $|(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})|^2 + 4(\vec{a}.\vec{b})^2$ is equal to ....

Sol. **(576)**
$|(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})|^2 + 4(\vec{a}.\vec{b})^2$
$|-\vec{b}\times\bar{a} + \bar{a}\times\bar{b}|^2 + 4(\bar{a}.\bar{b})^2$
$= 4a^2b^2\sin^2\theta + 4a^2b^2\cos^2\theta$
$= 4a^2b^2 = 4 \times 16 \times 9 = 576$

24. Let the abscissas of the two points P and Q be the roots of $2x^2 - rx + p = 0$ and the ordinates of P and Q be the roots of $x^2 - sx - q = 0$. If the equation of the circle described on PQ as diameter is $2(x^2 + y^2) - 11x - 14y - 22 = 0$, then $2r + s - 2q + p$ is equal to ....

Sol. **(7)**
Equation of circle will be
$(2x^2 - rx + p) + (2y^2 - 2sy - 2q) = 0$
$= 2(x^2 + y^2) - rx - 2sy + p - 2q = 0$ ... (1)
Comparing with
$2(x^2 + y^2) - 11x - 14y - 22 = 0$ ... (2)
$r = 11, s = 7, p - 2q = -22$
∴ $2r + s - 2q + p = 22 + 7 - 22 = 7$ Ans.

25. The number of values of x in the interval $\left(\frac{\pi}{4}, \frac{7\pi}{4}\right)$ for which $14\text{cosec}^2x - 2\sin^2x = 21 - 4\cos^2x$ holds, is ......

Sol. **(4)**
$14\ \text{cosec}^2x - 2\sin^2x = 21 - 4\cos^2x$
$14\ \text{cosec}^2x = 2\sin^2x + 21 - 4 + 4\sin^2x$
$\frac{14}{\sin^2x} = 6\sin^2x + 17$
Let $\sin^2x = t$
$14 = 6t^2 + 17t$
$6t^2 + 17t - 14 = 0$
$6t^2 + 21t - 4t - 14 = 0$
$(3t - 2)(2t + 7) = 0$
$\sin^2x = \frac{2}{3}$
$\sin x = \pm\sqrt{\frac{2}{3}}$
If $x \in \left(\frac{\pi}{4}, \frac{7\pi}{4}\right)$,

sinx = $\sqrt{\frac{2}{3}} \rightarrow$ 2 values of x

$\sin x = -\sqrt{\frac{2}{3}} \rightarrow$ 2 values of x

Ans. = 4 values of x

26. For a natural number n, let $\alpha_n = 19^n - 12^n$. then, the value of $\frac{31\alpha_9 - \alpha_{10}}{57\alpha_8}$ is ......

Sol. **(4)**

$\alpha_n = 19^n - 12^n$

$\frac{31\alpha_9 - \alpha_{10}}{57\alpha_8} = \frac{31(19^9 - 12^9) - (19^{10} - 12^{10})}{57(19^8 - 12^8)}$

$= \frac{19^9(12) - 12^9(19)}{57(19^8 - 12^8)}$

$= \frac{19 \times 12(19^8 - 12^8)}{57(19^8 - 12^8)}$

= 4

27. Let f : R → R be a function defined by $f(x) = \left(2\left(1 - \frac{x^{25}}{2}\right)(2 + x^{25})\right)^{\frac{1}{50}}$. If the function g(x) = f(f(f(x))) + f(f(x)), then the greatest integer less than or equal to g(1) is ......

Sol. **(2)**

$f(x) = \left(2\left(1 - \frac{x^{25}}{2}\right)(2 + x^{25})\right)^{\frac{1}{50}}$

$f(1) = \left(2\left(\frac{1}{2}\right)(3)\right)^{\frac{1}{50}} = 3^{\frac{1}{50}}$

$f(f(1)) = \left\{2\left(1 - \frac{\sqrt{3}}{2}\right)(2 + \sqrt{3})\right\}^{\frac{1}{50}} = 1$

$\therefore$ g(1) = f(1) + 1 = $3^{\frac{1}{50}} + 1$

[g(1)] = 2

28. Let the lines

$L_1 : \vec{r} = \lambda(\hat{i} + 2\hat{j} + 3\hat{k}), \lambda \in R$

$L_2 : \vec{r} = (\hat{i} + 3\hat{j} + \hat{k}) + \mu(\hat{i} + \hat{j} + 5\hat{k}), \mu \in R$

intersect at the point S. If a plane ax + by – z + d = 0 passes through S and is parallel to both the lines L_1 and L_2, then the value of a + b + d is equal to ......

Sol. **(5)**

$L_1 : \bar{r} = (0,0,0) + \lambda(1,2,3)$

$L_2 : \bar{r} = (1,3,1) + \mu(1,1,5)$

direction of $\vec{n}$ of plane = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 1 & 1 & 5 \end{vmatrix} = 7\hat{i} - 2\hat{j} - \hat{k}$

for point of intersection of L_1 and L_2 :

$\lambda = 1 + \mu$ .... (1)

$2\lambda = 3 + \mu$ .... (2)

$3\lambda = 1 + 5\mu$ ... (3)

Solving (1), (2) and (3),

$\lambda = 2, \mu = 1$

$\therefore$ S(2, 4, 6)

Equation of plane 7(x – 2) – 2 (y – 4) – (z – 6) = 0

7x – 2y – z – 14 + 8 + 6 = -0

7x – 2y – z = 0

a = 7, b = -2, d = 0

a + b + d = 5 Ans.

29. Let A be a 3 × 3 matrix having entries from the set {-1, 0, 1}. The number of all such matrices A having sum of all the entries equal to 5, is ....

Sol. (414)

$a_{11} + a_{12} + a_{13} + ... + a_{33} = 5$

Case 1 : five 1 and four 0 $\Rightarrow \frac{9!}{5!4!} = 126$

Case 2 : six 1, one '−1' and two '0' $\Rightarrow \frac{9!}{6!2!} = 252$

Case 3 : seven 1, two – 1, $\Rightarrow \frac{9!}{7!2!} = 36$

Ans. = 126 + 252 + 36 = 414

30. The greatest integer less than or equal to the sum of first 100 terms of the sequence $\frac{1}{3}, \frac{5}{9}, \frac{19}{27}, \frac{65}{81}, ...$ is equal to .....

Sol. (98)

$S = \frac{1}{3} + \frac{5}{9} + \frac{19}{27} + \frac{65}{81} + ...$ upto 100 terms

$= \frac{3-2}{3} + \frac{3^2-2^2}{3^2} + \frac{3^3-2^3}{3^3} + \frac{3^4-2^4}{3^4} + ...$ upto 100 terms

$= 100 - \left\{\frac{2}{3} + \frac{2^2}{3^2} + \frac{2^3}{3^3} + ...\text{upto 100 terms}\right\}$

$= 100 - 2\left[1 - \left(\frac{2}{3}\right)^{100}\right]$

$S = 98 + 2\left(\frac{2}{3}\right)^{100}$

$\therefore [s] = 98$

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