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QUESTION PAPER WITH SOLUTION

MATHEMATICS
25^{th} June 2022 | Shift - 2

SECTION - A

1. Let A = {x ∈ R : |x + 1| < 2) and B = {x ∈ R : |x – 1| ≥ 2}. Then which one of the following statements is NOT true ?

(A) A – B = (-1, 1) (B) B – A = R – (-3, 1) (C) A ∩ B = (-3, -1] (D) A ∪ B = R – [1, 3)

Sol. **(B)**

$A : x \in (-3,1)$ $B: x \in (-\infty, 1] \cup [3, \infty)$

(A) A – B = (–1, 1)

(B) B – A = (–∞, –3] ∪ [3,∞) = $\mathbb{R} - (-3,3)$

(C) $A \cap B = (-3, -1]$

(D) $A \cup B = (-\infty, 1) \cup [3, \infty)$

$= \mathbb{R} - [1,3)$

So, option B is incorrect

2. Let a, b, ∈ R be such that the equation $ax^2 - 2bx + 15 = 0$ has a repeated root α . If α and β are the roots of the equation $x^2 - 2bx + 21 = 0$, then $\alpha^2 + \beta^2$ is equal to :

(A) 37 (B) 58 (C) 68 (D) 92

Sol. **(B)**

$ax^2 - 2bx + 15 = 0$ (α , α)

$\Rightarrow 2\alpha = \frac{2b}{a}$

$\Rightarrow \alpha = \frac{b}{a}$

Also, $\alpha^2 = \frac{15}{a} \Rightarrow \frac{b^2}{a^2} = \frac{15}{a}$

$\Rightarrow b^2 = 15a$

Now given $x^2 - 2bx + 21 = 0$ (α , β)

α + β = 2b, $\alpha\beta = 21$

Now, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= 4b^2 - 42$

Put $\alpha = \frac{b}{a}$ in $x^2 - 2bx + 21 = 0$

$\frac{b^2}{a^2} - \frac{2b^2}{a} + 21 = 0$

$b^2 - 2ab^2 + 21a^2 = 0$

$15a - 2a \times 15a + 21a^2 = 0$ ($\because b^2 = 15a$)

$15a - 9a^2 = 0$

3a(5 – 3a) = 0 ⇒ a = 0 (rejected) or a = 5/3

Now , $\alpha^2 + \beta^2 = 4b^2 - 42$

= 4(15a) – 42

$= 60 \times \frac{5}{3} - 42 = 58$

3. Let z_1 and z_2 be two complex numbers such that $\bar{z}_1 = i\bar{z}_2$ and $\arg\left(\frac{z_1}{\bar{z}_2}\right) = \pi$. Then

(A) $\arg z_2 = \frac{\pi}{4}$ (B) $\arg z_2 = -\frac{3\pi}{4}$ (C) $\arg z_1 = \frac{\pi}{4}$ (D) $\arg z_1 = -\frac{3\pi}{4}$

Sol. **(C)**

$\bar{z}_1 = i\bar{z}_2$

$z_1 = -iz_2$

$\arg(\frac{z_1}{\bar{z}_2}) = \pi$

$\arg(-i\frac{z_2}{\bar{z}_2}) = \pi$ $\arg(z_2) = \theta$

$-\frac{\pi}{2} + \theta + \theta = \pi$

$2\theta = \frac{3\pi}{2}$

$\arg(z_2) = \theta = \frac{3\pi}{4}$, $\arg z_1 = \frac{\pi}{4}$

4. The system of equations

-kx + 3y – 14z = 25

-15x + 4y – kz = 3

-4x + y + 3z = 4

is consistent for all k in the set

(A) R (B) R - {-11, 13} (C) R – {13} (D) R – {-11, 11}

Sol. **(D)**

$$\Delta = \begin{vmatrix} -k & 3 & -14 \\ -15 & 4 & -k \\ -4 & 1 & 3 \end{vmatrix} \neq 0$$

$k^2 \neq 121 \Rightarrow k \neq -11, 11$

5. $\lim_{x\to\frac{\pi}{2}} \tan^2 x\left((2\sin^2 x + 3\sin x + 4)^{\frac{1}{2}} - (\sin^2 x + 6\sin x + 2)^{\frac{1}{2}}\right)$ is equal to

(A) $\frac{1}{12}$ (B) $-\frac{1}{18}$ (C) $-\frac{1}{12}$ (D) $\frac{1}{6}$

Sol. **(A)**

$\lim_{x\to\frac{\pi}{2}} \tan^2 x\left\{\frac{2\sin^2+3\sin x+4-\sin^2 x-6\sin x-2}{\sqrt{2\sin^2 x+3\sin x+4}+\sqrt{\sin^2 x+6\sin x+2}}\right\}$

$= \lim_{x\to\frac{\pi}{2}} \tan^2 x\left\{\frac{\sin^2 x-3\sin x+2}{3+3}\right\}$

$= \frac{1}{6}\lim_{h\to 0} \tan^2\left(\frac{\pi}{2}-h\right)\left\{\sin^2\left(\frac{\pi}{2}-h\right) - 3\sin\left(\frac{\pi}{2}-h\right) + 2\right\}$

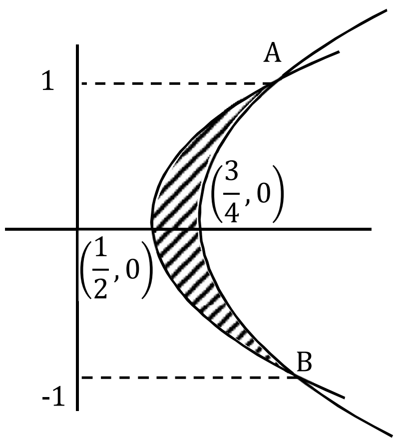
$= \frac{1}{6}\lim_{h\to 0} \frac{1}{\tan^2 h}\{\cos^2 h - 3\cos h + 2\}$

$= \frac{1}{6}\lim_{h\to 0} \frac{(\cos h-1)(\cos h-2)}{h^2} = \frac{1}{12}$

6. The area of the region enclosed between the parabolas $y^2 = 2x – 1$ and $y^2 = 4x – 3$ is

(A) $\frac{1}{3}$ (B) $\frac{1}{6}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

Sol. **(A)**



2x – 1 = 4x – 3

x = 1

So, A(1, 1), B(1, - 1)

Area = $\int_{-1}^{1}\left(\frac{3+y^2}{4} - \frac{1+y^2}{2}\right)dy$

$= 2\int_{0}^{1}\left\{\frac{y^2+3}{4} - \frac{y^2+1}{2}\right\}$

$= 2\left[\frac{1}{4}\left(\frac{y^3}{3}+3y\right)_0^1 - \frac{1}{2}\left(\frac{y^3}{3}+y\right)_0^1\right]$

$= 2\left[\frac{1}{4}\left(\frac{1}{3}+3\right) - \frac{1}{2}\left(\frac{1}{3}+1\right)\right]$

$= \frac{10}{6} - \frac{4}{3} \Rightarrow \frac{2}{6} = \frac{1}{3}$

7. The coefficient of x^{101} in the expression $(5 + x)^{500} + x(5 + x)^{499} + x^2 (5 + x)^{498} + ... + x^{500}$, $x > 0$, is

(A) $^{501}C_{101}(5)^{399}$ (B) $^{501}C_{101}(5)^{400}$ (C) $^{501}C_{100}(5)^{400}$ (D) $^{500}C_{101}(5)^{399}$

Sol. **(A)**

$S = (5 + x)^{500} + x(5 + x)^{499} + x^2 (5 + x)^{498} + ... + x^{500}$, $(x > 0)$

$$S = (5+x)^{500}\left\{\frac{\left(\frac{x}{x+5}\right)^{501}-1}{\frac{x}{x+5}-1}\right\}$$

$$S = (5+x)^{500}\left(\frac{x^{501}-(x+5)^{501}}{-5(x+5)^{500}}\right)$$

$$s = \frac{1}{5}\left((x+5)^{501} - x^{501}\right)$$

coeff. of $x^{101} = \frac{1}{5}\ {}^{501}C_r x^{501-r}(5)^r$

$501 - r = 101 \Rightarrow r = 400$

$\Rightarrow \frac{1}{5}\ {}^{501}C_{400}(5)^{400}$

$\Rightarrow {}^{501}C_{101}(5)^{400} \times \frac{1}{5} \Rightarrow {}^{501}C_{101}(5)^{399}$

8. The sum $1 + 2.3 + 3.3^2 + ... + 10.3^9$ is equal to :

(A) $\frac{2.3^{12}+10}{4}$ (B) $\frac{19.3^{10}+1}{4}$ (C) $5.3^{10} - 2$ (D) $\frac{9.3^{10}+1}{2}$

Sol. **(B)**

$S = 1 + 2.3 + 3.3^2 + ... + 10.3^9$

$3s = \quad 3 + 2.3^2 + ... + 9.3^9 + 10.3^{10}$

$-2s = 1 + 3 + 3^2 + ... + 3^9 - 10(3)^{10}$

$-2s = \left(\frac{3^{10}-1}{2}\right) - 10(3)^{10}$

$$s = \frac{20(3^{10})-(3^{10}-1)}{2\times 2}$$

$$S = \frac{19.(3^{10})+1}{4}$$ Ans.

9. Let P be the plane passing through the intersection of the planes $\vec{r}.(\hat{\imath}+3\hat{\jmath}-\hat{k}) = 5$ and $\vec{r}.(2\hat{\imath}-\hat{\jmath}+\hat{k}) = 3$, and the point (2, 1, - 2). Let the position vectors of the points X and Y be , $\hat{\imath}-2\hat{\jmath}+4\hat{k}$ and $5\hat{\imath}-\hat{\jmath}+2\hat{k}$ respectively. Then the points

(A) X and X + Y are on the same side of P (B) Y and Y – X are on the opposite sides of P

(C) X and Y are on the opposite sides of P (D) X + Y and X – Y are on the same side of P

Sol. **(C)**

$P_1 + \lambda P_2 = 0$

$(x + 3y - z - 5) + \lambda(2x - y + z - 3) = 0$

It passes through (2, 1, -2)

$2 + \lambda(-2) = 0 \Rightarrow \lambda = 1$

so, equation of required plane is

$P : 3x + 2y - 8 = 0$

(A) $\left.\begin{array}{r} X = (1,-2,4) \\ X+Y = (6,-3,6)\end{array}\right\}$ $P|_X . P|_{X+Y} = (3 - 4 - 8)(18 - 6 - 8) < 0$

$\Rightarrow$ X & X + Y are on opposite side

(B) $\left.\begin{array}{r} Y = (5,-1,2) \\ Y-X = (4,1,-2)\end{array}\right\}$ $P|_Y . P|_{Y-X} = (15 - 2 - 8)(12 + 2 - 8) > 0$

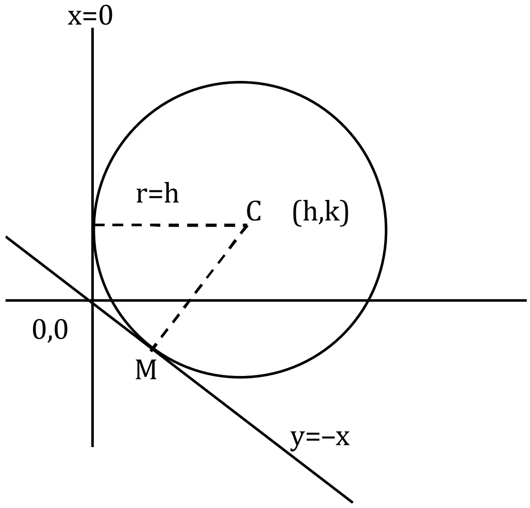
$\Rightarrow$ Y & Y - X are on same side

(C) $\left.\begin{matrix} X=(1,-2,4) \\ Y=(5,-1,2) \end{matrix}\right\} P|_X . P|_Y$ = (3 – 4 – 8) (15 – 2 – 8) < 0
⇒ X & Y are on opposite side

(D) $\left.\begin{matrix} X+Y=(6,-3,6) \\ X-Y=(-4,-1,2) \end{matrix}\right\} \Rightarrow P|_{X+Y} . P|_{X-Y}$ = (18 – 6 – 8)(-12 – 2- 8) < 0
⇒ X + Y & X - Y are on opposite side

10. A circle touches both the y – axis and the line x + y = 0 .Then the locus of its center is
(A) $y=\sqrt{2}x$ (B) $x=\sqrt{2}y$ (C) $y^2 - x^2 = 2xy$ (D) $x^2 - y^2 = 2xy$

Sol. **(D)**



CM = r = h

$\frac{|h+k|}{\sqrt{2}} = h$

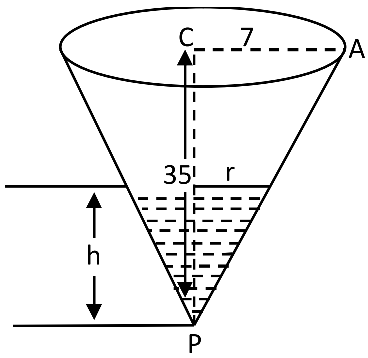
$h^2 + k^2 + 2hk = 2h^2$

$h^2 - k^2 - 2hk = 0$

$x^2 - y^2 = 2xy$

11. Water is being filled at the rate of 1 cm^3/ sec in a right circular conical vessel (vertex downwards) of height 35 cm and diameter 14 cm. When the height of the water level is 10 cm, the rate (in cm^2/sec) at which the wet conical surface area of the vessel increases is
(A) 5 (B) $\frac{\sqrt{21}}{5}$ (C) $\frac{\sqrt{26}}{5}$ (D) $\frac{\sqrt{26}}{10}$

Sol. **(C)**



$\frac{7}{r} = \frac{35}{h}$

$\frac{h}{r} = 5$

h = 5r

$\frac{dv}{dt} = 1$

$\frac{1}{3}\frac{d}{dt}(\pi r^2(5r)) = 1$

$\frac{5\pi}{3} \times 3r^2 \frac{dr}{dt} = 1$ $\qquad \frac{dr}{dt} = \frac{1}{5\pi r^2}$

Now, Area = $\pi rl = \pi r\sqrt{r^2+h^2}$

$A = \pi r\sqrt{r^2+25r^2}$

$A = \pi r(\sqrt{26}r)$

$\frac{dA}{dt} = \sqrt{26}\pi \times 2r\frac{dr}{dt}$

Now where h = 10 $\Rightarrow$ 5r = 10

r = 2

$\frac{dA}{dt} = \pi\sqrt{26}\times 2\times 2\times\frac{1}{5\pi(4)}$

$\frac{dA}{dt} = \frac{\sqrt{26}}{5}$

12. If $b_n = \int_0^{\frac{\pi}{2}}\frac{\cos^2 nx}{\sin x}dx, n\in N$, then

(A) $b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in an A.P. with common difference - 2

(B) $\frac{1}{b_3-b_2}, \frac{1}{b_4-b_3}, \frac{1}{b_5-b_4}$ are in an A.P. with common difference 2

(C) $b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in a G. P.

(D) $\frac{1}{b_3-b_2}, \frac{1}{b_4-b_3}, \frac{1}{b_5-b_4}$ are in an A.P. with common difference -2.

Sol. (D)

$b_n = \int_0^{\frac{\pi}{2}}\frac{\cos^2(n\pi)}{\sin x}dx, n\in N$

$b_n - b_{n-1} = \int_0^{\frac{\pi}{2}}\frac{\cos^2 n\pi - \cos^2(n-1)x}{\sin x}$

$= \int_0^{\frac{\pi}{2}}\frac{\sin^2(n-1)x - \sin^2 nx}{\sin x}$

$= \int_0^{\frac{\pi}{2}}\frac{\sin(2nx-x)(-\sin x)}{\sin x}$

$= -\int_0^{\frac{\pi}{2}}\sin(2n-1)x$

$= \frac{\cos(2n-1)x}{2n-1}\Big|_0^{\pi/2}$

$= \frac{-1}{2n-1}$

Now, $b_3 - b_2 = -\frac{1}{5}$

$b_4 - b_3 = \frac{-1}{7}$ $\Rightarrow \frac{1}{b_3-b_2}, \frac{1}{b_4-b_3}, \frac{1}{b_5-b_4}$ are in A.P. with common difference -2

$b_5 - b_4 = \frac{-1}{9}$

13. If y = y(x) is the solution of the differential equation $2x^2\frac{dy}{dx} - 2xy + 3y^2 = 0$ such that $y(e) = \frac{e}{3}$, then y(1) is equal to

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{3}{2}$ (D) 3

Sol. (B)

$\frac{dy}{dx} - \frac{y}{x} + \frac{3}{2}\left(\frac{y}{x}\right)^2 = 0$

$y^{-2}\frac{dy}{dx} - \frac{1}{xy} + \frac{3}{2x^2} = 0$

$\frac{1}{y} = t \Rightarrow -\frac{1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$

$-\frac{dt}{dx}-\frac{t}{x}+\frac{3}{2x^2}=0$
$\frac{dt}{dx}+\frac{t}{x}=\frac{3}{2x^2}$
I.F. $= e^{\int \frac{dt}{x}} = e^{\ell nx} = x$
tx $= \int \frac{3}{2x}dx$
$\frac{x}{y}=\frac{3}{2}\ell nx + C$
$\Downarrow x = e, y = \frac{e}{3}$
$\frac{e}{\frac{e}{3}} = \frac{3}{2}+c \Rightarrow c = 3-\frac{3}{2}=\frac{3}{2}$
So, $\frac{x}{y}=\frac{3}{2}\ell nx+\frac{3}{2}$
at x = 1, $\frac{1}{y}=\frac{3}{2} \Rightarrow y = \frac{2}{3}$

14. If the angle made by the tangent at the point $(x_0.\ y_0)$ on the curve
x = 12(t + sint cost),
$y = 12(1+\text{sint})^2, 0 < t < \frac{\pi}{2}$, with the positive x – axis is $\frac{\pi}{3}$, then y_0 is equal to :
(A) $6(3+2\sqrt{2})$ (B) $3(7+4\sqrt{3})$ (C) 27 (D) 48

Sol. **(C)**
x = 12(t + sint cost)
$y = 12\ (1+\text{sint})^2$
$0 < t < \frac{\pi}{2}$
$\frac{dy}{dx}=\frac{12\times 2(1+\sin t)\cos t}{12\{1-\sin^2 t+\cos^2 t\}}$

$\frac{dy}{dx}=\frac{2(1+\sin t)\cos t}{(1+\cos 2t)}=\frac{2(1+\sin t)\cos t}{2\cos^2 t}$

Now, given $\tan\theta = \tan\frac{\pi}{3}=\sqrt{3}$
$\Rightarrow \sqrt{3}=\frac{1+\sin t}{\cos t}$
$\Rightarrow \sqrt{3}\cos t-\sin t = 1$
$\Rightarrow \frac{\sqrt{3}}{2}\cos t-\frac{\sin t}{2}=\frac{1}{2}$
$\Rightarrow \cos\left(t+\frac{\pi}{6}\right)=\cos\frac{\pi}{3}$
$\Rightarrow t+\frac{\pi}{6}=\frac{\pi}{3}$ $\Rightarrow t = \frac{\pi}{6}$
Now, $y_0 = 12\left(1+\sin\frac{\pi}{6}\right)^2$
$= 12\left\{1+\frac{1}{2}\right\}^2 \Rightarrow 9\times 3 = 27$

15. The value of 2sin(12°) – sin(72°) is :
(A) $\frac{\sqrt{5}(1-\sqrt{3})}{4}$ (B) $\frac{1-\sqrt{5}}{8}$ (C) $\frac{\sqrt{3}(1-\sqrt{5})}{2}$ (D) $\frac{\sqrt{3}(1-\sqrt{5})}{4}$

Sol. **(D)**
$2\sin 12^o - \sin 72^o$
$\sin 12^o - \sin 72^o + \sin 12^o$
$\sin 12^o - \{\sin 72^o - \sin 12^o\}$
$\sin 12^o - \{2\sin 30^o \cos 42^o\}$

$\cos 78^\circ - \cos 42^\circ = -2 \sin 18^\circ \sin 60^\circ$

$= -2 \times \frac{\sqrt{5}-1}{4} \times \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}(1-\sqrt{5})}{4}$

16. A biased die is marked with numbers 2, 4, 8, 16, 32, 32 on its faces and the probability of getting a face with mark n is 1/n. If the die is thrown thrice, then the probability, that the sum of the numbers obtained is 48, is :

(A) $\frac{7}{2^{11}}$ (B) $\frac{7}{2^{12}}$ (C) $\frac{3}{2^{10}}$ (D) $\frac{13}{2^{12}}$

Sol. **(D)**

P(sum=48) = P(16) ×P(16)×P(16)

+ P(32) ×P(8)×P(8)

$= \frac{1}{16} \times \frac{1}{16} \times \frac{1}{16} + 3_{C_1} \times \frac{2}{32} \times \frac{1}{8} \times \frac{1}{8}$

$= \frac{1}{16^3} + \frac{6\times 2}{16\times 16\times 16}$

$= \frac{13}{16^3} = \frac{13}{2^{12}}$

17. The negation of the Boolean expression $((\sim q) \wedge p) \Rightarrow \left((\sim p) \vee q\right)$ is logically equivalent to :

(A) P ⇒ q (B) q ⇒ p (C) ~(p ⇒ q) (D) ~(q ⇒ p)

Sol. **(C)**

$\sim [(\overset{P}{(\sim q) \wedge p}) \Rightarrow (\overset{Q}{(\sim p) \vee q})]$

$P \wedge \sim Q \Rightarrow (\sim q \wedge p) \wedge \sim \{(\sim p) \vee q\}$

$= (\sim q \wedge p) \wedge (p \wedge \sim q)$

$= (p \wedge \sim q) \wedge (p \wedge \sim q)$

$= (p \wedge \sim q) = \sim (\sim p \vee q)$

$= \sim (p \Rightarrow q)$

18. If the line y = 4 + kx, k > 0, is the tangent to the parabola y = x- x^2 at the point P and V is the vertex of the parabola, then the slope of the line through P and V is :

(A) $\frac{3}{2}$ (B) $\frac{26}{9}$ (C) $\frac{5}{2}$ (D) $\frac{23}{6}$

Sol. **(C)**

$y = x - x^2$

$v\left(-\frac{b}{2a}, \frac{-D}{4a}\right) \equiv \left(\frac{-1}{-2}, \frac{-(1)}{4(-1)}\right) = \left(\frac{1}{2}, \frac{1}{4}\right)$

$y = x - x^2$

$x^2 - x + kx + 4 = 0$

$x^2 + x(k-1) + 4 = 0$

D = 0 $\Rightarrow (k-1)^2 - 4^2 = 0$

$\Rightarrow k - 1 = 4, -4$

$\Rightarrow k = 5, -3$

$\Rightarrow k = 5 \quad (\because k > 0)$

Now, equation of tangent is y = 4 + 5x

$5x + 4 = x - x^2$

$x^2 + 4x + 4 = 0 \Rightarrow x = -2$

$y = -2 - 4 = -6$

So, P(–2,–6), $v\left(\frac{1}{2},\frac{1}{4}\right)$

$m_{PV} = \frac{\frac{1}{4}+6}{\frac{1}{2}+2} = \frac{25}{4} \times \frac{2}{5} = \frac{5}{2}$

19. The value of $\tan^{-1}\left(\frac{\cos\left(\frac{15\pi}{4}\right)-1}{\sin\left(\frac{\pi}{4}\right)}\right)$ is equal to :

(A) $\frac{-\pi}{4}$ (B) $-\frac{\pi}{8}$ (C) $-\frac{5\pi}{12}$ (D) $-\frac{4\pi}{9}$

Sol. **(B)**

$\tan^{-1}\left\{\frac{\cos\left(3\pi+\frac{3\pi}{4}\right)-1}{\sin\left(\frac{3\pi}{4}\right)}\right\}$

$-\tan^{-1}\left\{\frac{1-\left(-\cos\frac{3\pi}{4}\right)}{\sin\frac{3\pi}{4}}\right\}$

$-\tan^{-1}\left\{\frac{2\cos^2\frac{3\pi}{8}}{2\sin\frac{3\pi}{8}\cos\frac{3\pi}{8}}\right\}$

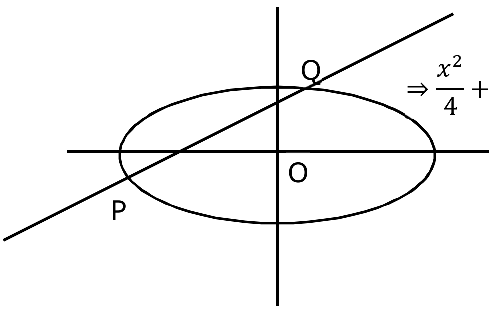
$-\frac{\pi}{2} + \cot^{-1}(\cot(\frac{3\pi}{8}))$

$-\frac{\pi}{2} + \frac{3\pi}{8} = \frac{-\pi}{8}$

20. The line y = x + 1 meets the ellipse $\frac{x^2}{4} + \frac{y^2}{2} = 1$ at two points P and Q. If r is the radius of the circle with PQ as diameter then $(3r)^2$ is equal to :

(A) 20 (B) 12 (C) 11 (D) 8

Sol. **(A)**



$\frac{x^2}{4} + \frac{(x+1)^2}{2} = 1$

$x^2 + 2x^2 + 4x + 2 = 4$

$3x^2 + 4x - 2 = 0$

$x = \frac{-4\pm\sqrt{16+24}}{6} = \frac{-4\pm2\sqrt{10}}{6}$

$x = \frac{-2+\sqrt{10}}{3}, \frac{-2-\sqrt{10}}{3}$

at $= \frac{-2+\sqrt{10}}{3}, y = \frac{-2+\sqrt{10}}{3} + 1 = \frac{1+\sqrt{10}}{3}$

at $x = \frac{-2-\sqrt{10}}{3}, y = \frac{1-\sqrt{10}}{3}$

$PQ = \sqrt{\left(\frac{2\sqrt{10}}{3}\right)^2 + \left(\frac{2\sqrt{10}}{3}\right)^2}$

$= \sqrt{2 \times \frac{40}{9}} = \frac{2}{3}\sqrt{20}$

$r = \frac{PQ}{2} = \frac{\sqrt{20}}{3}$

$(3r)^2 = 20$

21. Let $A = \begin{pmatrix} 2 & -2 \\ 1 & -1 \end{pmatrix}$ and $= \begin{pmatrix} -1 & 2 \\ -1 & 2 \end{pmatrix}$. Then the number of elements in the set {(n, m) : n, m ∈ {1, 2, ...10} and $nA^n + mB^m = I$} is

Sol. **(1)**

$A^2 = \begin{pmatrix} 2 & -2 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 2 & -2 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 2 & -2 \\ 1 & -1 \end{pmatrix} = A$

$B^2 = \begin{pmatrix} -1 & 2 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} -1 & 2 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ -1 & 2 \end{pmatrix} = B$

⇒ A & B are idempotent

Now, $nA^n + mB^m = nA + mB = I$

which gives m = n = 1

Only one set possible

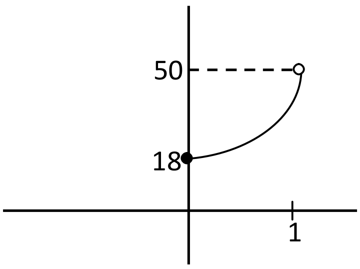
22. Let $f(x) = [2x^2 + 1]$ and $g(x) = \begin{cases} 2x - 3, & x < 0 \\ 2x + 3, & x \geq 0 \end{cases}$, where [t] is the greatest integer ≤ t. Then, in the open interval (-1, 1), the number of points where fog is discontinuous is equal to ....

Sol. **(62)**

$f(x) = [2x^2] + 1 \geq 1$

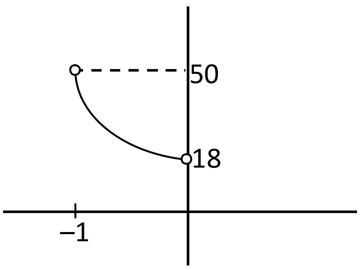
$g(x) = \begin{cases} 2x - 3\ , & x < 0 \\ 2x + 3, & x \geq 0 \end{cases}$

Now $fog(x) = [2(2x+3)^2] + 1$



49–18=31

$fog(x) = [2(2x-3)^2] + 1$



50 – 18 – 1 = 31

∴ 62 points of discontinuity

23. The value of b > 3 for which $12\int_3^b \frac{1}{(x^2-1)(x^2-4)}dx = \log_e\left(\frac{49}{40}\right)$ is equal to :

Sol. **(6)**

$12\int_3^b \frac{dx}{(x^2-1)(x^2-4)} = \ln\left(\frac{49}{40}\right)$

$\Rightarrow \frac{12}{3}\int_3^b \frac{(x^2-1)-(x^2-4)}{(x^2-1)(x^2-4)}dx = \ln\left(\frac{49}{40}\right)$

$\Rightarrow 4\left[\int_3^b \frac{dx}{x^2-4} - \int_3^b \frac{dx}{x^2-1}\right] = \ln\left(\frac{49}{40}\right)$

$\Rightarrow 4\left[\frac{1}{4}\left(\ln\left|\frac{x-2}{x+2}\right|\right)_3^b - \frac{1}{2}\left(\ln\left|\frac{x-1}{x+1}\right|\right)_3^b\right] = \ln\left(\frac{49}{40}\right)$

$\Rightarrow \ln\left|\frac{b-2}{b+2}\right| - \ln\left(\frac{1}{5}\right) - 2\ln\left|\frac{b-1}{b+1}\right| + 2\ln\left(\frac{1}{2}\right) = \ln\left(\frac{49}{40}\right)$

$\Rightarrow \ln\left[\left(\frac{b-2}{b+2}\right)\times\frac{(b+1)^2}{(b-1)^2}\times\frac{5}{4}\right] = \ln\left(\frac{49}{40}\right)$

$\Rightarrow \left(\frac{b-2}{b+2}\right)\times\frac{(b+1)^2}{(b-1)^2}\times\frac{5}{4} = \frac{49}{40}$

$\Rightarrow \frac{(b-2)(b^2+2b+1)}{(b+2)(b^2-2b+1)} = \frac{49}{50}$

$\Rightarrow \frac{b^3+2b^2+b-2b^2-4b-2}{b^3-2b^2+b+2b^2-4b+2} = \frac{49}{50}$

$(b^3 - 3b - 2)\,50 = 49(b^3 - 3b + 2)$

$b^3 - 150b + 147b - 100 - 98 = 0$

$b^3 - 3b - 198 = 0$

$\Rightarrow b = 6$

24. If the sum of the coefficients of all the positive even powers of x in the binomial expansion of $\left(2x^3+\frac{3}{x}\right)^{10}$ is $5^{10}-\beta.3^9$, then β is equal to ......

Sol. **(83)**

$\left(2x^3+\frac{3}{x}\right)^{10}$

$T_{r+1} = 10_{C_r}(2x^3)^{10-r}\left(\frac{3}{x}\right)^r$

$= 10_{C_r}(2)^{10-r}x^{30-4r}3^r$

at r = 0, 1, 2, 3, 4, 5, 6, 7 we will get even powers of 'x'.

$10_{C_0}(2)^{10} + 10_{C_1}(2)^9 3^1 + 10_{C_2}(2)^8 3^2 + \ldots + 10_{C_7}(2)^3 3^7$

$\because (2+3)^{10} = 10_{C_0}(2)^{10} + \ldots + 10_{C_7}(2)^3(3)^7 + 10_{C_8}(2)^2 3^8 + \ldots + 10_{C_{10}}(3)^{10}$

So, sum of the co-efficients of all the positive even powers of x

$= 5^{10} - \{10_{C_8}2^2\times 3^8 + 10_{C_9}(2)^1(3)^9 + 10_{C_{10}}(3)^{10}\}$

$= 5^{10} - 3^9\{\frac{10\times 9}{2}\times\frac{4}{3} + 10\times 2 + 3\}$

$= 5^{10} - 3^9(60+23) = 5^{10} - 3^9\times 83$

So, β = 83

25. If the mean deviation about the mean of the numbers 1,2,3..... n, where n is odd, is $\frac{5(n+1)}{n}$, then n is equal to :

Sol. **(21)**

$Mean = \frac{1+2+....+n}{n} = \frac{n+1}{2}$

n = 2k–1 ⇒ Mean = k

$M.D.(Mean) = \frac{|1-k|+|2-k|+....+|n-k|}{n}$

$$= \frac{2((k-1)+(k-2)+...+1)+0}{n}$$
$$= \frac{2(k-1)k}{2n} = \frac{k(k-1)}{n}$$
$$\Rightarrow \left(\frac{n+1}{2}\right)\left(\frac{n-1}{2n}\right) = \frac{5(n+1)}{n}$$
$\Rightarrow (n - 1) = 20$
$\Rightarrow n = 21$

26. Let $\vec{b} = \hat{\imath} + \hat{\jmath} + \lambda\hat{k}, \lambda \in R$. If $\vec{a}$ is a vector such that $\vec{a} \times \vec{b} = 13\hat{\imath} - \hat{\jmath} - 4\hat{k}$ and $\vec{a}.\vec{b} + 21 = 0$, then $(\vec{b} - \vec{a}).(\hat{k} - \hat{\jmath}) + (\vec{b} + \vec{a}).(\hat{\imath} - \hat{k})$ is equal to :

Sol. **(14)**

$\vec{a} \times \vec{b} = 13\hat{\imath} - \hat{\jmath} - 4\hat{k}$
$\vec{b} \times (\vec{a} \times \vec{b}) = \vec{b} \times (13\hat{\imath} - \hat{\jmath} - 4\hat{k})$
$$\vec{a}|\vec{b}|^2 - (\vec{a}\cdot\vec{b})\vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & \lambda \\ 13 & -1 & -4 \end{vmatrix}$$
$(\lambda^2 + 2)\vec{a} + 21\vec{b} = \hat{\imath}(\lambda - 4) - \hat{\jmath}(-4 - 13\lambda) + \hat{k}(-14)$ ...(1)
Now, let $\vec{a} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$
$\vec{a} \times \vec{b} = 13\hat{\imath} - \hat{\jmath} - 4\hat{k}$
$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x & y & z \\ 1 & 1 & \lambda \end{vmatrix} = (\lambda y - z)\hat{\imath} - \hat{\jmath}(\lambda x - z) + \hat{k}(x - y) = 13\hat{\imath} - \hat{\jmath} - 4\hat{k}$$

$\lambda y - z = 13$, $\quad z - \lambda x = -1$, $\quad x - y = -4$
$\lambda(y - x) = 12 \quad \Rightarrow \lambda = 3$

Put in (1)
$11\vec{a} + 21\vec{b} = -\hat{\imath} + 43\hat{\jmath} - 14\hat{k}$
$11\vec{a} + 21\hat{\imath} + 21\hat{\jmath} + 21\lambda\hat{k} = -\hat{\imath} + 43\hat{\jmath} - 14\hat{k}$
$11\vec{a} + 21\hat{\imath} + 21\hat{\jmath} + 63\hat{k} = -\hat{\imath} + 43\hat{\jmath} - 14\hat{k}$
$11\vec{a} + 22\hat{\imath} - 22\hat{\jmath} + 77\hat{k} = 0$
$\vec{a} = -2\hat{\imath} + 2\hat{\jmath} - 7\hat{k}$
$(\vec{b} - \vec{a}) \cdot (\hat{k} - \hat{\jmath}) + (\vec{a} + \vec{b})(\hat{\imath} - \hat{k})$
$= [((1,1,3) - (-2,2,-7)) \cdot (\hat{k} - \hat{\jmath})] + [((1,1,3) + (-2,2,-7)) \cdot (\hat{\imath} - \hat{k})]$
$= [(3\hat{\imath} - \hat{\jmath} + 10\hat{k}) \cdot (\hat{k} - \hat{\jmath})] + [(-\hat{\imath} + 3\hat{\jmath} - 4\hat{k}) \cdot (\hat{\imath} - \hat{k})]$
$= (1 + 10) + (-1 + 4) = 14$

27. The total number of three - digit numbers, with one digit repeated exactly two times, is

Sol. **(243)**

Case I : When two zero
a 0 0 $\quad a \in \{1, 2, ...,9\}$
so such numbers = 9

Case II: When one zero
a 0 a $\quad a \in \{1, 2, ...,9\}$
a a 0
such numbers = 9×2 = 18

Case III : When no zero

a a b

a b a

b a a

such numbers = 3 × 9 × 8 = 216

total = 9 + 18 + 216 = 243

28. Let f(x) = |(x – 1)(x^2 – 2x – 3)| +x – 3, x ∈ R. If m and M are respectively the number of points of local minimum and local maximum of f in the interval (0, 4), then m + M is equal to :

Sol. **(3)**

$$f(x) = \begin{cases} (x^2-1)(x-3)+(x-3), x \in (0,1] \cup [3,4) \\ -(x^2-1)(x-3)+(x-3), x \in [1,3] \end{cases}$$

$$\Rightarrow f'(x) \begin{cases} 3x^2-6x, x \in (0,1) \cup (3,4) \\ -3x^2+6x+2, x \in (1,3) \end{cases}$$

f(x) is non-derivable at x = 1 and x = 3

also f'(x) = 0 at x = 1 + $\sqrt{\frac{5}{3}}$ ⇒ m + M = 3

29. Let the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be $\frac{5}{4}$. If the equation of the normal at the point $\left(\frac{8}{\sqrt{5}},\frac{12}{5}\right)$ on the hyperbola is $8\sqrt{5}x+\beta y=\lambda$, then $\lambda-\beta$ is equal to :

Sol. **(85)**

$$b^2 = a^2\left(\frac{25}{16}-1\right) = a^2 \times \frac{9}{16}$$

$$\frac{x^2}{a^2}-\frac{y^2 \times 16}{9a^2}=1$$

It passes through $\left(\frac{8}{\sqrt{5}},\frac{12}{5}\right)$

$$\frac{64}{5a^2}-\frac{144\times16}{25\times9a^2}=1$$

320 – 256 = 25a^2

64 = 25a^2

$a=\frac{8}{5}$ and $b^2=\frac{9a^2}{16}$

$$\Rightarrow b=\frac{3}{4}\times\frac{8}{5}=\frac{6}{5}$$

Now $\frac{x^2}{\left(\frac{8}{5}\right)^2}-\frac{y^2}{\left(\frac{6}{5}\right)^2}=1$

equation of normal

$$\frac{a^2x}{x_1}+\frac{b^2y}{y_1}=a^2+b^2$$

$$\frac{64}{25}\times\frac{x\sqrt{5}}{8}+\frac{36}{25}\times\frac{y\times5}{12}=\frac{64+36}{25}$$

$$\frac{8x\sqrt{5}}{25}+\frac{15y}{25}=\frac{100}{25}$$

$$\Rightarrow \beta=15, \lambda=100$$

$$\Rightarrow \lambda-\beta=85$$

30. Let ℓ_1 be the line in xy – plane with x and y intercepts $\frac{1}{8}$ and $\frac{1}{4\sqrt{2}}$ respectively, and ℓ_2 be the line in zx – plane with x and z intercepts $-\frac{1}{8}$ and $-\frac{1}{6\sqrt{3}}$ respectively. If d is the shortest distance between the line ℓ_1 and ℓ_2, then d^{-2} is equal to :

Sol. **(51)**

$l_1 : 8x + 4\sqrt{2}y = 1$
$l_2 : -8x - 6\sqrt{3}z = 1$

$l_1 : a_1 = \left(\frac{1}{8}, 0,0\right)$ & $a_2 = \left(0, \frac{1}{4\sqrt{2}}, 0\right)$
$P_1 : a_2 - a_1 = \langle\frac{-1}{8}, \frac{1}{4\sqrt{2}}, 0\rangle$

$l_2 : b_1 = \left(-\frac{1}{8}, 0,0\right)$ & $b_2 = \left(0,0, \frac{-1}{6\sqrt{3}}\right)$
$P_2 : b_2 - b_1 = \langle\frac{1}{8}, 0, \frac{-1}{6\sqrt{3}}\rangle$

$$\vec{n} = P_1 \times P_2 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -\frac{1}{8} & \frac{1}{4\sqrt{2}} & 0 \\ \frac{1}{8} & 0 & \frac{-1}{6\sqrt{3}} \end{vmatrix}$$

$$\vec{n} = \hat{\imath}\left(\frac{-1}{24\sqrt{6}}\right) - \hat{\jmath}\left(\frac{1}{48\sqrt{3}}\right) - \hat{k}\left(\frac{1}{32\sqrt{2}}\right)$$

$$\vec{n} = \frac{-1}{96\sqrt{6}}\langle 4, 2\sqrt{2}, 3\sqrt{3}\rangle$$

$\vec{c} = (a_1 - b_1) = \left(\frac{1}{4}, 0,0\right)$

$$d = \left|\frac{\vec{c} \cdot \vec{n}}{|\vec{n}|}\right|$$

$= \left|\frac{1}{\sqrt{16+8+27}}\right| = \frac{1}{\sqrt{51}}$

$\Rightarrow \frac{1}{d^2} = 51$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