JEE MAIN
QUESTION PAPER
WITH SOLUTION

38000+
SELECTIONS SINCE 2007

MATHEMATICS
27th June 2022 | Shift - 1

MOTION®
JEE (Main+Advanced) | NEET | NTSE | Olympiads | Boards

Umeed **Rank** Ki Ho Ya **Selection** Ki,
JEET NISCHIT HAI!

MOST PROMISING RANKS
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NEET / AIIMS

- AIR-1 TO 10 — 25 TIMES
- AIR-11 TO 25 — 37 TIMES
- AIR-26 TO 50 — 43 TIMES
- AIR-51 TO 100 — 78 TIMES

JEE MAIN+ADVANCED

- AIR-1 TO 10 — 8 TIMES
- AIR-11 TO 25 — 6 TIMES
- AIR-26 TO 50 — 18 TIMES
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NATION'S BEST SELECTION
PERCENTAGE (%) RATIO

STUDENT QUALIFIED IN NEET

2021	3276 / 3411 = 93.12%
2020	2663 / 2843 = 93.66%
2019	2041 / 2212 = 92.27%

STUDENT QUALIFIED IN JEE ADVANCED

2021	1256 / 2994 = 41.95%
2020	994 / 2538 = 39.16%
2019	769 / 2105 = 36.53%

STUDENT QUALIFIED IN JEE MAIN

2021	2994 / 4087 = 73.25%
2020	2538 / 3554 = 71.44%
2019	2288 / 3316 = 68.99%

NITIN VIJAY (NV Sir)
Founder & CEO

SECTION - A

1. The area of the polygon, whose vertices are the non – real roots of the equation $\bar{z} = iz^2$ is

(A) $\frac{3\sqrt{3}}{4}$ (B) $\frac{3\sqrt{3}}{2}$ (C) $\frac{3}{2}$ (D) $\frac{3}{4}$

Sol. **A**

$\Rightarrow$ Let z = x + iy ; x, y $\in$ R

Now $\bar{z} = iz^2$

then $x - iy = i(x^2 - y^2 + 2xyi)$

$x - iy = i(x^2 - y^2) - 2xy$

Comparing both sides

$\Rightarrow x = -2xy$ & $-y = x^2 - y^2$

$\Rightarrow x(1 + 2y) = 0$

$x = 0$ or $y = -\frac{1}{2}$

Put $x = 0$ in $-y = x^2 - y^2$

We get $y = y^2$

$\Rightarrow y = 0, 1$

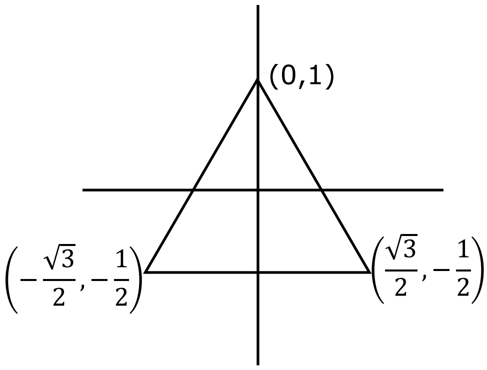
Similarly

Put $y = -\frac{1}{2}$ in $-y = x^2 - y^2$

$\Rightarrow \frac{1}{2} = x^2 - \frac{1}{4}$

$x = \pm\frac{\sqrt{3}}{2}$

$z = (0, i, \frac{\sqrt{3}}{2} - \frac{1}{2}i, -\frac{\sqrt{3}}{2} - \frac{1}{2}i)$



Area $= \frac{1}{2} \cdot (\sqrt{3})\left(\frac{3}{2}\right)$

$= \frac{3\sqrt{3}}{4}$

2. Let the system of linear equations x + 2y + z = 2, α x + 3y – z = α , -α x + y + 2z = - α be inconsistent. Then α is equal to :

(A) $\frac{5}{2}$ (B) $\frac{-5}{2}$ (C) $\frac{7}{2}$ (D) $-\frac{7}{2}$

Sol. **D**

$$\Delta = \begin{vmatrix} 1 & 2 & 1 \\ \alpha & 3 & -1 \\ -\alpha & 1 & 2 \end{vmatrix}$$

$= (6 + 1) - 2((2\alpha - \alpha) + 1(\alpha + 3\alpha)$

$= 7 - 2\alpha + 4\alpha$

$= 7 + 2\alpha$

$\Delta = 0 \Rightarrow \alpha = -\frac{7}{2}$

$$\Delta_1 \begin{vmatrix} 2 & 2 & 1 \\ \alpha & 3 & -1 \\ -\alpha & 1 & 2 \end{vmatrix}$$

$= 14 + 2\alpha$

$= 14 + 2\left(\frac{-7}{2}\right)$ $\left\{\because \alpha = \frac{-7}{2}\right\}$

$\Delta_1 = 7 \neq 0$

3. If $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$, where a, b,c are in A.P. and $|a| < 1, |b| < 1, |c| < 1, abc \neq 0$, then
(A) x, y,z are in A.P. (B)x, y, z are in G.P. (C)$\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P. (D)$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1 - (a + b + c)$

Sol. **C**
$x = 1 + a + a^2 = \ldots\ldots\ldots$
$x = \frac{1}{1-a} \Rightarrow a = 1 - \frac{1}{x}$
$y = \frac{1}{1-b} \Rightarrow b = 1 - \frac{1}{y}$
$z = \frac{1}{1-c} \Rightarrow c = 1 - \frac{1}{z}$
a, b, c are in A.P.
$\Rightarrow 1 - \frac{1}{x}, 1 - \frac{1}{y}, 1 - \frac{1}{z}$ are in A.P.
$\Rightarrow -\frac{1}{x}, -\frac{1}{y}, -\frac{1}{z}$ are in A.P.
$\Rightarrow \frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.

4. Let $\frac{dy}{dx} = \frac{ax-by+a}{bx+cy+a}$ where a, b, c are constants, represent a circle passing through point (2, 5) then the shortest distance of the point (11, 6) from this circle is :
(A) 10 (B)8 (C)7 (D)5

Sol. **B**
Let equation of circle is
$x^2 + y^2 + 2gx + 2fy + c = 0$
$\Rightarrow \frac{dy}{dx} = \frac{-(2x+2g)}{(2y+2f)}$
Comparing with $\frac{dy}{dx} = \frac{ax-by+a}{bx+cy+a}$
$\Rightarrow$ b= 0, a =–2, c = 2
$\Rightarrow -2g = -2 \Rightarrow g = 1$ $2f = -2$
$f = -1$
Now circle will be
$x^2 + y^2 + 2x - 2y + c = 0$
its passes through (2,5)
which will give c = –23
so circle will be $x^2 + y^2 + 2x - 2y - 23 = 0$
centre C = (–1,1)
and radius 5
Now P is (11,6)
So minimum distance of P from circle will be
$\sqrt{(11+1)^2 + (6-1)^2} - 5$
= 13 – 5
= 8

5. Let a be an integer such that $\lim_{x\to 7} \frac{18-[1-x]}{[x-3a]}$ exists, where [t] is greatest integer ≤ t, then a is equal to :
(A) -6 (B)- 2 (C)2 (D)6

Sol. **A**
$\lim_{x\to 7} \frac{18-[1-x]}{[x]-3a}$
L.H.L. $\lim_{x\to 7^-} \frac{18-[1-x]}{[x]-3a}$
$= \frac{18-(-6)}{6-3a}$
$= \frac{24}{6-3a}$

R.H.L. $\lim_{x\to 7^+} \frac{18[1-x]}{[x]-3a}$

$= \frac{18-(-7)}{7-3a}$

$= \frac{25}{7-3a}$

Now L.H.L. = R.H.L.

$\frac{24}{6-3a} = \frac{25}{7-3a}$

$\Rightarrow 168 - 72a = 150 - 75a$

$\Rightarrow 18 = -3a$

$\Rightarrow a = -6$

6. The number of distinct real roots of $x^4 - 4x + 1 = 0$ is

(A) 4 (B)2 (C)1 (D)0

Sol. **B**

Let $f(x) = x^4 - 4x + 1$

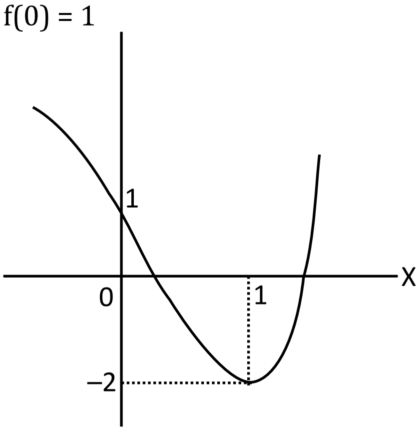
$f'(x) = 4x^3 - 4$

$f'(x) = 0 \Rightarrow x = 1$

x = 1 is point of minima

$f(1) = -2$

$f(0) = 1$



Hence 2 solutions

7. The lengths of the sides of a triangle are $10 + x^2$, $10 + x^2$ and $20 - 2x^2$. if for x = k , the area of the triangle is maximum, then $3k^2$ is equal to :

(A) 5 (B)8 (C)10 (D)12

Sol. **C**

$a = 20 - 2x^2$, $b = 10 + x^2$, $c = 10 + x^2$

$= \frac{a+b+c}{2}$

= 20

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{20(2x^2)(10-x^2)(10-x^2)}$

$= 2\sqrt{10}\sqrt{x^2(10-x^2)^2}$

$= 2\sqrt{10}|x(10-x^2)|$

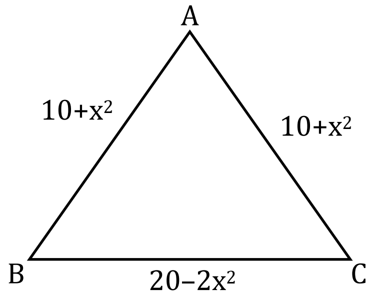
$= 2\sqrt{10}|10x - x^3|$

$S = 10x - x^3$

$\frac{ds}{dx} = 10 - 3x^2$

$\frac{ds}{dx} = 0$

$\Rightarrow 3x^2 = 10$



8. If $\cos^{-1}(\frac{y}{2}) = \log_e(\frac{x}{5})^5, |y| < 2$, then :

(A) $x^2y'' + xy' - 25y = 0$ (B) $x^2y'' - xy' - 25y = 0$

(C) $x^2y'' - xy' + 25y = 0$ (D) $x^2y'' + xy' + 25y = 0$

Sol. **D**

$\cos^{-1}\left(\frac{y}{2}\right) = \log_e\left(\frac{x}{5}\right)^5$

$\cos^{-1}\left(\frac{y}{2}\right) = 5\log_e\left(\frac{x}{5}\right)$

$\frac{-1}{\sqrt{1-\frac{y^2}{4}}} \cdot \frac{y'}{2} = 5 \cdot \frac{1}{\frac{x}{5}} \times \frac{1}{5}$

$\Rightarrow \frac{-y'}{\sqrt{4-y^2}} = \frac{5}{x}$

$-xy' = 5\sqrt{4-y^2}$

$-xy'' - y' = 5.\frac{1}{2\sqrt{4-y^2}}(-2y\,y')$

$\Rightarrow xy'' + y' = \frac{5y' \cdot y}{\sqrt{4-y^2}}$

$xy'' + y' = 5 \cdot \left(\frac{-5}{x}\right)y$

$x^2y'' + xy' = -25y$

9. If $\int \frac{(x^2+1)e^x}{(x+1)^2} dx = f(x)e^x + C$, where C is a constant, then $\frac{d^3f}{dx^3}$ at x = 1 is equal to :

(A) $\frac{-3}{4}$ (B) $\frac{3}{4}$ (C) $-\frac{3}{2}$ (D) $\frac{3}{2}$

Sol. **B**

$\int (\frac{x^2+1}{(x+1)^2})e^x \cdot dx$

$\int (\frac{x^2-1+2}{(x+1)^2})e^x \cdot dx$

$\int (\frac{x-1}{x+1} + \frac{2}{(x+1)^2})e^x \cdot dx$

$\int (f(x) + f'(x))e^x dx$

$= f(x)\, e^x + c$

Where $f(x) = \frac{x-1}{x+1}$

$f'(x) = \frac{2}{(x+1)^2}$

$f''(x) = \frac{-4}{(x+1)^3}$

$f''' = \frac{12}{(x+1)^4}$

$f'''(1) = \frac{12}{16}$

$= \frac{3}{4}$

10. The value of the integral $\int_{-2}^{2} \frac{|x^3+x|}{(e^{x|x|}+1)}$ is equal to :

(A) $5e^2$ (B) $3e^{-2}$ (C) 4 (D) 6

Sol. **D**

$f(x) = \frac{|x^3+x|}{(e^{x|x|}+1)}dx$

$\int_{-2}^{2} f(x)\, dx = \int_0^2 (f(x) + f(-x))dx$

$= \int_0^2 (\frac{|x^3+x|}{(e^{x|x|}+1)} + \frac{|-x^3-x|}{(e^{-x|-x|}+1)})dx$

$=\int_0^2 \left(\frac{|x^3+x|}{(e^{x|x|}+1)}+\frac{|x^3+x|}{(e^{-x|x|}+1)}\right)dx$

$=\int_0^2 \left(\frac{x^3+x}{(e^{x^2}+1)}+\frac{x^3+x}{(e^{-x^2}+1)}\right)dx$

$I=\int_0^2 \left(\frac{x^3+x}{1+e^{x^2}}+\frac{e^{x^2}(x^3+x)}{1+e^{x^2}}\right)dx$

$=\int_0^2 (x^3+x)dx$

$=\left[\frac{x^4}{4}+\frac{x^2}{2}\right]_0^2$

$= 4 + 2 = 6$

11. If $\frac{dy}{dx}+\frac{2^{x-y}(2^y-1)}{2^x-1}=0, x, y>0, y(A)=1$, then $y(B)$ is equal to :

(A) $2 + \log_2 3$ (B) $2 + \log_3 2$ (C) $2 - \log_3 2$ (D) $2 - \log_2 3$

Sol. **D**

$\frac{dy}{dx}+\frac{2^{x-y}(2^y-1)}{2^x-1}=0$

$x, y > 0, y(1) = 1, y(2) = ?$

$\frac{dy}{dx}=\frac{-2^x(2^y-1)}{2^y(2^x-1)}$

$\int \frac{2^y}{2^y-1}dy=-\int \frac{2^x}{2^x-1}dx$

$\frac{1}{\ln 2}\int \frac{2^y \ln 2}{2^y-1}dy=-\frac{1}{\ln 2}\int \frac{2^x \ln 2}{2^x-1}dx$

$\frac{1}{\ln 2}\ln|2^y-1|=\frac{-1}{\ln 2}\ln|2^x-1|+C$

At $x = 1, y = 1$

Putting this values in above relation we get $C = 0$

$\ln|2^y-1| + \ln|2^x-1| = 0$

$(2^x-1)(2^y-1) = 1$

$2^y - 1 = \frac{1}{2^x-1}$

At $x = 2$

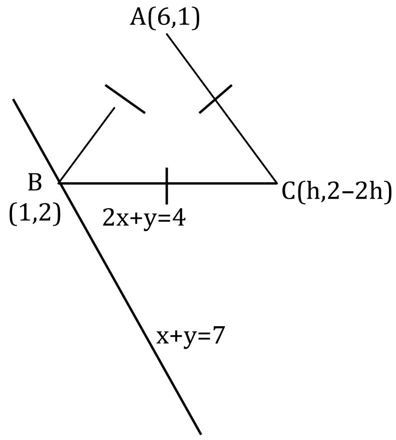
$2^y=\frac{1}{3}+1=\frac{4}{3}$

$y = \log_2 \frac{4}{3} = \log_2 4 - \log_2 3 = 2 - \log_2 3$

12. In an isosceles triangle ABC, the vertex A is (6, 1) and the equation of the base :BC is $2x + y = 4$. Let the point B lie on the line $x + 3y = 7$. If (α, β) is the centroid of ΔABC, then $15(\alpha+\beta)$ is equal to :

(A) 39 (B) 41 (C) 51 (D) 63

Sol. **C**



Point B(1,2)

Now let C be $(h, 4 - 2h)$

(As C lies on $2x + y = 4$)

$\because \Delta$ is isosceles with base BC

$\therefore AB = AC$

$\sqrt{25+1}=\sqrt{(6-h)^2+(2h-3)^2}$

$\sqrt{26}=\sqrt{36+h^2-12h+4h^2+9-12h}$

$26 = 5h^2 - 24h + 45 \Rightarrow 5h^2 - 24h + 19 = 0$

$\Rightarrow 5h^2 - 5h - 19h + 19 = 0$

$h=\frac{19}{5}$ or $h = 1$

Thus $C\left(\frac{19}{5},\frac{-18}{5}\right)$

Centroid $\left(\frac{6+1+\frac{19}{5}}{3},\frac{1+2-\frac{18}{5}}{3}\right)$

$\left(\frac{35+19}{15},\frac{15-18}{15}\right)$

$\left(\frac{54}{15},\frac{-3}{15}\right)$

$\alpha = \frac{54}{15}; \beta = \frac{-3}{15}$

$15(\alpha + \beta) = 51$

13. Let the eccentricity of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b}=1$, a > b , be 1/4. if this ellipse passes through the point $\left(-4\sqrt{\frac{2}{5}},3\right)$, then $a^2 + b^2$ is equal to :

(A) 29 (B)31 (C)32 (D)34

Sol. **B**

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ a>b

$e^2 = 1-\frac{b^2}{a^2}$

$\frac{1}{16}=1-\frac{b^2}{a^2}$

$\frac{b^2}{a^2}=1-\frac{1}{16}=\frac{15}{16} \Rightarrow b^2=\frac{15}{16}a^2$

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$\frac{16\times\frac{2}{5}}{a^2}+\frac{9}{b^2}=1$

$\frac{32}{5a^2}+\frac{9}{b^2}=1$

$\frac{32}{5a^2}+\frac{9}{\frac{15}{16}a^2}=1$

$\frac{80}{5a^2}=1$

$16 = a^2$

$b^2 = 15$

14. If two straight lines whose direction cosines are given by the relations $l + m - n = 0$, $3l^2 + m^2 + cn = 0$ are parallel, then the positive value of c is :

(A) 6 (B)4 (C)3 (D)2

Sol. **A**

$1 + m - n = 0$

$3l^2 + m^2 + cl(l+m) = 0$ $\because n = l + m$

$3l^2 + m^2 + cl^2 + clm = 0$

$(3 + c)l^2 + clm + m^2 = 0$

$(3+c)(\frac{l}{m})^2 + c(\frac{l}{m}) + 1 = 0$ .....(1)

$\because$ lies are parallel

Roots of (1) must be equal

$\Rightarrow D = 0$

$c^2 - 4(3 + c) = 0$

$c^2 - 4c - 12 = 0$

$(c - 6)(c + 2) = 0$

$c = 6$ or $c = -2$

+ve value of c = 6

15. Let $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} - 3\hat{j} + 2\hat{k}$. Then the number of vectors $\vec{b}$ such that $\vec{b} \times \vec{c} = \vec{a}$ and $|\vec{b}| \in \{1, 2, ..., 10\}$ is

(A) 0 (B)1 (C)2 (D)3

Sol. **A**

$\vec{a} = i + j - k$

$\vec{c} = 2i - 3j + 2k$

$\vec{b} \times \vec{c} = \vec{a}$

$|\vec{b}| \in \{1,2......10\}$

$\because \vec{b} \times \vec{c} = \vec{a}$

$\Rightarrow \vec{a}$ is perpendicular to $\vec{b}$ as well as $\vec{a}$ is perpendicular to $\vec{c}$

Now $\vec{a} \cdot \vec{c}$-= 2 – 3 – 2 = –3 ≠ 0

This $\vec{b} \times \vec{c} = \vec{a}$ is not possible .

No. of vectors $\vec{b}$ = 0

16. Five numbers x_1, x_2, x_3, x_4, x_5 are randomly selected from the numbers 1, 2, 3, ..., 18 and are arranged in the increasing order $(x_1 < x_2 < x_3 < x_4 < x_5)$. The probability that $x_2 = 7$ and $x_4 = 11$ is :

(A) $\frac{1}{136}$ (B)$\frac{1}{72}$ (C)$\frac{1}{68}$ (D)$\frac{1}{34}$

Sol. **C**

No. of ways to select and arrange x_1, x_2, x_3, x_4, x_5 from 1, 2, 3....... 18

$n(s) = {}^{18}C_5$

x_1 (x_2) x_3 (x_4) x_5

7 11

$n(E) = {}^6C_1 \times {}^3C_1 \times {}^7C_1$

$P(E) = \frac{6\times3\times7}{{}^{18}C_5}$

$\frac{1}{17\times4} = \frac{1}{68}$

17. Let X be a random variable having binomial distribution B(7, p). If P(X = 3) = 5P(X = 4), then the sum of the mean and the variance of X is :

(A) $\frac{105}{16}$ (B)$\frac{7}{16}$ (C)$\frac{77}{36}$ (D)$\frac{49}{16}$

Sol. **C**

B(7, p)

n = 7 p = p

given

P(x = 3) = 5P(x = 4)

${}^7C_3 \times p^3 (1 - p)^4 = 5.{}^7C_4(1-p)^3$

$\frac{{}^7C_3}{5\times{}^7C_4} = \frac{p}{1-p}$

1 – p = 5p

6p = 1

$p = \frac{1}{6} \Rightarrow q = \frac{5}{6}$

n = 7

Mean = np = $7 \times \frac{1}{6} = \frac{7}{6}$

Var = npq = $7 \times \frac{1}{6} \times \frac{5}{6} = \frac{35}{36}$

Sum

$= \frac{7}{6} + \frac{35}{36}$

$= \frac{42+35}{36} = \frac{77}{36}$

18. The value of $cos\left(\frac{2\pi}{7}\right)+cos\left(\frac{4\pi}{7}\right)+cos\left(\frac{6\pi}{7}\right)$ is equal to :

(A) - 1 (B)$\frac{-1}{2}$ (C)$\frac{-1}{3}$ (D)$-\frac{1}{4}$

Sol. **B**

$cos\frac{2\pi}{7}+cos\frac{4\pi}{7}+cos\frac{6\pi}{7}$

$=\frac{sin\left(3\times\frac{\pi}{7}\right)}{sin\frac{\pi}{7}}\times cos\left(\frac{\frac{2\pi}{7}+\frac{6\pi}{7}}{2}\right)$

$=\frac{2sin\left(\frac{3\pi}{7}\right)}{2sin\frac{\pi}{7}}\times cos\left(\frac{4\pi}{7}\right)$

$=\frac{sin\left(\frac{7\pi}{7}\right)+sin\left(\frac{-\pi}{7}\right)}{2sin\frac{\pi}{7}}$

$=\frac{-sin\frac{\pi}{7}}{2sin\frac{\pi}{7}}$

$=-\frac{1}{2}$

19. $sin^{-1}\left(sin\frac{2\pi}{3}\right)+cos^{-1}\left(cos\frac{7\pi}{6}\right)+tan^{-1}\left(tan\frac{3\pi}{4}\right)$ is equal to :

(A) $\frac{11\pi}{12}$ (B)$\frac{17\pi}{12}$ (C)$\frac{31\pi}{12}$ (D)$-\frac{3\pi}{4}$

Sol. **A**

$sin^{-1}\left(sin\frac{2\pi}{3}\right)+cos^{-1}\left(cos\frac{7\pi}{6}\right)+tan^{-1}tan\left(\frac{3\pi}{4}\right)$

Now,

$sin^{-1}sin(\frac{2\pi}{3})=\pi-\frac{2\pi}{3}=\frac{\pi}{3}$

$cos^{-1}\left(cos\frac{2\pi}{6}\right)=2\pi-\frac{7\pi}{6}=\frac{5\pi}{6}$

$tan^{-1}tan\left(\frac{3\pi}{4}\right)=\frac{3\pi}{4}-\pi=\frac{-\pi}{4}$

So,

$sin^{-1}\left(sin\frac{2\pi}{3}\right)+cos^{-1}cos\frac{7\pi}{6}+tan^{-1}tan\frac{3\pi}{4}$

$=\frac{11\pi}{12}$

20. The boolean expression $((\sim p\wedge q))\vee q$ is equivalent to :

(A) $q\rightarrow(p\wedge q)$ (B)p → q (C)p → (p → q) (D)$p\rightarrow(p\vee q)$

Sol. **D**

$(\sim(p\wedge q))\vee q$

$=(\sim p\vee\sim q)\vee q$

$=\sim p\vee\sim q\vee q$

$=\sim p\vee$ t

= this statement is a tautology option D

p ⇒ $(p\vee$ q) is also a tautology.

OR

P	q	$P^\wedge q$	$\sim(P^\wedge q)$	$\sim(P^\wedge q)\vee q$	$p\vee q$	$p\rightarrow(p\vee q)$
T	T	T	F	T	T	T
T	F	F	T	T	T	T
F	T	F	T	T	T	T
F	F	F	T	T	F	T

SECTION B

21. Let f : R → R be a function defined by $f(x)=\frac{2e^{2x}}{e^{2x}+e}$, then $f\left(\frac{1}{100}\right)+f\left(\frac{2}{100}\right)+f\left(\frac{3}{100}\right)+....+f\left(\frac{99}{100}\right)$ is equal to ....

Sol. **99**

$f(x)+f(1-x)=\frac{2e^{2x}}{e^{2x}+e}+\frac{2e^{2-2x}}{e^{2-2x}+e}=2\left[\frac{e^{2x}}{e^{2x}+e}+\frac{e^{2}}{e^{2}+e^{2x+1}}\right]$

$=2\left[\frac{e^{2x-1}}{e^{2x-1}+1}+\frac{1}{1+e^{2x-1}}\right]=2$

$f\left(\frac{1}{100}\right)+f\left(\frac{2}{100}\right)+f\left(\frac{3}{100}\right)+....+f\left(\frac{99}{100}\right)$

$=\{f(\frac{1}{100})+f(\frac{99}{100})\}+\{f(\frac{2}{100})+f(\frac{98}{100})\}+$ ...... $+f\left\{\left(\frac{49}{100}\right)+f\left(\frac{51}{100}\right)\right\}+f\left(\frac{1}{2}\right)$

= (2 + 2 + 2 + ------ 49 times) + $\frac{2e}{e+e}$

= 98 + 1 = 99

22. If the sum of all the roots of the equation $e^{2x}-11e^{x}-45e^{-x}+\frac{81}{2}=0$ is $\log_e p$, then p is equal to ......

Sol. **45**

$e^{2x}-11e^{x}-45e^{-x}+\frac{81}{2}=0]$

$(e^x)^3-11(e^x)^2-45+\frac{81e^x}{2}=0]$

$e^x=t$

$2t^3-22t^2+81t-90=0$

$t_1\, t_2\, t_3=45$

$e^{x_1}.e^{x_2}.e^{x_3}=45$

$e^{x_1+x_2+x_3}=45$

$\log_e(e^{x_1+x_2+x_3})=\log_e 45$

$x_1+x_2+x_3=\log_e 45$

$\log_e P=\log_e 45$

P = 45

23. The positive value of the determinant of the matrix A, whose $Adj(Adj(A))=\begin{pmatrix}14 & 28 & -14\\ -14 & 14 & 28\\ 28 & -14 & 14\end{pmatrix}$, is .....

Sol. **14**

Adj (Adj A) = $\begin{bmatrix}14 & 18 & -14\\ -14 & 14 & 28\\ 28 & -14 & 14\end{bmatrix}$

$|Adj(AdjA)|=\begin{vmatrix}14 & 28 & -14\\ -14 & 14 & 28\\ 28 & -14 & 14\end{vmatrix}=14\times14\times14\begin{vmatrix}1 & 2 & -1\\ -1 & 1 & 2\\ 2 & -1 & 1\end{vmatrix}$

$=(14)^3\,[3-2(-5)-1(-1)]=(14)^3\,[14]=(14)^4$ $\{\because |Adj(AdjA)|=|A|^{(n-1)^2}\}$

$|A|^4=(14)^4\Rightarrow|A|=14$

24. The number of ways, 16 identical cubes, of which 11 are blue and rest are red, can be placed in a row so that between any two red cubes there should be at least 2 blue cubes, is ....

Sol. **56**

16 cubes → 11 Blue, 5 Red

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 11$

$x_1, x_6 \geq 0$, $x_1, x_3, x_4, x_5 \geq 2$

$x_2 = t_2 + 2$

$x_3 = t_3 + 2$

$x_4 = t_4 + 2$

$x_5 = t_5 + 2$

$x_1, t_2, t_3, t_4, t_5, x_6 \geq 0$

No. of solutions = ${}^{6+3-1}C_3 = {}^8C_3 = 56$

25. If the coefficient of x^{10} in the binomial expansion of $\left(\frac{\sqrt{x}}{5^{\frac{1}{4}}} + \frac{\sqrt{5}}{x^{\frac{1}{3}}}\right)^{60}$ is $5^k l$, where $l, k \in N$ and l is co-prime to 5, then k is equal to .....

Sol. **5**

$\left(\frac{\sqrt{x}}{5^{1/4}} + \frac{\sqrt{5}}{x^{1/3}}\right)^{60}$

$T_{r+1} = {}^{60}C_r \left(\frac{x^{1/2}}{5^{1/4}}\right)^{60-r} \left(\frac{5^{1/2}}{x^{1/3}}\right)^r$

$= {}^{60}C_r\, 5^{\frac{3r-60}{4}} \cdot x^{\frac{180-5r}{6}}$

$\frac{180-5r}{6} = 10 \Rightarrow r = 24$

Coeff. of $x^{10} = {}^{60}C_{24} 5^3 = \frac{\lfloor 60}{\lfloor 24 \lfloor 36} 5^3$

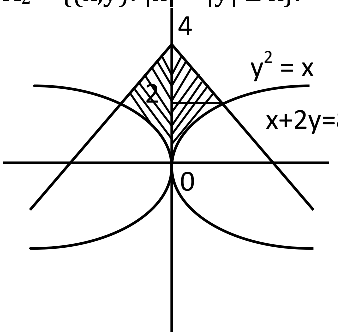
Powers of 5 in $= {}^{60}C_{24} . 5^3 = \frac{5^{14}}{5^4 \times 5^8} \times 5^3 = 5^5$

26. Let $A_1 = \{(x, y): |x| \leq y^2, |x| + 2y \leq 8\}$ and $A_2 = \{(x, y): |x| + |y| \leq k\}$. If 27 (Area A_1) = 5(Area A_2), then k is equal to :

Sol. **6**

$A_1 = \{(x,y): |x| \leq y^2, |x| + 2y \leq 8\}$ and

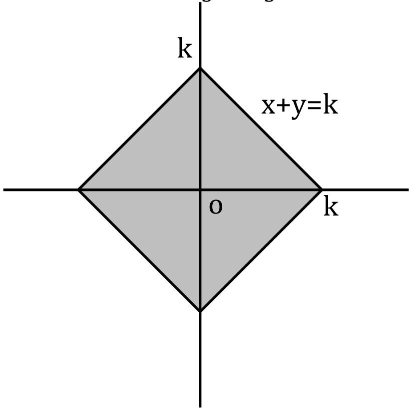
$A_2 = \{(x,y): |x| + |y| \leq k\}$.



area (A_1) = $2\left[\int_0^2 y^2 dy + \int_2^4 (8 - 2y) dy\right]$

$= 2\left[\left(\frac{y^3}{3}\right)_0^2 + (8y - y^2)_2^4\right]$

area $(A_1) = 2 \times \frac{20}{3} = \frac{40}{3}$



Area $(A_2) = 4 \times \frac{1}{2}k^2$
Area $(A_2) = 2k^2$
Now
27(Area A_1) = 5 (Area A_2)
$9 \times 4 = k^2$
k = 6

27. If the sum of the first ten term of the series $\frac{1}{5}+\frac{2}{65}+\frac{3}{325}+\frac{4}{1025}+\frac{5}{2501}+\ldots$ is $\frac{m}{n}$, where m and n are co-prime number, then m + n is equal to ....

Sol. **276**

$\frac{1}{5}+\frac{2}{65}+\frac{3}{325}+\frac{4}{1025}+\frac{5}{2501}+\ldots.$

$T_n = \frac{n}{4n^4+1}$

$= \frac{n}{(2n^2+1)^2-(2n)^2} = \frac{n}{(2n^2+2n+1)(2n^2-2n+1)}$

$= \frac{1}{4}\left[\frac{1}{2n^2-2n+1} - \frac{1}{2n^2+2n+1}\right]$

$S_{10} = \sum_{n=1}^{10} T_n = \frac{1}{4}\left[\frac{1}{1}-\frac{1}{5}+\frac{1}{5}-\frac{1}{13}+\ldots.\frac{1}{200+20+1}\right]$

$= \frac{1}{4}\left[1-\frac{1}{221}\right] = \frac{1}{4}\times\frac{220}{221} = \frac{55}{221} = \frac{m}{n}$

m + n = 55 + 221 = 276

28. A rectangle R with end points of one of its side as (1, 2) and (3, 6) is inscribed in a circle. if the equation of a diameter of the circle is 2x –y + 4 = 0, then the area of R is ......

Sol. **16**

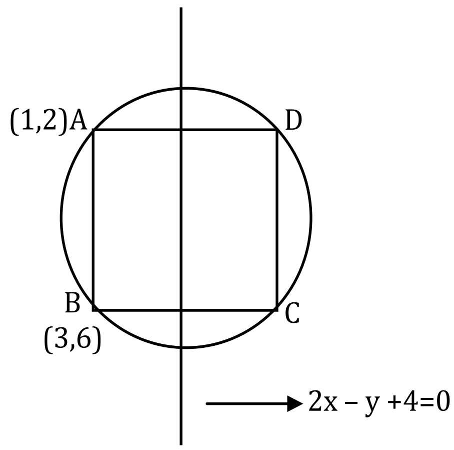
Eq. of line AB
y = 2x
Slope of AB = 2
Slope of given diameter = 2
So the diameter is parallel to AB
Distance between diameter and line AB

$-\left(\frac{4}{\sqrt{2^2+1^2}}\right) = \frac{4}{\sqrt{5}}$

Thus BC = $2 \times \frac{4}{\sqrt{5}} = \frac{8}{\sqrt{5}}$

$AB = \sqrt{(1-3)^2+(2-6)^2} = \sqrt{20} = 2\sqrt{5}$

Area = AB × BC = $\frac{8}{\sqrt{5}} \times 2\sqrt{5} = 16$



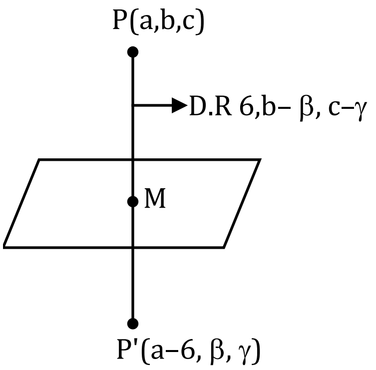
29. A circle of radius 2 unit passes through the vertex and the focus of the parabola $y^2 = 2x$ and touches the parabola $y = \left(x - \frac{1}{4}\right)^2 + \alpha$ where $\alpha > 0$, Then $(4\alpha - 8)^2$ is equal to ......

Sol. **63**

Vertex and focus of parabola $y^2 = 2x$
are V (0,0) and $S\left(\frac{1}{2}, 0\right)$ respectively
Let equation of circle be
$(x - h)^2 + (y - k)^2 = 4$
$\because$ Circle passes through (0,0)
$\Rightarrow h^2 + k^2 = 4$ ....(1)
$\because$ Circle passes through $\left(\frac{1}{2}, 0\right)$
$\left(\frac{1}{2} - h\right)^2 + k^2 = 4$
$\Rightarrow h^2 + k^2 - h = \frac{15}{4}$ ......(2)
On solving (1) and (2)
$4 - h = \frac{15}{4}$
$h = 4 - \frac{15}{4} = \frac{1}{4}$
$k = +\frac{\sqrt{63}}{4}$
$k = -\frac{\sqrt{63}}{4}$ is rejected as circle with centre
$\left(\frac{1}{4}, -\frac{\sqrt{63}}{4}\right)$ can't touch given parabola.
Equation of circle is
$\left(x - \frac{1}{4}\right)^2 + \left(y - \frac{\sqrt{63}}{4}\right)^2 = 4$
From figure
$\alpha = 2 + \frac{\sqrt{63}}{4} = \frac{8+\sqrt{63}}{4}$
$4\alpha - 8 = \sqrt{63}$
$(4\alpha - 8)^2 = \sqrt{63}$

30. Let the mirror image of the point (a, b, c) with respect to the plane $3x - 4y + 12z + 19 = 0$ be $(a - 6, \beta, \gamma)$, if $a + b + c = 5$, then $7\beta - 9\gamma$ is equal to ......

Sol. **137**



$M = \left(a - 3, \frac{\beta+b}{2}, \frac{\gamma+c}{2}\right)$
Since M lies on $3x - 4y + 12z + 19 = 0$
$\Rightarrow 3a - 2b + 6c - 2\beta + 6\gamma + 10 = 0$ ....(1)

Since PP' is parallel to normal of the plane then

$M = \left(a - 3, \frac{\beta+b}{2}, \frac{\gamma+c}{2}\right)$

$\Rightarrow \beta = b + 8, \quad \gamma = c - 24$

$a + b + c = 5 \Rightarrow a + \beta - 8 + \gamma + 24 = 5$

$\Rightarrow a = -\beta - \gamma - 11$

Now putting these values in (1) we get

$3(-\beta-\gamma-11)-2(\beta-8) + 6(\gamma + 24) - 2\beta + 6\gamma + 10 = 0$

$\Rightarrow 7\beta - 9\gamma = 137$