JEE MAIN 2023
Paper with Solution

Mathamatics | 1st Feb 2023 _ Shift-2

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Nation's Best SELECTION
Percentage (%) Ratio

Student Qualified in NEET

(2022)
4837/5356 = **90.31%**
(2021)
3276/3411 = **93.12%**

Student Qualified in JEE ADVANCED

(2022)
1756/4818 = **36.45%**
(2021)
1256/2994 = **41.95%**

Student Qualified in JEE MAIN

(2022)
4818/6653 = **72.41%**
(2021)
2994/4087 = **73.25%**

NITIN VIIJAY (NV Sir)
Founder & CEO

SECTION - A

61. Let $\alpha x = \exp(x^{\beta}y^{\gamma})$ be the solution of the differential equation $2x^2y\,dy-\left(1-xy^2\right)dx=0$, $x>0$, $y(2)=\sqrt{\log_e 2}$. Then α + β + γ equals :

(1) 1 (2) –1 (3) 3 (4) 0

Sol. **1**

$$2x^2ydy-\left(1-xy^2\right)dx=0$$

$$\Rightarrow 2x^2y\frac{dy}{dx}-1+xy^2=0$$

$$\Rightarrow 2y\frac{dy}{dx}-\frac{1}{x^2}+\frac{y^2}{x}=0$$

$$\Rightarrow 2y\frac{dy}{dx}+\frac{y^2}{x}=\frac{1}{x^2}\text{(L.D.E)}$$

$$y^2=t\Rightarrow 2y\frac{dy}{dx}=\frac{dt}{dx}$$

$$\frac{dt}{dx}+\frac{t}{x}=\frac{1}{x^2}$$

$$f=e^{\int\frac{1}{x}dx}=x$$

$$\Rightarrow t\times x=\int x\cdot\frac{1}{x^2}dx$$

$$\Rightarrow y^2\cdot x=\ln x+c$$

$$\text{Also,}y(2)=\sqrt{\log_e 2}$$

$$\log_e^2 2=\log_e 2+c\Rightarrow c=\log_e 2$$

$$\Rightarrow y^2x=\ln x+\ln 2$$

$$\Rightarrow y^2x=\ln 2x$$

$$2x=\exp\left(x^1y^2\right)$$

Compare it with given solution we get,

$$\alpha=2,\beta=1,\gamma=2$$

$$\Rightarrow \alpha+\beta-\gamma=2+1-2=1$$

62. The sum $\sum_{n=1}^{\infty}\frac{2n^2+3n+4}{(2n)!}$ is equal to :

(1) $\frac{13e}{4}+\frac{5}{4e}$ (2) $\frac{11e}{2}+\frac{7}{2e}-4$ (3) $\frac{11e}{2}+\frac{7}{2e}$ (4) $\frac{13e}{4}+\frac{5}{4e}-4$

Sol. **4**

$$\sum_{n=1}^{\infty}\frac{2n^2+3n+4}{(2n)!}$$

$$\frac{1}{2}\sum_{n=1}^{\infty}\frac{2n(2n-1)+8n+8}{(2n)!}$$

$$\frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{(2n-2)!}+2\sum_{n=1}^{\infty}\frac{1}{(2n-1)!}+4\sum_{n=1}^{\infty}\frac{1}{(2n)!}$$

$$e=1+1+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\ldots.$$

$$e^{-1}=1-1+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}+\ldots.$$

$$\left(e+\frac{1}{e}\right)=2\left(1+\frac{1}{2!}+\frac{1}{4!}+\ldots\ldots\right)$$

$$e-\frac{1}{e}=\left(1+\frac{1}{3!}+\frac{1}{5!}+\ldots..\right)$$

Now

$$\frac{1}{2}\left(\sum_{n=1}^{\infty}\frac{1}{(2n-2)!}\right)+2\sum_{n=1}^{\infty}\frac{1}{(2n-1)!}+4\sum_{n=1}^{\infty}\frac{1}{(2n)!}$$

$$=\frac{1}{2}\left[\frac{e+\frac{1}{e}}{2}\right]+2\left[\frac{e-\frac{1}{e}}{2}\right]+4\left(\frac{e+\frac{1}{e}-2}{2}\right)$$

$$=\frac{\left(e+\frac{1}{e}\right)}{4}+e-\frac{1}{e}+2e+\frac{2}{e}-4$$

$$=\frac{13}{4}e+\frac{5}{4e}-4$$

63. Let $\vec{a}=5\hat{i}-\hat{j}-3k$ and $\vec{b}=\hat{i}+3\hat{j}+5k$ be two vectors. Then which one of the following statements is TRUE ?

(1) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the projection vector is same as of $\vec{b}$.

(2) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$.

(3) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-17}{\sqrt{35}}$ and the direction of the projection vector is same as of $\vec{b}$.

(4) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-17}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$.

Sol. **Bonus**

Projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}.\vec{b}}{|\vec{b}|}$

$$\Rightarrow \frac{(5\hat{i}-\hat{j}-3\hat{k})\cdot(\hat{i}+3\hat{j}+5\hat{k})}{\sqrt{1^2+3^2+5^2}} = \frac{5-3-15}{\sqrt{35}}$$

$$\Rightarrow \frac{-13}{\sqrt{35}}$$

Ans. (4)

64. Let $\vec{a} = 2\hat{i} - 7\hat{j} + 5k$, $\vec{b} = \hat{i} + k$ and $\vec{c} = \hat{i} + 2\hat{j} - 3k$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$ and $\vec{r} \cdot \vec{b} = 0$, then $|\vec{r}|$ is equal to :

(1) $\dfrac{11}{5}\sqrt{2}$ (2) $\dfrac{\sqrt{914}}{7}$ (3) $\dfrac{11}{7}\sqrt{2}$ (4) $\dfrac{11}{7}$

Sol. **3**

$\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$

$\Rightarrow (\vec{r} - \vec{c}) \times \vec{a} = 0 \Rightarrow \vec{r} - \vec{c} = \lambda\vec{a}$($\vec{r} - \vec{c}$&$\vec{a}$are parallel)

$\Rightarrow \vec{r} = \vec{c} + \lambda\vec{a}$

$\Rightarrow \vec{r} \cdot \vec{b} = \vec{c} \cdot \vec{b} + \lambda\vec{a} \cdot \vec{b}$

$0 = (1-3) + \lambda(2+5) \Rightarrow \lambda = \dfrac{2}{7}$

Hence, $\vec{r} = \vec{c} + \dfrac{2\vec{a}}{7}$

$\vec{r} \Rightarrow \dfrac{11}{7}\hat{i} - \dfrac{11}{7}\hat{k}$

$$|\vec{r}| = \sqrt{\left(\frac{11}{7}\right)^2 + \left(-\frac{11}{7}\right)^2} \Rightarrow r = \frac{11\sqrt{2}}{7}$$

65. Let $f : \mathbb{R} - \ 0,1 \ \to \mathbb{R}$ be a function such that $f(x) + f\left(\dfrac{1}{1-x}\right) = 1 + x$. Then $f(2)$ is equal to

(1) $\dfrac{9}{2}$ (2) $\dfrac{7}{4}$ (3) $\dfrac{9}{4}$ (4) $\dfrac{7}{3}$

Sol. **3**

For $x = 2, \Rightarrow f(2) + f(-1) = 3$ .... (1)

For $x = \dfrac{1}{2} \Rightarrow f\left(\dfrac{1}{2}\right) + f(-1) = \dfrac{3}{2}$ .... (2)

For $x = -1 \Rightarrow f(-1) + f\left(\dfrac{1}{2}\right) = 0$ .... (3)

$(2)-(3) \Rightarrow f(2)-f(-1)=\frac{3}{2}$ .... (4)

$(1)+(4) \Rightarrow 2f(2)=\frac{9}{2} \Rightarrow f(2)=\frac{9}{4}$

Ans. 3

66. Let P(S) denote the power set of S={1, 2, 3, ………, 10}. Define the relations R_1 and R_2 on P(S) as AR_1B if $\left(A\cap B^C\right)\cup\left(B\cap A^C\right)=\varnothing$ and AR_2B if $A\cup B^C=B\cup A^C, \forall A,B\in P(S)$. Then :

(1) only R_1 is an equivalence relation
(2) only R_2 is an equivalence relation
(3) both R_1 and R_2 are equivalence relations
(4) both R_1 and R_2 are not equivalence relations

Sol. **3**

$S=\{1,2,3,......10\}$

$P(S)=$ power set of S

$AR_1B \Rightarrow (A\cap\vec{B})\cup(\vec{A}\cap B)=\phi$

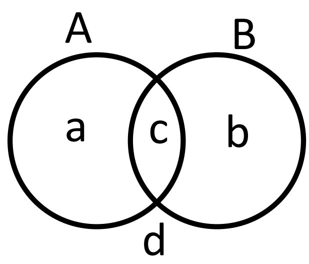
R1 is reflexive, symmetric

For transitive

$(A\cap\vec{B})\cup(\vec{A}\cap B)=\phi; \{a\}=\phi=\{b\}\ A=B$

$(B\cap\vec{C})\cup(\vec{B}\cap C)=\phi \therefore B=C$

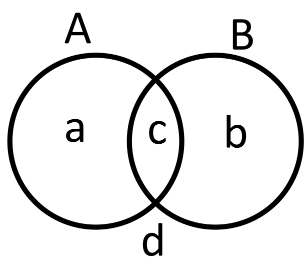
$\therefore A=C$ equivalence



$R_2 \equiv A\cup\vec{B}=\vec{A}\cup B$

$R_2 \rightarrow$ reflexive, symmetric

for transitive



$A\cup\vec{B}=\vec{A}\cup B \Rightarrow \{a,c,d\}=\{b,c,d\}$

$\{a\}=\{b\} \therefore A=B$

$B\cup\vec{C}=\vec{B}\cup C \Rightarrow B=C$

$\therefore A=C \quad \therefore A\cup\vec{C}=\vec{A}\cup C \therefore$ Equivalence

67. The area of the region given by $\{(x,y)\ :\ xy\ \le 8,\ 1 \le y \le x^2\}$ is :

(1) $16\log_e 2 + \frac{7}{3}$ (2) $16\log_e 2 - \frac{14}{3}$ (3) $8\log_e 2 - \frac{13}{3}$ (4) $8\log_e 2 + \frac{7}{6}$

Sol. **2**

$$A = \int_1^4 \left(\frac{8}{y} - \sqrt{y}\right) dy$$

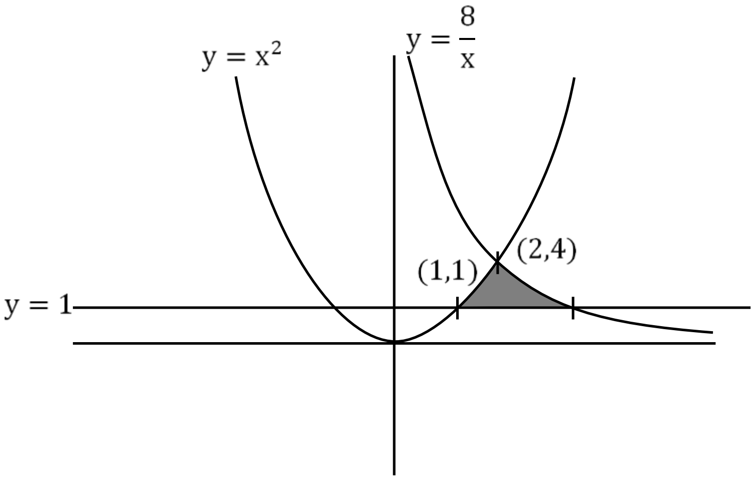
$$A = 8[\ln y]_1^4 - \frac{2}{3}\left(y^{\frac{3}{2}}\right)_1^4$$

$$\Rightarrow 8\ln 4 - \frac{2}{3}\left(4^{3/2} - 1\right)$$

$$\Rightarrow 8\ln 4 - \frac{2}{3}(8-1)$$

$$\Rightarrow 16\ln 2 - \frac{14}{3}$$

Ans. (2)



68. If $A = \frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$, then :

(1) $A^{30} + A^{25} + A = I$ (2) $A^{30} = A^{25}$ (3) $A^{30} + A^{25} - A = I$ (4) $A^{30} - A^{25} = 2I$

Sol. **3**

$$4 = \frac{1}{2}\begin{bmatrix} 1 & f_3 \\ -f^3 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{f_3}{2} \\ -\frac{f_3}{2} & \frac{1}{2} \end{bmatrix}$$

$$|4 - \lambda I| = 0$$

$$\begin{vmatrix} \frac{1}{2}-\lambda & \frac{f_3}{2} \\ -\frac{f_3}{2} & \frac{1}{2}-\lambda \end{vmatrix}=0 \qquad \Rightarrow \lambda^2+\frac{1}{4}-\lambda+\frac{3}{4}=0$$

$\lambda^2-\lambda+1=0 \qquad \Rightarrow A^2-A+1=0$

$\Rightarrow A^3-A^2+A=0$

and $A^4=(A-I)^2 \qquad \Rightarrow A^4=A^2+I-2A$

$\Rightarrow A^4=A-I+I-2A=-A$

$\boxed{A^4=-A}$

$\Rightarrow A^{30}=\left(A^4\right)^7 A^2=-A^4=-\left(A^4\right)A=-A^3=A-A^2$

$A^{25}=\left(A^4\right)^6 A=A^6A=A^7=A^4A^3=-AA^3=-A^4=A$

Put these values on all options, we, get,

$\Rightarrow A^{30}+A^{25}-A=I$

So, option (3) is correct.

69. Which of the following statements is a tautology ?

(1) $p\vee(p\wedge q)$

(2) $(p\wedge(p\to q))\to\sim q$

(3) $(p\wedge q)\to(\sim(p)\to q)$

(4) $p\to(p\wedge(p\to q))$

Sol. **3**

(i) $p\to(p\wedge(p\to q))$

$(\sim p)\vee(p\wedge(\sim p\vee q))$

$(\sim p)\vee(f\vee(p\wedge q))$

$\sim p\vee(p\wedge q)=(\sim p\vee p)\wedge(\sim p\vee q)$

$=\sim p\vee q$

(ii) $(p\wedge q)\to(\sim p\to q)$

$\sim(p\wedge q)\vee(p\vee q)=t$

$\{a,b,d\}\vee\{a,b,c\}=V$

Tautology

(iii) $(p\wedge(p\to q))\to\sim q$

$\sim(p\wedge(\sim p\vee q))\vee\sim q=\sim(p\wedge q)\vee\sim q=\sim p\vee\sim q$

Not tautology

(iv) $p\vee(p\wedge q)=p$

Not tautology.

So, option (2) is correct.

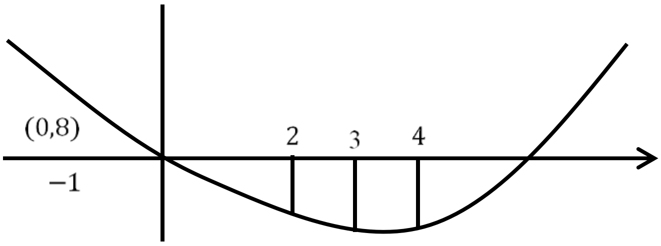
70. The sum of the absolute maximum and minimum values of the function $f(x)=\left|x^2-5x+6\right|-3x+2$ in the interval [–1,3] is equal to :

(1) 12 (2) 13 (3) 10 (4) 24

Sol. **4**

$$f(x)=\begin{cases} x^2-5x+6-3x+2.x\varepsilon(-\infty,2)\cup[2,\infty] \\ -\left(x^3-5x+6\right)-3x-2 \quad .x\varepsilon[2,3] \end{cases}$$

$$f(x)=\begin{cases} x^2-8x+8 & .x\varepsilon(-\infty,2)\cup[3,\infty] \\ -x^2+2x-4 & .x\varepsilon[2,3] \end{cases}$$



Absolute maximum $=|f(-1)|=\left|(-1)^2-8(-1)+8\right|=17$

Absolute minimum $=\left|f(3)\right|=7$

Sum=17+7=24

71. Let the plane P pass through the intersection of the planes 2x + 3y – z = 2 and x + 2y + 3z = 6 and be perpendicular to the plane 2x + y – z = 0. If d is the distance of P form the point (–7,1,1,) then d^2 is equal to :

(1) $\frac{250}{83}$ (2) $\frac{250}{82}$ (3) $\frac{15}{53}$ (4) $\frac{25}{83}$

Sol. **1**

Plane P, is passing through intersection of the two planes, so,

$$2x+3y-z-2+\lambda(x+2y+3z-6)=0$$

$$x(2+\lambda)+y(3+2\lambda)+z(3\lambda-1)-2-6\lambda=0$$

It is perpendicular with plane, $2x+y-2+1=0$

So, $(2+\lambda)2+(3+2\lambda)1+(3\lambda-1)(-1)=0$

$\boxed{\lambda=-8}$

So, plane $p_1-6x-13y-25z+46=0$

distance of plane p from the point $(-7,1,1)$

$$d=\frac{|+42-13-25+46|}{\sqrt{36+169+625}}=\frac{50}{\sqrt{30}}=$$

$$d^2=\frac{2500}{830}=\frac{250}{83}$$

Ans. (1)

72. The number of integral values of k, for which one root of the equation $2x^2-8x+k=0$ lies in the interval (1,2) and its other root lies in the interval (2,3), is :

(A) 3 (2) 0 (3) 2 (4) 1

Sol. **4**

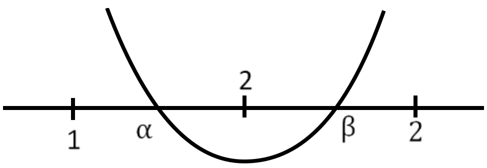
$\Rightarrow f(1)\cdot f(2)<0$

$(k-6)(k-8)<0$

$\text{Also}, f(2)\cdot f(3)<0$

$(k-8)(k-6)<0$

$k\varepsilon(6,8)\Rightarrow$ Integral value of k is 7. Ans: (4)



73. Let $P(x_0,y_0)$ be the point on the hyperbola $3x^2-4y^2=36$, which is nearest to the line 3x + 2y = 1. Then $\sqrt{2}(y_0-x_0)$ is equal to :

(A) –9 (2) –3 (3) 3 (4) 9

Sol. **1**

$3x^2-4y^2=36 \quad 3x+2y=1$

$$m=-\frac{3}{2}$$

$$m=+\frac{3\sec\theta}{\sqrt{12}\cdot\tan\theta}$$

$$\Rightarrow\frac{3}{\sqrt{12}}\times\frac{1}{\sin\theta}=\frac{-3}{2}$$

$$\sin\theta=-\frac{1}{\sqrt{3}}$$

$(\sqrt{12}\cdot\sec\theta,3\tan\theta)$

$$\left(\sqrt{12}\cdot\frac{\sqrt{3}}{\sqrt{2}},-3\times\frac{1}{\sqrt{2}}\right)\Rightarrow\left(\frac{6}{\sqrt{2}},\frac{-3}{\sqrt{2}}\right)=(x_0,y_0)$$

$$\Rightarrow\sqrt{2}(y_0-x_0)=\sqrt{2}\left(\frac{-3}{\sqrt{2}}-\frac{6}{\sqrt{2}}\right)=-9$$

74. Two dice are thrown independently. Let A be the event that the number appeared on the 1st die is less than the number appeared on the 2nd die, B be the event that the number appeared on the 1st die is even and that on the second die is odd, and C be the event that the number appeared on the 1st die is odd and that on the 2nd is even. Then :

(1) the number of favourable cases of the events A, B and C are 15,6 and 6 respectively

(2) the number of favourable cases of the event $(A \cup B) \cap C$ is 6

(3) B and C are independent

(4) A and B are mutually exclusive

Sol. **2**

$A=\{(1,2),(1,3),(1,4),(1,5),(1,6),(2,3),(2,4),(2,5),(2,6)(3,4),(3,5),(3,6)(4,5),(4,6),(5,6)\}$

$n(A)=15$

$B=\{(2,1),(2,3),(2,5),(4,1),(4,3),(4,5),(6,1),(6,3),(6,5)\}$

$n(B)=9$

$C=\{(1,2),(1,4),(1,6),(3,2),(3,4),(3,6),(5,2),(5,4),(5,6)\}$

$n(C)=9$

$((A\cup B)\cap C)=\{(1,2),(1,4),(1,6),(3,4),(3,6),(5,6)\}$

$\Rightarrow n((A\cup B)\cap C)=6$

75. If $y(x)=x^x, x>0$, then $y''(2)-2y'(2)$ is equal to :

(1) $4\log_e 2+2$ (2) $8\log_e 2-2$ (3) $4(\log_e 2)^2+2$ (4) $4(\log_e 2)^2-2$

Sol. **4**

$y=x^x$

$\ln y = x\ln x$

$\frac{1}{y}y'=1+\ln x$

$y'=y(1+\ln x)$

$y'=x^x(1+\ln x)$

$\text{At} x=2 \text{we have} y=4$

So $y'(2)=4(1+\ln 2)$ ……….. (2)

$\text{And} y'' = y'(1+\ln x)+\frac{y}{x}$

$y'(2) = y'(1+\ln 2)+2$

$y'(2) = y'(2) = y'(\ln 2)+2$

$y''(2) = 2y'(2) = (\ln 2-1)y'(2)+2$

$= 4(\ln 2-1)(\ln 2+2)+2$

$= 4(\ln 2)^2 - 2$

76. Let $S=\left\{x\in\mathbb{R} : 0<x<1 \text{ and } 2\tan^{-1}\left(\frac{1-x}{1+x}\right)=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right\}$.

If n(S) denotes the number of elements in S then :

(1) n(S) = 2 and only one element in S is less then $\frac{1}{2}$.

(2) n(S) = 1 and the element in S is more then $\frac{1}{2}$.

(3) n(S)=0

(4) n(S)=1 and the element in S is less than $\frac{1}{2}$.

Sol. **1**

Put $x=\tan\theta \quad \theta\in\left(0,\frac{\pi}{4}\right)$

$2\tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)=\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$2\tan^{-1}\left[\tan\left(\frac{\pi}{4}-\theta\right)\right]=\cos^{-1}[\cos(2\theta)]$

$\Rightarrow 2\left(\frac{\pi}{4}-\theta\right)=2\theta \Rightarrow \theta=\frac{\pi}{8}$

$\Rightarrow x=\tan\frac{\pi}{8}=\sqrt{2}-1\simeq 0.414$

77. The value of the integral $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x+\frac{\pi}{4}}{2-\cos 2x}dx$ is :

(1) $\frac{\pi^2}{12\sqrt{3}}$ (2) $\frac{\pi^2}{6\sqrt{3}}$ (3) $\frac{\pi^2}{6}$ (4) $\frac{\pi^2}{3\sqrt{3}}$

Sol. **2**

$$I=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{xdx}{2-\cos 2x}+\frac{\pi}{4}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{xdx}{2-\cos 2x}$$

$$=0+\frac{\pi}{4}\cdot 2\int_{0}^{\frac{\pi}{4}}\frac{xdx}{2-\cos 2x}$$

$$=\pi/2\int_{0}^{\frac{\pi}{4}}\frac{\sec^2 xdx}{1+3\tan^2 x}$$

Now,

$\tan x=t$

$$=\frac{\pi}{2}\int_{0}^{1}\frac{dt}{1+3t^2}$$

$$=\frac{\pi}{2}\left[\frac{\tan^{-1}(\sqrt{3}t)}{\sqrt{3}}\right]_0^1$$

$$=\frac{\pi}{2\sqrt{3}}\left(\frac{\pi}{3}\right)=\frac{\pi^2}{6\sqrt{3}}$$

78. For the system of linear equations $\alpha x+y+z=1$, $x+\alpha y+z=1$, $x+y+\alpha z=\beta$, which one of the following statements is NOT correct ?

(1) It has infinitely many solutions if $\alpha=2$ and $\beta=-1$

(2) It has no solution if $\alpha=-2$ and $\beta=1$

(3) $x+y+z=\frac{3}{4}$ if $\alpha=2$ and $\beta=1$

(4) It has infinitely many solutions if $\alpha=1$ and $\beta=1$

Sol. **1**

$$\begin{vmatrix}\alpha & 1 & 1\\ 1 & \alpha & 1\\ 1 & 1 & \alpha\end{vmatrix}=0$$

$$\alpha\left(\alpha^2-1\right)-1(\alpha-1)+1(1-\alpha)=0$$

$$\alpha^3-3\alpha+2=0$$

$$\alpha^2(\alpha-1)+\alpha(\alpha-1)-2(\alpha-1)=0$$

$$(\alpha-1)\left(\alpha^2+\alpha-2\right)=0$$

$$\alpha=1,\alpha=-2,1$$

For $\alpha=1, \beta=1$

$$\left.\begin{matrix}x+y+z=1\\ x+y+z=b\end{matrix}\right\}\text{infinite solution}$$

For $\alpha=2, \beta=1$

$\Delta=4$

$$\Delta_1=\begin{vmatrix}1&1&1\\1&2&1\\1&1&2\end{vmatrix}=3-1-1 \qquad \Rightarrow x=\frac{1}{4}$$

$$\Delta_2=\begin{vmatrix}2&1&1\\1&1&1\\1&1&2\end{vmatrix}=2-1=1 \qquad \Rightarrow y=\frac{1}{4}$$

$$\Delta_3=\begin{vmatrix}2&1&1\\1&2&1\\1&1&1\end{vmatrix}=2-1=1 \qquad \Rightarrow z=\frac{1}{4}$$

For $\alpha=2 \Rightarrow$ unique solution

79. Let $9=x_1<x_2<.....<x_7......,x_7$ be in an A.P. with common difference d. If the standard deviation of $x_1 \cdot x_2.....,x_7$ is 4 and mean is $\overline{x}$, then $\overline{x}+x_6$ is equal to :

(1) $2\left(9+\frac{8}{\sqrt{7}}\right)$ (2) $18\left(1+\frac{1}{\sqrt{3}}\right)$ (3) 25 (4) 34

Sol. **4**

Mean $\Rightarrow \bar{x}=\frac{\sum_{i=1}^{7}x_i}{7}=\frac{\frac{7}{2}[2a+6d]}{7}=a+3d=x_4$

Variance $=\frac{\sum_{i=1}^{7}(x_i-\bar{x})^2}{7}=(4)^2 \Rightarrow \frac{\sum_{i=1}^{7}(x_i-x_4)^2}{7}=16$

$\Rightarrow \frac{(3d)^2+(2d)^2+d^2+0+d^2+(2d)^2+(3d)^2}{7}=16$

$=4d^2=16 \Rightarrow d=2$

$\Rightarrow \bar{x}=9+3(2)=15$

$\& x_0=a+5d=9+5(2)=19 \Rightarrow \bar{x}+x_0=34$

80. Let a, b be two real numbers such that ab < 0. IF the complex number $\frac{1+ai}{b+i}$ is of unit modulus and a + ib lies on the circle |z–1| = |2z|, then a possible value of $\frac{1+[a]}{4b}$, where [t] is greatest integer function, is :

(1) $-\frac{1}{2}$ (2) –1 (3) 1 (4) $\frac{1}{2}$

Sol. **Bonus**

$ab<0\left|\frac{1+ai}{b+i}\right|=1$

$|1+ia|=|b+i|$

$a^2+1=b^2+1\Rightarrow a=\pm b\Rightarrow b=-a \quad \text{as } ab<0$

(a,b) lies on $|z-1|=|2z|$

$|a+ib-1|=2|a+ib|$

$(a-1)^2+b^2=4\left(a^2+b^2\right)$

$(a-1)^2=a^2=4\left(2a^2\right)$

$1-2a=6a^2\Rightarrow 6a^2+2a-1=0$

$a=\frac{-2\pm\sqrt{28}}{12}=\frac{-1\pm\sqrt{7}}{6}$

$a=\frac{\sqrt{7}-1}{6}\ \&\ b=\frac{1-\sqrt{7}}{6}$

$[a]=0$

$\therefore \frac{1+[a]}{4b}=\frac{6}{4(1-\sqrt{7})}=-\left(\frac{1+\sqrt{7}}{4}\right)$

or $[a]=0$

Similarly it is not matching with $a=\frac{-1-\sqrt{7}}{6}$

No answer is matching.

SECTION - B

81. Let αx + βy + yz = 1 be the equation of a plane through the point (3,–2,5)and perpendicular to the line joning the points (1, 2, 3) and (–2, 3, 5). Then the value of αβy is equal to

Sol. **6**

Plane is perp. To the line joining the

(1,2,3) & (-2, 3, 5)

So, line will be along normal of plane.

$\vec{n}=(3,-1,-2)$ or $(-3,1,2)$

Compare it with eq. of plane, $\alpha x+\beta y+r_2=1$

$\alpha=3,\beta=-1,r=-2$ or $\alpha=-3,\beta=1,r=2$

So, $\alpha\beta r=6$ (in both cases)

Ans. 6

82. If the term without x in the expansion of $\left(x^{\frac{2}{3}}+\frac{a}{x^3}\right)^{22}$ is 7315, then |α| is equal to

Sol. **1**

$\Rightarrow$General Term, $T_{r+1}={}^{22}C_r\left(x^{\frac{2}{3}}\right)^{22-r}\cdot\left(\frac{\alpha}{x^3}\right)^r$

$T_{r+1}={}^{22}C_r\cdot x^{\frac{2(22-r)}{3}-3r}\alpha^r$

For term independent of x,

$\frac{2(22-r)}{3}-3r=0$

$44-2r=9r\Rightarrow 11r=44$

$T_{4+1}={}^{22}C_4\cdot\alpha^4=7315$

$7315.\alpha^4=7315\Rightarrow\alpha^4=1\Rightarrow|\alpha|=1$

Ans. 1

83. If the x – intercept of a focal chord of the parabola $y^2=8x+4y+4$ is 3 , then the length of this chord is equal to

Sol. **16**

$y^2=8x+4y+4$

$(y-2)^2=8(x+1)$

$y^2=4ax$

$a=2, X=x+1, Y=y-2$

focus $(1,2)$

$y-2=m(x-1)$

Put (3, 0) in the above line

$m=-1$

Length of focal chord = 16

84. Let the sixth term in the binomial expansion of $\left(\sqrt{\log_2\left(10-3^3\right)}+\sqrt[5]{2^{x\log_2 3}}\right)^m$, in the increasing powers of $2^{(x-2)\log_2 3}$, be 21. If the binomial coefficients of the second, third and fourth terms in the expansion are respectively the first, third and fifth terms of A.P., then the sum of the squares of all possible values of x is

Sol. **4**

$\Rightarrow$ Sixth Term, $T_{5+1} = {}^mC_5\left(10-3^x\right)^{\frac{m-5}{2}} 3^{x-2} = 21$

So, $2\,{}^mC_2 = {}^mC_1 + {}^mC_3$

$2\frac{m(m-1)}{2} = m + \frac{m(m-1)(m-2)}{6}$

$\Rightarrow m = 2,7$ (But $m = 2$ is inadmissible)

$\Rightarrow m = 7$

Now, $T_{5+1} = {}^7C_5\left(10-3^x\right)^{\frac{7-5}{2}} 3^{x-2} = 21$

$\Rightarrow \frac{10.3^x - \left(3^x\right)^2}{3^2} = 1$

$\left(3^x\right)^2 - 10.3^x + 9 = 0$

$3^x = 9,1$

$\Rightarrow x = 0,2$

Sum of squeals of values of $x = 0^2 + 2^2 = 4$

Ans. (4)

85. The point of intersection C of the plane 8x + y + 2z = 0 and the line joining the point A(–3,–61) and B(2,–4,–3) divides the line segment AB internally in the ratio k:. If a, b, c (|a|, |b|, |c|) are coprime are the direction ratios of the perpendicular form the point C on the line $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$, then |a + b + c|is equal to

Sol. **10**

Plane : $8x + y + 2z = 0$

Given line $AB: \frac{x-2}{5} = \frac{y-4}{10} = \frac{z+3}{-4} = \lambda$

Any point on line $(5\lambda+2, 10\lambda+4, -4\lambda-3)$

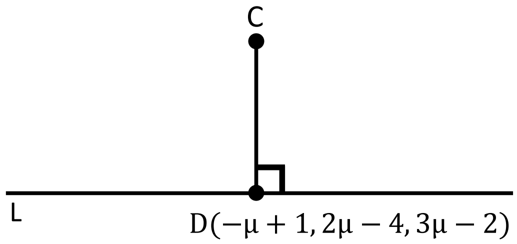
Point of intersection of line and plane

$8(5\lambda+2) + 10\lambda + 4 - 8\lambda - 6 = 0$

$\lambda = -\frac{1}{3}$

$C\left(\frac{1}{3}, \frac{2}{3}, -\frac{5}{3}\right)$

$L: \frac{x-1}{-1} = \frac{y+4}{2} = \frac{z+2}{3} = \mu$



$$\overline{CD}=\left(-\mu+\frac{2}{3}\right)\hat{i}+\left(2\mu-\frac{14}{3}\right)\hat{j}+\left(3\mu-\frac{1}{3}\right)\hat{k}$$

$$\left(-\mu+\frac{2}{3}\right)(-1)+\left(2\mu-\frac{14}{3}\right)2+\left(3\mu-\frac{1}{3}\right)3=0$$

$$\mu=\frac{11}{14}$$

$$\overrightarrow{CD}=\frac{-5}{42},\frac{-130}{42},\frac{85}{42}$$

Direction ratio $\rightarrow(-1,-26,17)$

$|a+b+c|=10$

86. The line x = 8 is the directrix of the ellipse ellipse $E:\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with the corresponding focus (2,0). If the tangent to E at the point P in the first quadrant passes through the point $\left(0,4\sqrt{3}\right)$ and intersects that x-axis at Q then $(3PQ)^2$ equl to

Sol. **39**

$$\frac{a}{e}=8 \quad .....(1)$$

$$ae=2 \quad .....(2)$$

$$8e=\frac{2}{e}$$

$$e^2=\frac{1}{4}\Rightarrow e=\frac{1}{2}$$

$$a=4$$

$$b^2=a^2\left(1-e^2\right)$$

$$=16\left(\frac{3}{4}\right)=12$$

$$\frac{x\cos\theta}{4}+\frac{y\sin\theta}{2\sqrt{3}}=1$$

$$\sin\theta=\frac{1}{2}$$

$$\theta=30^\circ$$

$P(2\sqrt{3},\sqrt{3})$

$Q\left(\frac{8}{\sqrt{3}},0\right)$

$(3PQ)^2 = 39$

87. The total number of six digit numbers, formed using the digits 4, 5, 9 only and divisible by 6 , is

Sol. **81**

We have,

For this, 4 will be fixed as unit place digit

Total number

Case I: $4's \to 6\text{times}$ 1

Case II: $4's \to 4\text{times}$

$5's \to 1\text{times}$ $\frac{5!}{3!} = 20$

$9's \to 1\text{times}$

Case III: $4's \to 3\text{times}$

$5's \to 3\text{times}$ $\frac{5!}{2!3!} = 10$

Case IV: $4's \to 3\text{times}$

$9's \to 3\text{times}$ $\frac{5!}{2!3!} = 10$

Case V: $4's \to 2\text{times}$

$5's \to 2\text{times}$ $\frac{5!}{2!2!} = 30$

$9's \to 2\text{times}$

Case VI: $4's \to 1\text{times}$

$5's \to 1\text{times}$ $\frac{5!}{4!} = 5$

$9's \to 4\text{times}$

Case VII: $4's \to 1\text{times}$

$5's \to 4\text{times}$ $\frac{5!}{4!} = 5$

$9's \to 1\text{times}$

Total numbers =81

88. Number of integral solutions to the equation x + y + z = 21, where $x \geq 1$, $y \geq 3$, $z \geq 4$, is equal to

Sol. **105**

$$^{15}C_2 = \frac{15\times14}{2} = 105$$

89. The sum of the common terms of the following three arithmetic progressions.

3, 7, 11, 15,…..,399

2, 5, 8, 11,……,359 and

2, 7, 12, 17,……, 197

is equal to

Sol. **321**

3,7,11,15,………..,399 $\quad d_1 = 4$

2,5,8,11,…………..,359 $\quad d_2 = 3$

2,7,12,17,……..,197 $\quad d_3 = 5$

$$\text{LCM}(d_1, d_2, d_3) = 60$$

Common terms are 47, 107, 167

Sum = 321

90. If $\int \frac{5\cos^x\left(1+\cos x\cos 3x+\cos^2 x+\cos^3 x+\cos 3x\right)dx}{1+5^{\cos x}} = \frac{k\pi}{16}$, then k is equal to

Sol. **13**

$$I = \int_0^{\pi} \frac{5^{\cos x}\left(1+\cos x\cos 3x+\cos^2 x+\cos^3 x\cos 3x\right)}{1+5^{\cos x}}dx$$

$$I = \int_0^{\pi} \frac{5^{-\cos x}\left(1+\cos x\cos 3x+\cos^2 x+\cos^3 x\cos 3x\right)}{1+5^{-\cos x}}dx$$

$$2I = \int_0^{\pi}\left(1+\cos x\cos 3x+\cos^2 x+\cos^3 x\cos 3x\right)dx$$

$$2I = 2\int_0^{\frac{\pi}{2}}\left(1+\cos x\cos 3x+\cos^2 x+\cos^3 x\cos 3x\right)dx$$

$$I = \int_0^{\frac{\pi}{2}}\left(1+\sin x(-\sin 3x)+\sin^2 x-\sin^3 x\sin 3x\right)dx$$

$$2I = \int_0^{\frac{\pi}{2}}\left(3+\cos 4x+\cos^3 x\cos 3x-\sin^3 x\sin 3x\right)dx$$

$$2I=\int_0^{\frac{\pi}{2}}3+\cos 4x+\left(\frac{\cos 3x+3\cos x}{4}\right)\cos 3x-\sin 3x\left(\frac{3\sin x-\sin 3x}{4}\right)dx$$

$$2I=\int_0^{\frac{\pi}{2}}\left(3+\cos 4x+\frac{1}{4}+\frac{3}{4}\cos 4x\right)dx$$

$$2I=\frac{13}{4}\times\frac{\pi}{2}+\frac{7}{4}\left(\frac{\sin 4x}{4}\right)_0^{\frac{\pi}{2}}\Rightarrow I=\frac{13\pi}{16}$$

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