JEE MAIN 2023
Paper with Solution

MATHS | 01ST FEB 2023 _ Shift-1

Umeed Rank Ki Ho Ya Selection Ki, JEET NISCHIT HAI!

Most Promising RANKS
Produced by MOTION Faculties

NEET / AIIMS

AIR-1 to 10
25 Times

AIR-11 to 50
83 Times

AIR-51 to 100
81 Times

JEE MAIN+ADVANCED

AIR-1 to 10
8 Times

AIR-11 to 50
32 Times

AIR-51 to 100
36 Times

NITIN VIJAY (NV Sir)
Founder & CEO

Nation's Best SELECTION
Percentage (%) Ratio

Student Qualified in NEET

(2022)
4837/5356 = **90.31%**
(2021)
3276/3411 = **93.12%**

Student Qualified in JEE ADVANCED

(2022)
1756/4818 = **36.45%**
(2021)
1256/2994 = **41.95%**

Student Qualified in JEE MAIN

(2022)
4818/6653 = **72.41%**
(2021)
2994/4087 = **73.25%**

Section A

61. If $y = y(x)$ is the solution curve of the differential equation
$\frac{dy}{dx} + y\tan x = x\sec x, 0 \le x \le \frac{\pi}{3}, y(0) = 1$, then $y\left(\frac{\pi}{6}\right)$ is equal to

(1) $\frac{\pi}{12} - \frac{\sqrt{3}}{2}\log_e\left(\frac{2\sqrt{3}}{e}\right)$ (2) $\frac{\pi}{12} - \frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$

(3) $\frac{\pi}{12} + \frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$ (4) $\frac{\pi}{12} + \frac{\sqrt{3}}{2}\log_e\left(\frac{2\sqrt{3}}{e}\right)$

Sol. **2**

Given D.E. is linear D.E.

I.F. = $e^{\int \tan x dx}$

= $e^{\ell n \sec x}$ = secx

Solution is –

y secx = $\int x\sec^2 x\, dx$

= x tanx – $\int \tan x\, dx$

$\Rightarrow$ y secx = xtanx – ℓn secx + c

Put y(0) = 1

1 = 0 – 0 + c $\Rightarrow$ c = 1

$$Y(x) = \frac{x\tan x}{\sec x} - \frac{\ell n \sec x}{\sec x} + \frac{1}{\sec x}$$

$$y\left(\frac{\pi}{6}\right) = \frac{\left(\frac{\pi}{6}\right)\left(\frac{1}{\sqrt{3}}\right)}{\left(\frac{2}{\sqrt{3}}\right)} - \frac{\ell n\left(\frac{2}{\sqrt{3}}\right)}{\left(\frac{2}{\sqrt{3}}\right)} + \frac{\sqrt{3}}{2}$$

$$= \frac{\pi}{12} - \frac{\sqrt{3}}{2}\ell n\left(\frac{2}{\sqrt{3}}\right) + \frac{\sqrt{3}}{2}\ell ne$$

$$= \frac{\pi}{12} - \frac{\sqrt{3}}{2}\ell n\left(\frac{2}{e\sqrt{3}}\right)$$

62. Let R be a relation on $\mathbb{R}$, given by

$$R = \{(a, b): 3a - 3b + \sqrt{7} \text{ is an irrational number }\}.$$

Then R is

(1) an equivalence relation

(2) reflexive and symmetric but not transitive

(3) reflexive but neither symmetric nor transitive

(4) reflexive and transitive but not symmetric

Sol. **3**

(a, a) $\in$ R $\Rightarrow$ 3a – 3a + $\sqrt{7}$

= $\sqrt{7}$ (irrational)

$\Rightarrow$ R is reflexive

Let a = $\frac{2\sqrt{7}}{3}$ and b = $\frac{\sqrt{7}}{3}$

(a, b) $\in$ R $\Rightarrow$ 2$\sqrt{7}$ – $\sqrt{7}$ + $\sqrt{7}$

$= 2\sqrt{7}$ (irration)

(b, a) $\in$ R $\Rightarrow \sqrt{7} - 2\sqrt{7} + \sqrt{7}$

$= 0$ (rational)

$\Rightarrow$ R is no symmetric

Let $a = \frac{2\sqrt{7}}{3}$, $b = \frac{\sqrt{7}}{3}$, $C = \frac{3\sqrt{7}}{3}$

(a ; b) $\in$ R $\Rightarrow 2\sqrt{7}$ (irrational)

(b ; c) $\in$ R $\Rightarrow \sqrt{7}$ (irrational)

(a, c) $\in$ R $\Rightarrow 2\sqrt{7} - 3\sqrt{7} + \sqrt{7}$

$= 0$ (rational)

R is not transitive

$\Rightarrow$ R is reflexive but neither symmetric nor transitive

63. For a triangle ABC, the value of $\cos 2A + \cos 2B + \cos 2C$ is least. If its inradius is 3 and incentre is M, then which of the following is NOT correct?

(1) perimeter of $\triangle ABC$ is $18\sqrt{3}$

(2) $\sin 2A + \sin 2B + \sin 2C = \sin A + \sin B + \sin C$

(3) $\overrightarrow{MA} \cdot \overrightarrow{MB} = -18$

(4) area of $\triangle ABC$ is $\frac{27\sqrt{3}}{2}$

Sol. **4**

Let P $= \cos 2A + \cos 2B + \cos 2C$

$= 2\cos(A + B)\cos(A - B) + 2\cos^2 C - 1$

$= 2\cos(\pi - C)\cos(A - B) + 2\cos^2 C - 1$

$= -2\cos C\,[\cos(A - B) + \cos(A + B)] - 1$

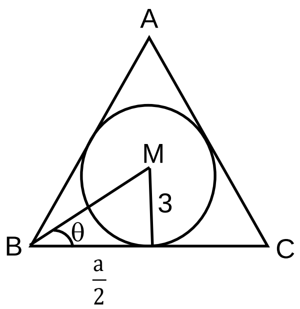
$= -1 - 4\cos A \cos B \cos C$

for P to be minimum

cosA cosB cos C must be maximum

$\Rightarrow \Delta ABC$ is equilateral triangle.

Let side length of triangle is a



$$\tan\theta = \frac{3}{a/2}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{6}{a} \quad \Rightarrow a = 6\sqrt{3}$$

area of triangle $= \frac{\sqrt{3}}{4}a^2$

$$= \frac{\sqrt{3}}{4}(108) = 27\sqrt{3}$$

64. Let S be the set of all solutions of the equation $\cos^{-1}(2x) - 2\cos^{-1}(\sqrt{1-x^2}) = \pi$, $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$. Then $\sum_{x \in S} 2\sin^{-1}(x^2 - 1)$ is equal to

(1) $\pi - 2\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (2) $\pi - \sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

(3) $\frac{-2\pi}{3}$ (4) 0

Sol. **Bonus**

$\cos^{-1}(2x) = \pi + 2\cos^{-1}\sqrt{1-x^2}$

Since $\cos^{-1}(2x) \in [0,\pi]$

R.H.S. $\geq \pi$

$\pi + 2\cos^{-1}\sqrt{1-x^2} = \pi$

$\Rightarrow \cos^{-1}\sqrt{1-x^2} = 0$

$\Rightarrow \sqrt{1-x^2} = 1$

$\Rightarrow x = 0$

but at x = 0

$\cos^{-1}(2x) = \cos^{-1}(0) = \frac{\pi}{2}$

no solution possible for given equation.

$x \in \phi$

65. Let S denote the set of all real values of λ such that the system of equations

$\lambda x + y + z = 1$

$x + \lambda y + z = 1$

$x + y + \lambda z = 1$

is inconsistent, then $\sum_{\lambda \in S}\left(|\lambda|^2 + |\lambda|\right)$ is equal to

(1) 4 (2) 12 (3) 6 (4) 2

Sol. **3**

Given system of equation is inconsistent

$\Rightarrow \Delta = 0$

$$\begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$\Rightarrow \lambda^3 - 3\lambda + 2 = 0$

$\Rightarrow (\lambda-1)^2(\lambda+2) = 0$

$\Rightarrow \lambda = 1, -2$

But for $\lambda = 1$ all planes are same

Then $\lambda = -2$

$\sum_{\lambda \in s}\left(|\lambda|^2 + |\lambda|\right) = 4 + 2 = 6$

66. In a binomial distribution B(n,p), the sum and the product of the mean and the variance are 5 and 6 respectively, then 6(n+p–q) is equal to

(1) 52 (2) 50 (3) 51 (4) 53

Sol. **1**

Given

np + npq = 5

$\Rightarrow$ np(1 + q) = 5 ....(i)

and (np) (npq) = 6

$\Rightarrow n^2p^2q = 6$ ....(ii)

$(i)^2 \div (ii)$

$$\frac{(1+9)^2}{9}=\frac{25}{6}$$

$\Rightarrow 6q^2 - 13q + 6 = 0$

$\Rightarrow q = \frac{2}{3}, \frac{3}{2}$ (reject)

$P = 1-\frac{2}{3}=\frac{1}{3}$

$$\frac{n}{3}\left(1+\frac{2}{3}\right)=5$$

$\Rightarrow$ n = 9

6(n + p – q) = 52

67. The combined equation of the two lines $ax + by + c = 0$ and $a'x + b'y + c' = 0$ can be written as $(ax + by + c)(a'x + b'y + c') = 0$.

The equation of the angle bisectors of the lines represented by the equation

$2x^2 + xy - 3y^2 = 0$ is

(1) $x^2 - y^2 - 10xy = 0$ (2) $x^2 - y^2 + 10xy = 0$

(3) $3x^2 + 5xy + 2y^2 = 0$ (4) $3x^2 + xy - 2y^2 = 0$

Sol. **1**

For pair of st. liens in form

$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$

equation of angle bisector is

$$\frac{x^2-y^2}{a-b}=\frac{xy}{h}$$

for $2x^2 + xy - 3y^2 = 0$

$a = 2, b = -3, h = \frac{1}{2}$

equation of angle bisector is

$$\frac{x^2-y^2}{5}=\frac{xy}{1/2}$$

$\Rightarrow x^2 - y^2 - 10xy = 0$

68. The area enclosed by the closed curve C given by the differential equation $\frac{dy}{dx}+\frac{x+a}{y-2}=0, y(1)=0$ is 4π.
Let P and Q be the points of intersection of the curve C and the y-axis. If normals at P and Q on the curve C intersect x-axis at points R and S respectively, then the length of the line segment RS is

(1) 2 (2) $\frac{4\sqrt{3}}{3}$ (3) $2\sqrt{3}$ (4) $\frac{2\sqrt{3}}{3}$

Sol. **2**

$$\frac{dy}{dx}+\frac{x+\alpha}{y-2}=0, y(1)=0$$

$$\frac{dy}{dx}=\frac{-(x+\alpha)}{y-2}$$

$$\int(y-2)dy=-\int(x+\alpha)dx$$

$$\frac{y^2}{2}-2y=-\left[\frac{x^2}{2}+\alpha x\right]+\lambda$$

$y(1) = 0$

$x = 1 \Rightarrow y = 0$

$$0-0=-\left[\frac{1}{2}+\alpha\right]+\lambda$$

$$\frac{y^2}{2}-2y=-\left[\frac{x^2}{2}+\alpha x\right]+\frac{1}{2}+\alpha$$

$$\frac{x^2+y^2}{2}=2y-\alpha x+\frac{1}{2}+\alpha$$

$x^2 + y^2 + 2\alpha x - 4y - 1 - 2\alpha = 0$

Area = 4π

$\pi r^2 = 4\pi$

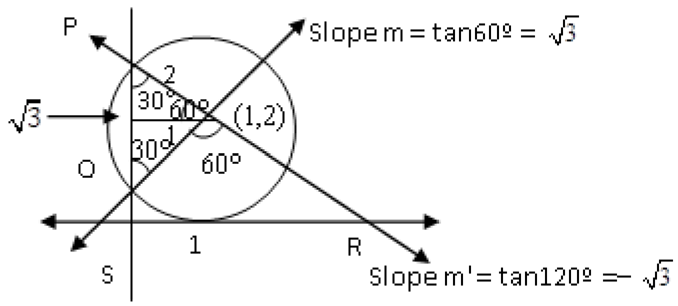
$r^2 = 4$

$\alpha^2 + 4 + 1 + 2\alpha = 4$

$\alpha^2 + 2\alpha + 1 = 0$

$(\alpha + 1)^2 = 0 \Rightarrow [\alpha = -1]$

$x^2 + y^2 - 2x - 4y + 1 = 0$



$y - 2 = \sqrt{3}(x-1)$ $y - 2 = -\sqrt{3}(x-1)$

$y = 0$ $y = 0$

$\frac{-2}{\sqrt{3}} = x - 1$

$1 + \frac{2}{\sqrt{3}} = x$

$1 - \frac{2}{\sqrt{3}} = x$

$R\left(1 + \frac{2}{\sqrt{3}}, 0\right)$

$S\left(1 - \frac{2}{\sqrt{3}}, 0\right)$

RS = $\left(1 + \frac{2}{\sqrt{3}}\right) - \left(1 - \frac{2}{\sqrt{3}}\right) = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$

69. The value of

$\frac{1}{1!50!} + \frac{1}{3!48!} + \frac{1}{5!46!} + \cdots. + \frac{1}{49!2!} + \frac{1}{51!1!}$ is :

(1) $\frac{2^{50}}{51!}$ (2) $\frac{2^{51}}{50!}$ (3) $\frac{2^{50}}{50!}$ (4) $\frac{2^{51}}{51!}$

Sol. **1**

S = $\frac{1}{1!50!} + \frac{1}{3!48!} + \frac{1}{5!46!} + \ldots\ldots + \frac{1}{49!2!} + \frac{1}{51!1!}$

$= \frac{1}{51!1!}\left(\frac{51!}{1!50!} + \frac{51!}{3!48!} + \frac{51!}{5!46!} + \ldots\ldots + \frac{51!}{49!2!} + \frac{51!}{51!0!}\right)$

$= \frac{1}{51!1!}\left({}^{51}C_{50} + {}^{51}C_{48} + {}^{51}C_{46} + \ldots\ldots. + {}^{51}C_2 + {}^{51}C_0\right)$

$\because {}^nC_0 + {}^nC_2 + {}^nC_4 + \ldots\ldots. = 2^{n-1}$

S = $\frac{2^{50}}{51!}$

70. The mean and variance of 5 observations are 5 and 8 respectively. If 3 observations are 1, 3, 5 then the sum of cubes of the remaining two observations is

(1) 1216 (2) 1072 (3) 1456 (4) 1792

Sol. **2**

Let remaining two observations are a and b

$5 = \frac{1+3+5+a+b}{5}$

$\Rightarrow$ a + b = 16 …(i)

$8 = \frac{1^2+3^2+5^2+a^2+b^2}{5} - 25$

$\Rightarrow$ $a^2 + b^2 = 130$

$a^2 + b^2 = 130$ …(ii)

$(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow$ 256 = 130 + 2ab

ab = 63

$a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

$= (16)^3 - 3(63)(16)$

$= 4096 - 3024$

$\Rightarrow a^3 + b^3 = 1072$

71. The sum to 10 terms of the series

$\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+\cdots$. is

(1) $\frac{55}{111}$ (2) $\frac{56}{111}$ (3) $\frac{58}{111}$ (4) $\frac{59}{111}$

Sol. **1**

$$T_n=\frac{n}{1+n^2+n^4}$$

$$=\frac{n}{(n^2-n+1)(n^2+n+1)}$$

$$=\frac{1}{2}\left[\frac{(n^2+n+1)-(n^2-n+1)}{(n^2-n+1)(n^2+n+1)}\right]$$

$$\Rightarrow T_n=\frac{1}{2}\left[\frac{1}{(n^2-n+1)}-\frac{1}{(n^2+n+1)}\right]$$

$$S_n=\sum_{n=1}^{10}T_n$$

$$=\frac{1}{2}\sum\left(\frac{1}{n^2-n+1}-\frac{1}{n^2+n+1}\right)$$

$$=\frac{1}{2}\left[\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{13}\right)\right.$$

$$\left.\ldots\ldots\ldots\ldots\ldots\ldots+\left(\frac{1}{91}-\frac{1}{111}\right)\right]$$

$$=\frac{1}{2}\left[1-\frac{1}{111}\right]=\frac{55}{111}$$

72. The shortest distance between the lines

$\frac{x-5}{1}=\frac{y-2}{2}=\frac{z-4}{-3}$ and $\frac{x+3}{1}=\frac{y+5}{4}=\frac{z-1}{-5}$ is

(1) $5\sqrt{3}$ (2) $7\sqrt{3}$ (3) $6\sqrt{3}$ (4) $4\sqrt{3}$

Sol. **3**

$$L_1:\frac{x-5}{1}=\frac{y-2}{2}=\frac{z-4}{-3}$$

$$\vec{a}_1=5\hat{i}+2\hat{j}+4\hat{k}$$

$$\vec{r}_1=\hat{i}+2\hat{j}-3\hat{k}$$

$$L_2:\frac{x+3}{1}=\frac{y+5}{4}=\frac{z-1}{-5}$$

$$\vec{a}_2=-3\hat{i}-5\hat{j}+\hat{k}$$

$$\vec{r}_2=\hat{i}+4\hat{j}-5\hat{k}$$

$$\vec{r}_1\times\vec{r}_2=\begin{vmatrix}\hat{i} & j & k\\ 1 & 2 & -3\\ 1 & 4 & -5\end{vmatrix}$$

$$=2\hat{i}+2\hat{j}+2\hat{k}$$

Shortest distance (d) = $\left|\dfrac{(\vec{r_1}\times\vec{r_2}).(\vec{a_1}-\vec{a_2})}{|\vec{r_1}\times\vec{r_2}|}\right|$

$= \dfrac{36}{2\sqrt{3}} = 6\sqrt{3}$

73. $\lim\limits_{n\to\infty}\left[\frac{1}{1+n}+\frac{1}{2+n}+\frac{1}{3+n}+\cdots+\frac{1}{2n}\right]$ is equal to

(1) $\log_e 2$ (2) $\log_e\left(\frac{3}{2}\right)$ (3) $\log_e\left(\frac{2}{3}\right)$ (4) 0

Sol. **1**

$\lim\limits_{n\to\infty}\left[\dfrac{1}{1+n}+\dfrac{1}{2+n}+\dfrac{1}{3+n}+......+\dfrac{1}{2n}\right]$

$= \lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\dfrac{1}{r+n}$

$= \lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\dfrac{1}{n}\left(\dfrac{1}{\frac{r}{n}+1}\right)$

$= \int_0^1\dfrac{dx}{x+1}$

$= \log_e(1+x)\Big|_0^1$

$= \log_e^2$

74. Let the image of the point $P(2,-1,3)$ in the plane $x+2y-z=0$ be Q. Then the distance of the plane $3x+2y+z+29=0$ from the point Q is

(1) $\frac{24\sqrt{2}}{7}$ (2) $2\sqrt{14}$ (3) $3\sqrt{14}$ (4) $\frac{22\sqrt{2}}{7}$

Sol. **3**

let Q(α, β, γ) is image of P(2, –1, 3) in the plane x + 2y – z = 0

$\dfrac{\alpha-2}{1}=\dfrac{\beta+1}{2}=\dfrac{\gamma-3}{-1} = \dfrac{-2(2-2-3)}{1^2+2^2+(-1)^2}=1$

α = 3, β = 1, γ = 2

Distance of Q(3, 1, 2) from

3x + 2y + z + 29 = 0

D = $\dfrac{|3(3)+2(1)+2+29|}{\sqrt{3^2+2^2+1^2}}$

$= \dfrac{42}{\sqrt{14}} = 3\sqrt{14}$

75. Let $f(x)=2x+\tan^{-1}x$ and $g(x)=\log_e\left(\sqrt{1+x^2}+x\right), x\in[0,3]$.
Then
(1) $\min f'(x)=1+\max g'(x)$
(2) $\max f(x)>\max g(x)$
(3) there exist $0<x_1<x_2<3$ such that $f(x)<g(x), \forall x\in(x_1,x_2)$
(4) there exists $\hat{x}\in[0,3]$ such that $f'(\hat{x})<g'(\hat{x})$

Sol. **2**

$f'(x) = 2+\dfrac{1}{1+x^2}>0$ for $x \in [0, 3]$

$f(x)\uparrow$ for $x \in [0, 3]$

$f(0) = 0,\ f(3) = 6 + \tan^{-1}(3)$

$$g'(x) = \frac{\frac{x}{\sqrt{x^2+1}}+1}{x+\sqrt{1+x^2}} = \frac{1}{\sqrt{1+x^2}}>0 \quad \text{for } x \in [0, 3]$$

$g(x)\uparrow$ for $x \in [0, 3]$

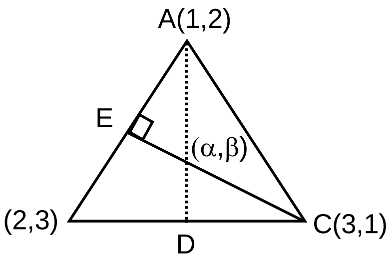
$g(0) = 0,\ g(3) = \log_e\left(\sqrt{10}+3\right)$

$\max f(x) > \max g(x)$

Option (2) correct

76. If the orthocentre of the triangle, whose vertices are (1,2) (2,3) and (3,1) is (α,β), then the quadratic equation whose roots are $\alpha + 4\beta$ and $4\alpha + \beta$, is
(1) $x^2 - 20x + 99 = 0$
(2) $x^2 - 19x + 90 = 0$
(3) $x^2 - 22x + 120 = 0$
(4) $x^2 - 18x + 80 = 0$

Sol. **1**



equation of AD : $x - 2y + 3 = 0$

equation of CE : $x + y - 4 = 0$

orthocenter (α, β) is $\left(\dfrac{5}{3},\dfrac{7}{3}\right)$

$\alpha + 4\beta = 11$ and $4\alpha + \beta = 9$

Quadratic equation is

$x^2 - (11 + 9)x + (11 \times 9) = 0$

$\Rightarrow x^2 - 20x + 99 = 0$

77. Let $S = \{x : x \in \mathbb{R} \text{ and } (\sqrt{3}+\sqrt{2})^{x^2-4} + (\sqrt{3}-\sqrt{2})^{x^2-4} = 10\}$
Then $n(S)$ is equal to
(1) 4 (2) 0 (3) 6 (4) 2

Sol. **1**

$$\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}+\left(\sqrt{3}-\sqrt{2}\right)^{x^2-4}=10$$

$$\Rightarrow \left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}+\frac{1}{\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}}=10$$

Let $\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}=t$

$t+\dfrac{1}{t}=10$

$\Rightarrow t^2 - 10t + 1 = 0$

$t=5+2\sqrt{6},\ 5-2\sqrt{6}$

If $t=5+2\sqrt{6}$

$\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}=\left(\sqrt{3}+\sqrt{2}\right)^{2}$

$\Rightarrow x^2 - 4 = 2$

$\Rightarrow x=\pm\sqrt{6}$

If $t=5-2\sqrt{6}$

$\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}=\left(\sqrt{3}+\sqrt{2}\right)^{-2}$

$\Rightarrow x^2 - 4 = -2$

$\Rightarrow x=\pm\sqrt{2}$

$S = \left\{\sqrt{6},-\sqrt{6},\sqrt{2},-\sqrt{2}\right\}$

$n(s) = 4$

78. If the center and radius of the circle $\left|\dfrac{z-2}{z-3}\right|=2$ are respectively (α,β) and γ.
then $3(\alpha + \beta + \gamma)$ is equal to
(1) 11 (2) 12 (3) 9 (4) 10

Sol. **2**

Put $z = x + iy$

$\dfrac{|(x-2)+iy|}{|(x-3)+iy|}=2$

$\Rightarrow (x-2)^2 + y^2 = 4((x-3)^2 + y^2)$

$\Rightarrow x^2 - 4x + 4 + y^2 = 4x^2 - 24x + 36 + 4y^2$

$\Rightarrow 3x^2 + 3y^2 - 20x + 32 = 0$

$\Rightarrow x^2 + y^2 - \dfrac{20}{3}x+\dfrac{32}{3}=0$

Center $(\alpha, \beta) = \left(\dfrac{10}{3},0\right)$

Radius $(\gamma) = \sqrt{\left(-\dfrac{10}{3}\right)^2-\dfrac{32}{3}} = \dfrac{2}{3}$

$3\left(\dfrac{10}{3}+0+\dfrac{2}{3}\right)=12$

79. Let $f(x)=\begin{vmatrix}1+\sin^2 x & \cos^2 x & \sin 2x\\ \sin^2 x & 1+\cos^2 x & \sin 2x\\ \sin^2 x & \cos^2 x & 1+\sin 2x\end{vmatrix}$, $x\in\left[\frac{\pi}{6},\frac{\pi}{3}\right]$. If α and β respectively are the maximum and the minimum values of f, then

(1) $\alpha^2+\beta^2=\frac{9}{2}$ (2) $\beta^2-2\sqrt{\alpha}=\frac{19}{4}$ (3) $\alpha^2-\beta^2=4\sqrt{3}$ (4) $\beta^2+2\sqrt{\alpha}=\frac{19}{4}$

Sol. **2**

$$f(\mathrm{x})=\begin{vmatrix}1+\sin^2 x & \cos^2 x & \sin 2x\\ \sin^2\mathrm{x} & 1+\cos^2 x & \sin 2x\\ \sin^2 x & \cos^2 x & 1+\sin x\end{vmatrix}$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$

$$=\begin{vmatrix}1+\sin^2 x & \cos^2 x & \sin 2x\\ -1 & 1 & 0\\ -1 & 0 & 1\end{vmatrix}$$

$= (1+\sin^2 x) - \cos^2 x(-1) + \sin 2x$

$f(x) = 2 + \sin 2x$

$$2x\in\left[\frac{\pi}{3},\frac{2\pi}{3}\right]\Rightarrow\frac{\sqrt{3}}{2}\le\sin 2x\le 1$$

$\alpha = 2 + 1 = 3$

$$\beta = 2+\frac{\sqrt{3}}{2}$$

$$\beta^2-2\sqrt{\alpha}=\left(2+\frac{\sqrt{3}}{2}\right)^2-2\sqrt{3}$$

$$=4+\frac{3}{4}+2\sqrt{3}-2\sqrt{3}$$

$$=\frac{19}{4}$$

80. The negation of the expression $q\vee((\sim q)\wedge p)$ is equivalent to

(1) $(\sim p)\vee(\sim q)$ (2) $p\wedge(\sim q)$ (3) $(\sim p)\vee q$ (4) $(\sim p)\wedge(\sim q)$

Sol. **4**

$\sim(q\vee(\sim q)\wedge p)$

$=\sim q\wedge(q\vee\sim p)$

$=(\sim q\wedge q)\vee(\sim q\wedge\sim p)$

$=F\vee(\sim q\wedge\sim p)=(\sim q)\wedge(\sim p)$

Section B

81. Let $\vec{v}=\alpha\hat{\imath}+2\hat{\jmath}-3\hat{k}$, $\vec{w}=2\alpha\hat{\imath}+\hat{\jmath}-\hat{k}$ and $\vec{u}$ be a vector such that $|\vec{u}|=\alpha>0$. If the minimum value of the scalar triple product $[\vec{u}\vec{v}\vec{w}]$ is $-\alpha\sqrt{3401}$, and $|\vec{u}\cdot\hat{\imath}|^2=\frac{m}{n}$ where m and n are coprime natural numbers, then $m+n$ is equal to

Sol. **3501**

$$\vec{v}\times\vec{w}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \alpha & 2 & -3\\ 2\alpha & 1 & -1\end{vmatrix}$$

$= \hat{i}-5\alpha\hat{j}-3\alpha\hat{k}$

$[u\ v\ w] = \vec{u}.(\vec{v}\times\vec{w})$

$=|\vec{u}|\,|\vec{v}\times\vec{w}|\cos\theta$

since $[u\ v\ w]$ is Least $\Rightarrow \cos\theta = -1$

$[u\ v\ w] = \left(|\vec{u}|\sqrt{1+25\alpha^2+9\alpha^2}\right)(-1)$

$\Rightarrow -\alpha\sqrt{1+34\alpha^2}=-\alpha\sqrt{3401}$

$\Rightarrow \alpha^2 = 100$

$\Rightarrow \alpha = 10 \qquad \{\because \alpha > 0\}$

$\vec{u}$ is parallel to $\vec{v}\times\vec{w}$

$\vec{u} = \lambda(\vec{v}\times\vec{w})$

$\vec{u}=\lambda(\hat{i}-50\hat{j}-30\hat{k})$

$|\vec{u}| = 10$

$|\lambda|\sqrt{3401}=10$

$|\lambda| = \dfrac{10}{\sqrt{3401}} \qquad \vec{u} = \pm\dfrac{10}{\sqrt{3401}}\left(\hat{i}-50\hat{j}-30\hat{k}\right)$

$|\vec{u}.\hat{i}|^2=\dfrac{100}{3401}=\dfrac{m}{n}$

$m + n = 100 + 3401 = 3501$

82. The number of words, with or without meaning, that can be formed using all the letters of the word ASSASSINATION so that the vowels occur together, is

Sol. **50400**

A – 3, I – 2, S – 4, N – 2, O –1, T–1

As vowels are together

Total words formed = $\left(\dfrac{8!}{4!2!}\right)\left(\dfrac{6!}{3!2!}\right)$

$= \left(\dfrac{8\times7\times6\times5}{2}\right)\left(\dfrac{6\times5\times4}{2}\right) = 50400$

83. The remainder, when $19^{200}+23^{200}$ is divided by 49 , is

Sol. **29**

$19^{200} + 23^{200} \quad a^{n+}b^{n}$

$19^3 = 6859 = 140 \times 49 - 1$

$= 49\lambda - 1$

$(19^3)^{66} = (49\lambda - 1)^{66}$

So, Remainder of 19^{198} divided by 49

is $(-1)^{66} = 1$

$19^2 = 361$ gives remainder 18
So, 19^{200} gives remainder 18
23^2 gives remainder 39
$(23)^3$ gives remainder 15
$(23)^4$ gives remainder 2
$((23)^4)^6$ gives remainder $(2)^6 = 64$
& 64 gives remainder 15
$(23)^{24} \longrightarrow 15$
$(23)^{25} \longrightarrow 2$
$((23)^{25})^8 \longrightarrow (2)^8 = 256 \longrightarrow 11$
So, Total remainder = 18 + 11 = 29

84. The number of 3-digit numbers, that are divisible by either 2 or 3 but not divisible by 7 , is
Sol. **514**
3 digit numbers divisible by either 2 or 3
P = n(divisible by 2) + n(divisible by 3) – n(divisible by 6)
P = 450 + 300 – 150
P = 600
Q = n(divisible by 14) + n(divisible by 21) – n(divisible by 42)
= 64 + 43 – 21 = 86
3 digit number divisible by either 2 or 3
But not divisible by –1 so P – Q = 600 – 86 = 514

85. Let $f: \mathbb{R} \to \mathbb{R}$ be a differentiable function such that $f'(x) + f(x) = \int_0^2 f(t)dt$.
If $f(0) = e^{-2}$, then $2f(0) - f(2)$ is equal to
Sol. **1**
Let $\int_0^2 f(t)dt = \lambda$
$f'(x) + f(x) = \lambda$
is linear Differential equation
I.f. $= e^{\int dx} = e^x$
$f(x).e^x = \int e^x \lambda dx$
$\Rightarrow f(x).e^x = \lambda e^x + C$
$\Rightarrow f(x) = \lambda + Ce^{-x}$
put $f(0) = e^{-2}$
$e^{-2} = \lambda + C \Rightarrow C = e^{-2} - \lambda$
$f(x) = \lambda + (e^{-2} - \lambda)\, e^{-x}$
$\lambda = \int_0^2 f(t)dt$
$= \int_0^2 (\lambda + (e^{-2} - \lambda)e^{-t})dt$
$\Rightarrow \lambda = \lambda + \lambda e^{-2} - e^{-4} + e^{-2}$
$\Rightarrow \lambda = e^{-2} - 1$
$f(x) = e^{-2} - 1 + e^{-x}$

$f(0) = e^{-2}$
$f(2) = 2e^{-2} - 1$
$2f(0) - f(2) = 1$

86. If $f(x) = x^2 + g'(1)x + g''(2)$ and $g(x) = f(1)x^2 + xf'(x) + f''(x)$, then the value of $f(4) - g(4)$ is equal to

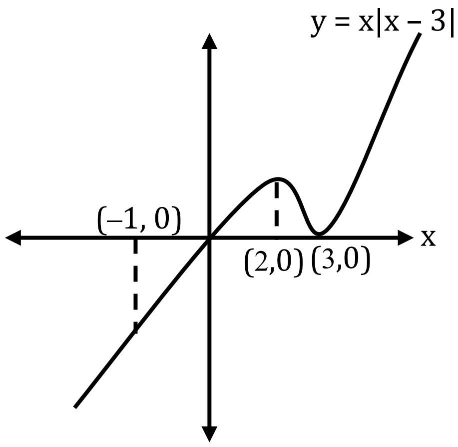
Sol. **14**

let $g'(1) = A$
$g''(2) = B$
$f(x) = x^2 + Ax + B$
$f(1) = A + B + 1$
$f'(x) = 2x + A$
$f''(x) = 2$
$g(x) = (A + B + 1)x^2 + x(2x + A) + 2$
$\Rightarrow g(x) = x^2(A + B + 2) + Ax + 2$
$g'(x) = 2x(A + B + 2) + A$
$g'(1) = A$
$\Rightarrow 2(A + B + 2) + A = A$
$A + B = -2 \quad \ldots.(i)$
$g''(x) = 2(A + B + 2)$
$g''(2) = B$
$\Rightarrow 2(A + B + 2) = B$
$\Rightarrow 2A + B = -4 \quad \ldots(ii)$
From (i) and (ii)
$A = -2$ and $B = 0$
$f(x) = x^2 - 2x$
$f(4) = 16 - 8 = 8$
$g(x) = -2x + 2$
$g(4) = -8 + 2 = -6$
$f(4) - g(4) = 8 - (-6) = 14$

87. Let A be the area bounded by the curve $y = x|x - 3|$, the x-axis and the ordinates $x = -1$ and $x = 2$. Then $12A$ is equal to

Sol. **62**

$$y = x|x-3| = \begin{cases} x(x-3); x \geq 3 \\ x(3-x); x < 3 \end{cases}$$



$$A = -\int_{-1}^{0} x(3-x)\,dx + \int_{0}^{2} x(3-x)\,dx$$

$$= \int_{-1}^{0} (x^2-3x)\,dx + \int_{0}^{2} (3x-x^2)\,dx$$

$$= \left[\frac{x^3}{3}-\frac{3x^2}{2}\right]_{-1}^{0} + \left[\frac{3x^2}{2}-\frac{x^3}{3}\right]_{0}^{2}$$

$$A = 0-\left(\frac{-1}{3}-\frac{3}{2}\right)+6-\frac{8}{3}=\frac{31}{6}$$

$$A = 12\left(\frac{31}{6}\right)=62$$

88. If $\int_0^1 (x^{21}+x^{14}+x^7)(2x^{14}+3x^7+6)^{1/7}dx = \frac{1}{l}(11)^{m/n}$ where $l, m, n \in \mathbb{N}$, m and n are coprime then $l+m+n$ is equal to

Sol. **63**

$$I = \int_0^1 (x^{21}+x^{14}+x^7)\,(2x^{14}+3x^7+6)^{\frac{1}{7}}\,dx$$

$$= \int_0^1 (x^{20}+x^{13}+x^6)\,(2x^{21}+3x^{14}+6x^7)^{\frac{1}{7}}\,dx$$

Put $2x^{21} + 3x^{14} + 6x^7 = t$

$\Rightarrow 42(x^{20} + x^{13} + x^6)dx = dt$

$\Rightarrow (x^{20} + x^{13} + x^6)dx = \dfrac{dt}{42}$

$$I = \int_0^{11} \frac{t^{\frac{1}{7}}}{42}dt$$

$$= \frac{1}{42}\left[\frac{t^{\frac{8}{7}}}{\frac{8}{7}}\right]_0^{11}$$

$$= \left(\frac{7}{8}\right)\left(\frac{1}{42}\right)(11)^{8/7}$$

$$= \frac{1}{48}(11)^{8/7} = \frac{1}{\ell}(11)^{m/n}$$

$\ell + m + n = 48 + 8 + 7 = 63$

89. Let $a_1 = 8, a_2, a_3, \ldots, a_n$ be an A.P. If the sum of its first four terms is 50 and the sum of its last four terms is 170 , then the product of its middle two terms is

Sol. **754**

$a_1 = 8$

d = common difference

$\dfrac{4}{2}$ [16 + 3d] = 50

$\Rightarrow$ d = 3

$\frac{4}{2}$ $[2a_n + 3(-d)] = 170$

$\Rightarrow 2(a_1 + (n-1)d) - 3d = 85$

$\Rightarrow 16 + 6(n-1) - 9 = 85$

$n - 1 = 13$

$n = 14$

Product of middle two terms = $T_7 \times T_8$

$= (a_1 + 6d)(a_1 + 7d)$

$= (8 + 18)(8 + 21)$

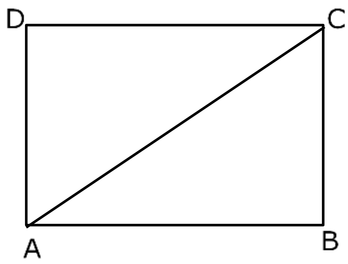
$= (26)(29) = 754$

90. $A(2,6,2), B(-4,0,\lambda), C(2,3,-1)$ and $D(4,5,0), |\lambda| \leq 5$ are the vertices of a quadrilateral $ABCD$. If its area is 18 square units, then $5 - 6\lambda$ is equal to

Sol. **11**

$\overrightarrow{AD} = 2\hat{i} - \hat{j} - 2\hat{k}$

$\overrightarrow{AC} = -3j - 3\hat{k}$



$$\overrightarrow{AD} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & -2 \\ 0 & -3 & -3 \end{vmatrix}$$

$= -3\hat{i} + 6\hat{j} - 6\hat{k}$

Area $(\Delta ADC) = \frac{1}{2}\left|\overrightarrow{AD} \times \overrightarrow{AC}\right|$

$= \frac{1}{2}\sqrt{9 + 36 + 36} = \frac{9}{2}$

$\overrightarrow{AB} = -6\hat{i} - 6\hat{j} + (\lambda - 2)\hat{k}$

$\overrightarrow{AC} = -3\hat{j} - 3\hat{k}$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -6 & -6 & \lambda - 2 \\ 0 & -3 & -3 \end{vmatrix}$$

$= (12 + 3\lambda)\,\hat{i} - 18\hat{j} + 18\hat{k}$

area $(\Delta ABC) = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$

$= \frac{3}{2}\sqrt{(4 + \lambda)^2 + 36 + 36}$

Area$(\Delta ABCD)$ = ar(ΔADC) + ar(ΔABC)

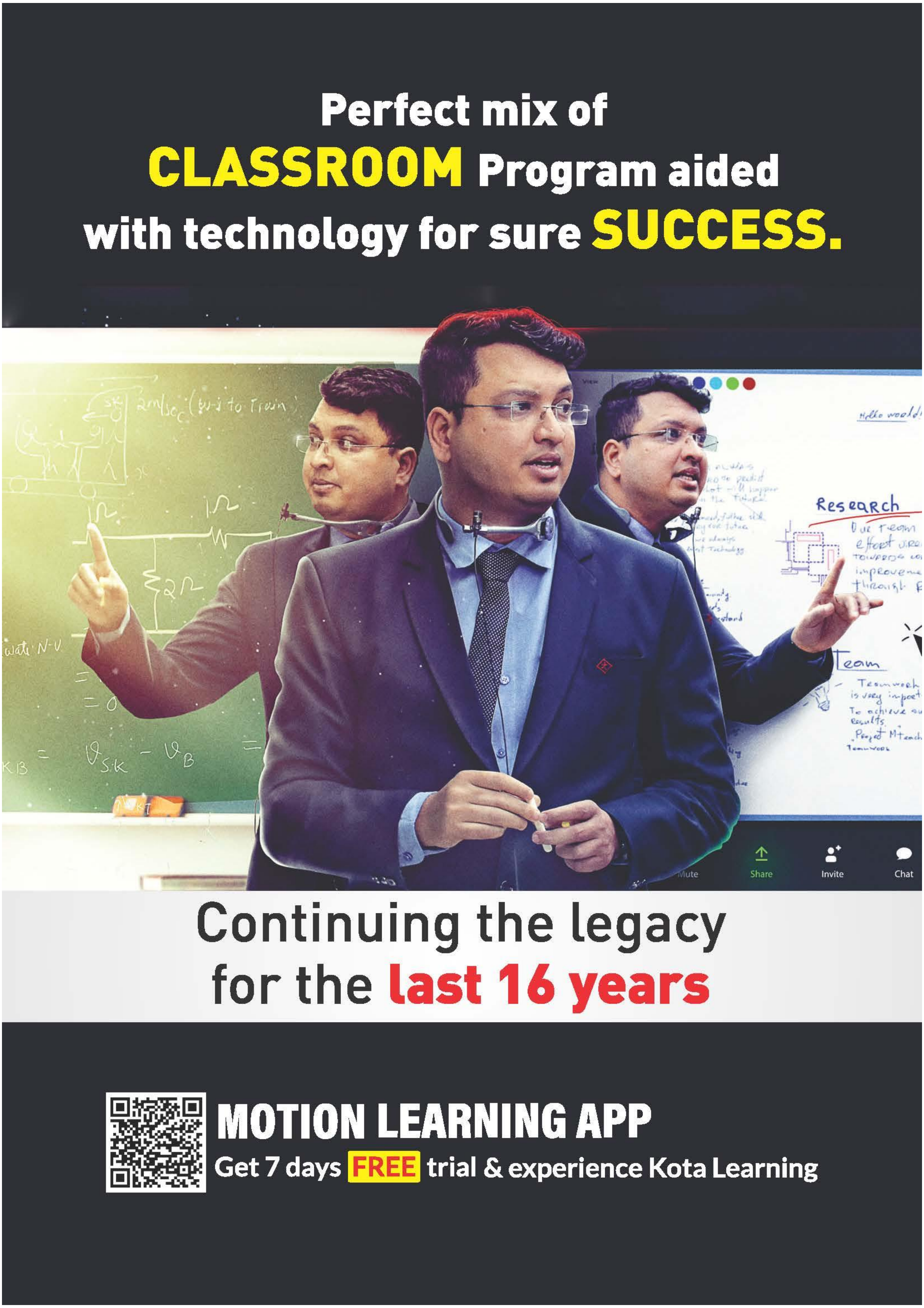
$\Rightarrow 18 = \frac{9}{2} + \frac{3}{2}\sqrt{(4 + \lambda)^2 + 72}$

$\Rightarrow (4 + \lambda)^2 = 9$

$4 + \lambda = 3$ or $4 + \lambda = -3$

$\Rightarrow \lambda = -1$ or $\lambda = -7$ (reject)

$5 - 6\lambda = 5 + 6 = 11$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