JEE MAIN 2023
Paper with Solution

MATHEMATICS | 25th Jan 2023 _ Shift-2

Umeed Rank Ki Ho Ya Selection Ki, JEET NISCHIT HAI!

Most Promising RANKS
Produced by MOTION Faculties

NEET / AIIMS

AIR-1 to 10
25 Times

AIR-11 to 50
83 Times

AIR-51 to 100
81 Times

JEE MAIN+ADVANCED

AIR-1 to 10
8 Times

AIR-11 to 50
32 Times

AIR-51 to 100
36 Times

Nation's Best SELECTION
Percentage (%) Ratio

Student Qualified in NEET

(2022)
4837/5356 = **90.31%**
(2021)
3276/3411 = **93.12%**

Student Qualified in JEE ADVANCED

(2022)
1756/4818 = **36.45%**
(2021)
1256/2994 = **41.95%**

Student Qualified in JEE MAIN

(2022)
4818/6653 = **72.41%**
(2021)
2994/4087 = **73.25%**

NITIN VIIJAY (NV Sir)
Founder & CEO

SECTION - A

61. Let $\Delta, \nabla \in \{\wedge, \vee\}$ be such that $(p \to q)\Delta(p \nabla q)$ is a tautology. Then

(1) $\Delta = \vee, \nabla = \vee$ (2) $\Delta = \vee, \nabla = \wedge$ (3) $\Delta = \wedge, \nabla = \vee$ (4) $\Delta = \wedge, \nabla = \wedge$

Sol. **(1)**

p	q	$p \to q$	$p \vee q$	$(p \to q) \vee (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	T

62. If the four points, whose position vectors are $3\hat{i} - 4\hat{j} + 2\hat{k}$, $\hat{i} + 2\hat{j} - \hat{k}$, $-2\hat{i} - \hat{j} + 3\hat{k}$ and $5\hat{i} - 2\alpha\hat{j} + 4\hat{k}$ are coplanar, then α is equal to

(1) $\frac{73}{17}$ (2) $\frac{107}{17}$ (3) $\frac{-73}{17}$ (4) $\frac{-107}{17}$

Sol. **(1)**

$$\underbrace{3\hat{i} - 4\hat{j} + 2\hat{k}}_{P},\ \underbrace{\hat{i} + 2\hat{j} - \hat{k}}_{Q},\ \underbrace{-2\hat{i} - \hat{j} + 3\hat{k}}_{R},\ \underbrace{5\hat{i} - 2\alpha\hat{j} + 4\hat{k}}_{S}$$

$$\overrightarrow{PQ} = -2\hat{i} + 6\hat{j} - 3\hat{k}$$

$$\overrightarrow{QR} = -3\hat{i} - 3\hat{j} + 4\hat{k}$$

$$\overrightarrow{RS} = 7\hat{i} + (1 - 2\alpha)\hat{j} + \hat{k}$$

$$\left[\overrightarrow{PQ}\ \overrightarrow{QR}\ \overrightarrow{RS}\right] = 0$$

$$\begin{vmatrix} -2 & 6 & -3 \\ -3 & -3 & 4 \\ 7 & 1-2\alpha & 1 \end{vmatrix} = 0$$

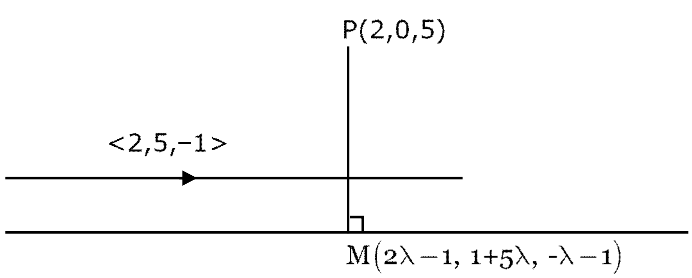
$$-2(-3 + 8\alpha - 4) - 6(-31) - 3(6\alpha - 3 + 21) = 0$$

$$\alpha = \frac{73}{17}$$

63. The foot of perpendicular of the point $(2,0,5)$ on the line $\frac{x+1}{2} = \frac{y-1}{5} = \frac{z+1}{-1}$ is (α, β, γ). Then, which of the following is NOT correct?

(1) $\frac{\beta}{\gamma} = -5$ (2) $\frac{\gamma}{\alpha} = \frac{5}{8}$ (3) $\frac{\alpha}{\beta} = -8$ (4) $\frac{\alpha\beta}{\gamma} = \frac{4}{15}$

Sol. **(1)**



$\overrightarrow{PM}(2,5,-1)=0$

$(2\lambda-3,\ 5\lambda+1,\ -\lambda-6)\cdot(2,5,-1)=0$

$4\lambda-6+25\lambda+5+\lambda+6=0$

$\boxed{\lambda=-\frac{1}{6}}$

Now, $\alpha=2\left(-\frac{1}{6}\right)-1=-\frac{4}{3}$

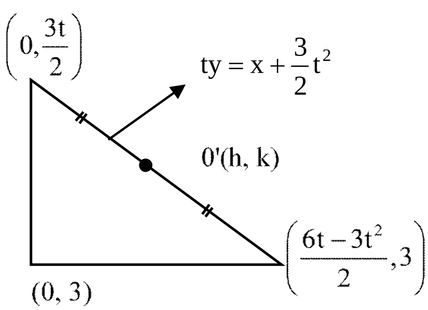
$\beta=\frac{1}{6}$

$\gamma=-\frac{5}{6}$

64. The equations of two sides of a variable triangle are x= 0 and y=3, and its third side is a tangent to parabola $y^2=6x$. The locus of its circumcentre is:

(1) $4y^2-18y-3x-18=0$
(2) $4y^2-18y-3x+18=0$
(3) $4y^2-18y+3x+18=0$
(4) $4y^2+18y+3x+18=0$

Sol. **(3)**



$2h=\frac{6t-3t^2}{2}$

$4h=6t-3t^2$ .....(i)

$\&\ 2k=\frac{3t+6}{2}$

$\boxed{\frac{4k-6}{3}=t}$

$4h=8k-12-\frac{1}{3}\left(16k^2-48k+36\right)$

$12h=24k-36-16k^2+48k-36$

$4y^2-18y+3x+18=0$

65. Let $f(x)=2x^n+\lambda,\lambda\in\mathbb{R},n\in\mathbb{N}$, and $f(4)=133, f(5)255$.
Then the sum of all the positive integer divisors of $(f(3)-f(2))$ is

(1) 60
(2) 59
(3) 61
(4) 58

Sol. **(1)**

$133 = 2(4^n)+\lambda$

$255 = 2(5^n)+\lambda$

$122 = 2[5^n-4^n]$

$\boxed{5^n - 4^n = 61}$

$\Downarrow$

$\boxed{n = 3}$

Now,

$f(3) = 2(3)^3 + \lambda$

$f(2) = 2(2)^3 + \lambda$

$f(3) – f(2) = 38 = 2 \times 19$

$(2^0 + 2^1)(19^0 + 19)$

$= 60$

66. $\sum_{k=0}^{6} {}^{51}C_3$ is equal to

(1) ${}^{51}C_4 - {}^{45}C_4$ (2) ${}^{52}C_3 - {}^{45}C_3$ (3) ${}^{52}C_4 - {}^{45}C_4$ (4) ${}^{51}C_3 - {}^{45}C_3$

Sol. **(3)**

${}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{46}C_3 + {}^{45}C_3$

add and subtract ${}^{45}C_4$

$\left({}^{45}C_4 + {}^{45}C_3\right) + {}^{46}C_3 + {}^{47}C_3 + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3 - {}^{45}C_4 \quad \left({}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r\right)$

${}^{52}C_4 - {}^{45}C_4$

$\Rightarrow$ [C]

67. Let the function $f(x) = 2x^3 + (2p-7)x^2 + 3(2p-9)x - 6$ have a maxima for some value of $x < 0$ and a minima for some value of $x > 0$. Then, the set of all values of p is

(1) $\left(0, \frac{9}{2}\right)$ (2) $\left(-\infty, \frac{9}{2}\right)$ (3) $\left(-\frac{9}{2}, \frac{9}{2}\right)$ (4) $\left(\frac{9}{2}, \infty\right)$

Sol. **(2)**

$f(x) = 2x^3 + (2p-7)x^2 + 3(2p-9)x - 6$

$f'(x) = 6x^2 + (4p-14)x + 6p - 27 = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix}$

let $\alpha > 0$ & $\beta < 0$

Products of roots $< 0 \Rightarrow$ (2)

68. Let $A = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix}$, where $i = \sqrt{-1}$.

If $M = A^TBA$, then the inverse of the matrix $AM^{2023}A^T$ is

(1) $\begin{bmatrix} 1 & 0 \\ -2023i & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$ (3) $\begin{bmatrix} 1 & 0 \\ 2023i & 1 \end{bmatrix}$ (4) $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$

Sol. **(4)**

Now, $M^2 = \left(A^TBA\right)\left(A^TBA\right) = A^TB^2A$ $\boxed{AA^T = I}$

$\Rightarrow M^{2023} = A^TB^{2023}A$

Let $D = AM^{2023}A^T = A\,A^TB^{2023}AA^T$ $\boxed{AA^T = I}$

$D = B^{2023}$

Now, $B^2 = \begin{bmatrix} 1- & i \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1- & i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2i \\ 0 & 1 \end{bmatrix}$

Now, $B^{2023} = \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$

$D^{-1} = \begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$

69. Let $\vec{a} = -\hat{i} - \hat{j} + \hat{k}$, $\vec{a} \cdot \vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{i} - \hat{j}$. Then $\vec{a} - 6\vec{b}$ is equal to

(1) $3(\hat{i} - \hat{j} + \hat{k})$ (2) $(\hat{i} + \hat{j} - \hat{k})$ (3) $3(\hat{i} + \hat{j} + \hat{k})$ (4) $3(\hat{i} - \hat{j} - \hat{k})$

Sol. **(3)**

$$\vec{a} \times (\vec{a} \times \vec{b}) = \begin{vmatrix} i & j & k \\ -1 & -1 & 1 \\ 1 & -1 & 0 \end{vmatrix} = \hat{i} + \hat{j} + 2\hat{k}$$

$$\vec{a} - 3\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$$

$$-3\vec{b} = 2\hat{i} + 2\hat{j} + \hat{k}$$

$$-6\vec{b} = 4\hat{i} + 4\hat{j} + 2\hat{k}$$

Now, $\vec{a} - 6\vec{b} = 3(\hat{i} + \hat{j} + \hat{k})$

70. The integral $16\int_1^2 \frac{dx}{x^3(x^2+2)^2}$ is equal to

(1) $\frac{11}{12} - \log_e 4$ (2) $\frac{11}{6} - \log_e 4$ (2) $\frac{11}{6} + \log_e 4$ (4) $\frac{11}{12} + \log_e 4$

Sol. **(2)**

$$16\int_1^2 \frac{dx}{x^3x^4\left(1+\frac{2}{x^2}\right)^2}$$

Let, $1 + \frac{2}{x^2} = t \Rightarrow -\frac{4}{x^3}dx = dt$

$$\frac{-4}{4}\int_3^{3/2} \frac{(t-1)^2}{t^2}dt = \int_{3/2}^{3} \frac{t^2 - 2t + 1}{t^2}dt$$

$$\Rightarrow \int_{3/2}^{3}\left(1 - \frac{2}{t} + \frac{1}{t^2}\right)dt$$

$$\Rightarrow 3 - \frac{3}{2} - 2\left(\ln 3 - \ln 3/2\right) - \frac{1}{3} + \frac{2}{3}$$

$$\Rightarrow \frac{11}{6} - 2\ln 2 \Rightarrow \frac{11}{6} - \ln 4$$

71. Let T and C respectively be the transverse and conjugate axes of the hyperbola $16x^2 - y^2 + 64x + 4y + 44 = 0$. Then the area of the region above the parabola $x^2 = y + 4$, below the transverse axis T and on the right of the conjugate axis C is:

(1) $4\sqrt{6} + \frac{28}{3}$ (2) $4\sqrt{6} - \frac{44}{3}$ (3) $4\sqrt{6} + \frac{44}{3}$ (4) $4\sqrt{6} - \frac{28}{3}$

Sol. **(1)**

$16(x^2 + 4x) - (y^2 - 4y) + 44 = 0$

$16\{(x+2)^2 - 4\} - (y - 2)^2 + 4 + 44 = 0$

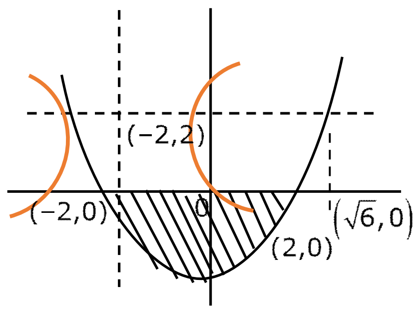
$16(x + 2)^2 - (y - 2)^2 = 16$

$$\frac{(x+2)^2}{1} - \frac{(y-2)^2}{16}$$

Area = $\int_{-2}^{\sqrt{6}} (y_2 - y_1)\,dx$

$= \int_{-2}^{\sqrt{6}} \left(2 - (x^2 - 4)\right) dx$

$= 4\sqrt{6} + \dfrac{28}{3}$



72. Let N be the sum of the numbers appeared when two fair dice are rolled and let the probability that $N-2,\ \sqrt{3N},\ N+2$ are in geometric progression be $\frac{k}{48}$. Then the value of k is

(1) 8 (2) 16 (3) 2 (4) 4

Sol. **(4)**

$3N = N^2 - 4$

$N^2 - 3N - 4 = 0$

$\boxed{N=4}$

Sum should be equal to 4 so possible outcomes are {(1,3), (2,2), (3,1)}

$\Rightarrow$ Prob = $\dfrac{3}{36} = \dfrac{1}{12} = \dfrac{k}{48}$

$\boxed{K=4}$

73. If the function $f(x) = \begin{cases} (1+|\cos x|)^{\frac{\lambda}{|\cos x|}} & , 0 < x < \frac{\pi}{2} \\ \mu & , x = \frac{\pi}{2} \\ \dfrac{\cot 6x}{e^{\cot 4x}} & \frac{\pi}{2} < x < \pi \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then $9\lambda + 6\log_e \mu + \mu^6 - e^{6\lambda}$

is equal to

(1) 10 (2) $2e^4 + 8$ (3) 11 (4) 8

Sol. $f\left(\dfrac{\pi^+}{2}\right) = e^{\lim_{h\to 0}\frac{\cot 6h}{\cot 4h}} \Rightarrow 2/3$

$f\left(\dfrac{\pi^-}{2}\right) = \lim_{h\to 0}(1+\sin h)\dfrac{\lambda}{\sin h}$

$= \dfrac{\lambda}{0}$

$\Rightarrow$ limit DNE (does not exist)

74. The number of functions $f : \{1,2,3,4\} \to \{a \in \mathbb{Z} \,|a| \le 8\}$ satisfying $f(n) + \frac{1}{n} f(n+1) = 1, \forall n \in \{1,2,3\}$ is

(1) 1 (2) 4 (3) 2 (4) 3

Sol. **(2)**

$f:\{1,2,3,4\}\to\{a\in z:|a|\le 8\}$

$f(n)+\frac{1}{n}f(n+1)=1\ \forall n\in\{1,2,3\}$

$f(n+1)=n(1-f(n))$

Put n = 1, $f(2)=1-f(1)$

Put n = 2, $f(3)=2(1-f(2))=2f(1)$

Put n = 3, $f(4)=3(1-f(3))=3(1-2f(1))$

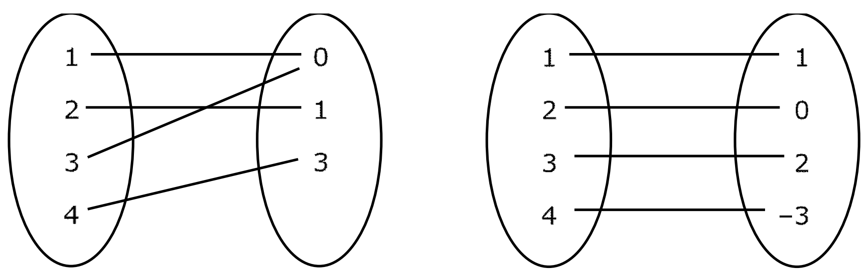
$f(4)=3-6f(1)$

Now : $f(2)=1-f(1)$

$f(3)=2f(1)$

$f(4)=3-6f(1)$

Case – I Take f(1) = 0 Case – II Take f(1) = 1



No. of function = 2

Ans : 4

75. Let $y = y(t)$ be a solution of the differential equation $\frac{dy}{dt}+\alpha y=\gamma e^{-\beta t}$ where, $\alpha > 0, \beta > 0$ and $\gamma > 0$. Then $\lim_{t\to\infty} y(t)$

(1) is −1 (2) is 1 (3) does not exist (4) is 0

Sol. **(4)**

$\frac{dy}{dt}+\alpha y=\gamma e^{-\beta t}$

L.D.E (Linear differential equation)

I.F. = $e^{\int \alpha\cdot dt}=e^{\alpha t}$

$y(e^{\alpha t})=\int \gamma e^{-\beta t}\cdot e^{\alpha t}\cdot dt$

$\Rightarrow ye^{\alpha t} = \gamma \dfrac{e^{(\alpha-\beta)t}}{\alpha-\beta} + C$

$\Rightarrow y(t) = \dfrac{\gamma}{\alpha-\beta} e^{-\beta t} + C \cdot e^{-\alpha t}$

$\lim_{t\to\infty} y(t) = \lim_{t\to\infty} \left\{ \dfrac{\gamma}{\alpha-\beta} e^{-\beta t} + c \cdot e^{-\alpha t} \right\}$

= 0 + 0

$\Rightarrow \lim_{t\to\infty} y(t) = 0$

76. Let z be a complex number such that $\left|\frac{z-2i}{z+i}\right| = 2, z \neq -i$. Then z lies on the circle of radius 2 and centre

(1) (2,0) (2) (0,2) (3) (0, –2) (4) (0,0)

Sol. $\left|\dfrac{x+i(y-2)}{x+i(y+1)}\right| = 2$

$x^2 + (y-2)^2 = 4\left(x^2 + (y+1)^2\right)$

$3x^2 + 4y^2 + 4 + 8y - y^2 - 4 + 4y = 0$

$3\left(x^2 + y^2\right) + 12y = 0$

$x^2 + y^2 + 4y = 0$

C(0,–2)

77. Let A, B, C be 3 × 3 matrices such that A is symmetric and B and C are skew-symmetric.

Consider the statements

(S1) $A^{13} B^{26} - B^{26} A^{13}$ is symmetric

(S2) $A^{26}C^{13} - C^{13} A^{26}$ is symmetric

Then,

(1) Only S2 is true (2) Both S1 and S2 are false

(3) Only S1 is true (4) Both S1 and S2 are true

Sol. **(1)**

$A^T = A$, $B^T = -B$, $C^T = -C$

(S_1) : $(A^{13}B^{26} - B^{26}A^{13})^T$

$= (A^{13}B^{26})^T - (B^{26}A^{13})^T$

$= (B^T)^{26} (A^T)^{13} - (A^T)^{13} (B^T)^{26}$

$= (-B)^{26} (A)^{13} - (A)^{13} (-B)^{26}$

$= B^{26} A^{13} - A^{13} B^{26}$

$= -(A^{13} B^{26} - B^{26} A^{13})$

$(S_1 \to \text{false})$

(S_2) : $(A^{26} C^{13} - C^{13} A^{26})^T$

$= (A^{26} C^{13})^T - (C^{13} A^{26})^T$

$= (C^T)^{13} (A^T)^{26} - (A^T)^{26} (C^T)^{13}$

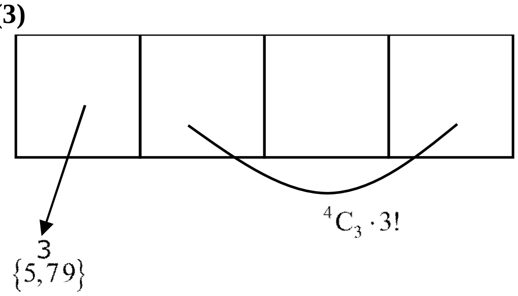
$= -C^{13}A^{26} - A^{26}(-C)^{13}$
$= A^{26}C^{13} - C^{13}A^{26}$

($S_2 \rightarrow$ True)

78. The number of numbers, strictly between 5000 and 10000 can be formed using the digits 1,3,5,7,9 without repetition, is

(1) 12 (2) 120 (3) 72 (4) 6

Sol. **(3)**



No. of ways = 3.4 × 3! =3.4! = 72

79. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by

$f(x) = \log_{\sqrt{m}}\{\sqrt{2}(\sin x - \cos x) + m - 2\}$, for some m, such that the range of f is [0,2]. Then the value of m is

(1) 5 (2) 4 (3) 3 (4) 2

Sol. **(1)**

$\because -\sqrt{2} \le \sin x - \cos x \le \sqrt{2}$

$\Rightarrow -2 \le \sqrt{2}(\sin x - \cos x) \le 2$

$\Rightarrow m - 4 \le \sqrt{2}(\sin x - \cos x) + m - 2 \le m$

$\Rightarrow \log_{\sqrt{m}}^{(m-4)} \le \log_{\sqrt{m}}^{\{\sqrt{2}(\sin x - \cos x) + m - 2\}} \le \log_{\sqrt{m}}^{m}$

$\Downarrow$

0

$\Rightarrow \log_{\sqrt{m}}^{(m-4)} = 0$

$\Rightarrow \boxed{m = 5}$

80. The shortest distance between the lines $x + 1 = 2y = -12z$ and $x = y + 2 = 6z - 6$ is

(1) $\frac{3}{2}$ (2) 2 (3) $\frac{5}{2}$ (4) 3

Sol. **(2)**

$\frac{x+1}{1} = \frac{y}{\frac{1}{2}} = \frac{z}{\frac{-1}{12}}$, $\quad \frac{x}{1} = \frac{y+2}{1} = \frac{z-1}{\frac{1}{6}}$

$$d = \left| \frac{(\vec{b} - \vec{a}) \cdot (\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|} \right|$$

$\vec{a} = (-1, 0, 0)$, $\quad \vec{b} = (0, -2, 1)$

$\vec{p} = \left(1, \frac{1}{2}, \frac{-1}{12}\right)$, $\vec{q} = \left(1, 1, \frac{1}{6}\right)$

$\vec{b} - \vec{a} = \hat{i} - 2\hat{j} + \hat{k}$

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & \frac{1}{2} & -\frac{1}{12} \\ 1 & 1 & \frac{1}{6} \end{vmatrix}$$

$$= \hat{i}\left(\frac{1}{12} + \frac{1}{12}\right) - \hat{j}\left(\frac{1}{6} + \frac{1}{12}\right) + \hat{k}\left(1 - \frac{1}{2}\right)$$

$$= \frac{\hat{i}}{6} - \frac{\hat{j}}{4} + \frac{\hat{k}}{2}$$

$$|\vec{p} \times \vec{q}| = \sqrt{\frac{1}{36} + \frac{1}{16} + \frac{1}{4}} = \frac{7}{12}$$

$$d = \left| \frac{(\hat{i} - 2\hat{j} + \hat{k}) \cdot \left(\frac{\hat{i}}{6} - \frac{\hat{j}}{4} + \frac{\hat{k}}{2}\right)}{\frac{7}{12}} \right|$$

$$d = \left| \frac{\frac{1}{6} + \frac{1}{2} + \frac{1}{2}}{\frac{7}{12}} \right| = \frac{\frac{7}{6}}{\frac{7}{12}} = 2$$

SECTION - B

81. 25% of the population are smokers. A smoker has 27 times more chances to develop lung cancer than a non smoker. A person is diagnosed with lung cancer and the probability that this person is a smoker is $\frac{k}{10}$. Then the value of k is.

Sol. **9**

$$P(\text{smoker}) = \frac{1}{4}$$

$$P(\text{non smoker}) = \frac{3}{4}$$

Probability that a smoker has lung cancer

$$P\left(\frac{C}{S}\right) = 27\,P\left(\frac{C}{NS}\right)$$

Probability that a person is smoker when he has lung cancer

$$= \frac{P(S)\cdot P\left(\frac{C}{S}\right)}{P(S)\cdot P\left(\frac{C}{S}\right) + P(NS)\cdot P\left(\frac{C}{NS}\right)}$$

$$= \frac{\frac{1}{4}\times P\left(\frac{C}{S}\right)}{\frac{1}{4}\times P\left(\frac{C}{S}\right) + \frac{3}{4}P\left(\frac{C}{NS}\right)}$$

$$= \frac{\frac{1}{4}\times 27P\left(\frac{C}{NS}\right)}{\frac{1}{4}\times 27P\left(\frac{C}{NS}\right) + \frac{3}{4}P\left(\frac{C}{NS}\right)}$$

$$\frac{27}{30} = \frac{k}{10}$$

$$\boxed{k = 9}$$

82. The remainder when $(2023)^{2023}$ is divided by 35 is

Sol. **7**

2023 = 289 × 7

2023 is a multiple of 7

n = $(2023)^{2023}$ is multiple of 7

and $(2023)^{2023} = (-2)^{2023} = -2(2^2)^{1011}$

$= -2(5-1)^{1011}$

$= -2\left[{}^{5}C_0 5^{1011} - {}^{5}C_1 5^{1010} + \ldots\ldots - {}^{1011}C_{1011}\right]$

$(2023)^{2023}$ when divided by 5

gives remainder 2

If n= $(2023)^{2023}$ divided by 35 = 7 × 5

n = 7k

n – 7 = 7 (k – 1) → n 7 is multiple of 7

and n = 5 m + 2

so n – 7 = 5m – 5 = multiple of 5

so n – 7 is multiple of 35 so when n is divided by 35, reminder = 7

83. Let $a \in \mathbb{R}$ and let α, β be the roots of the equation $x^2 + 60^{\frac{1}{4}}x + a = 0$

If $\alpha^4 + \beta^4 = -30$, then the product of all possible values of a is

Sol. (45)

$\alpha + \beta = -60^{\frac{1}{4}}$ and $\alpha\beta = a$

$\alpha^2 + \beta^2 = 60^{\frac{1}{2}} - 2a$

$\alpha^4 + \beta^4 + 2\alpha^2\beta^2 = 60\ 4a^2 - 4a \cdot 60^{\frac{1}{2}}$

$-30 + 2a^2 = 60 + 4a^2 - 4a\sqrt{60}$

$a^2 - 2a\sqrt{60} + 45 = 0$

$\boxed{\text{Product} = 45}$

84. For the two positive numbers a, b is a, b and $\frac{1}{18}$ are in a geometric progression, while $\frac{1}{a}$, 10 and $\frac{1}{b}$ are in an arithmetic progression, then 16a + b is equal to

Sol. (3)

$b^2 = \frac{a}{18}$

$20 = \frac{1}{a} + \frac{1}{b}$

$a = \frac{b}{20b - 1}$

$b^2 = \frac{1}{18} \times \frac{b}{20b - 1}$

$360b^2 - 18b - 1 = 0$

$360b^2 - 30b + 12b - 1 = 0$

$(12b - 1)(30b + 1) = 0$

$b = \frac{1}{12}, \frac{-1}{30}$ (rejected)

$a = \frac{1}{8}$

16 a + 12 b = 2 + 1 = 3

85. If m and n respectively are the numbers of positive and negative values of q in the interval [–p, p] that satisfy the equation $\cos 2\theta \cos\frac{\theta}{2} = \cos 3\theta \cos\frac{9\theta}{2}$, then mn is equal to

Sol. **25**

$$2\cos 2\theta\cos\frac{\theta}{2} = 2\cos 3\theta\cos\frac{9\theta}{2}$$

$$\cos\frac{5\theta}{2}+\cos\frac{3\theta}{2} = \cos\frac{15\theta}{2}+\cos\frac{3\theta}{2}$$

$$\cos\frac{5\theta}{2}-\cos\frac{15\theta}{2} = 0$$

$$\sin 5\theta = 0 \text{ or } \sin\frac{5\theta}{2} = 0$$

$$\theta = \frac{n\pi}{5} \text{ or } \frac{2n\pi}{5}$$

$$\theta = 0, \pm\frac{\pi}{5}, \pm\frac{2\pi}{5}, \pm\frac{3\pi}{5}, \pm\frac{4\pi}{5}, \pm\pi$$

m = n = 5

$$\boxed{mn = 25}$$

86. If the shortest distance between the line joining the points (1,2,3) and (2,3,4), and the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-2}{0}$ is a, then $28a^2$ is equal to

Sol. **18**

A (1, 2, 3) B (2, 3, 4)

Equation of line AB

$$\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1}$$

Given line

$$\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-2}{0}$$

shortest distance $= \left|\frac{(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_2\times\vec{b}_1)}{|\vec{b}_1\times\vec{b}_2|}\right|$

$$= \left|\frac{(3\hat{j}-\hat{k})\cdot(\hat{i}+2\hat{j}-3\hat{k})}{\sqrt{1+4+9}}\right|$$

$$\alpha = \frac{3}{\sqrt{14}}$$

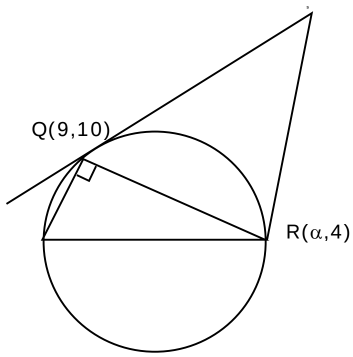
$$28\alpha^2 = 28\times\frac{9}{14} = 18$$

87. Points P(–3,2),Q(9,10) and (a,4) lie on a circle C with PR as its diameter, The tangents to C at the points Q and R intersect at the point S. If S lies on the line 2x – ky = 1, then k is equal to

Sol. **(3)**

Equation of circle is

$(x + 3)(x - \alpha) + (y - 2)(y - 4) = 0$



Q lies on it

$12(9 - \alpha) + 8 \times 6 = 0$

$\boxed{\alpha = 13}$

$x^2 + y^2 - 10x - 6y - 31 = 0$

Equation of Tangent at Q

$x.9 + y.10 - 5(x + 9) - 3(y + 10) - 31 = 0$

$4x + 7y = 106$ ....(1)

Equation of Tangent at R

$x.13 + y.4 - 5(x + 13) - 3(y + 4) - 31 = 0$

$8x + y = 108$ ....(2)

Solution (1) and (2)

$s = \left(\frac{25}{2}, 8\right)$

which lies on $2x - ky = 1$

$\boxed{k = 3}$

88. Suppose Anil's mother wants to give 5 whole fruits to Anil from a basket of 7 red apples, 5 white apples and 8 oranges. If in the selected 5 fruits, at least 2 oranges, at least one red apple and at least one white apple must be given, then the number of ways, Anil's mother can offer 5 fruits to Anil is

Sol. **6860**

Three cases are possible

1R 1W 3O + 2R 1W 2O + 1R 2W 2O

${}^7C_1 \cdot {}^5C_1 \cdot {}^8C_3 + {}^7C_2 \cdot {}^5C_1 \cdot {}^8C_2 + {}^7C_1 \cdot {}^5C_2 \cdot {}^8C_2$

= 6860

89. If $\int_{\frac{1}{3}}^{3} |\log_e x| dx = \frac{m}{n} \log_e \left(\frac{n^2}{e}\right)$, where m and n are coprime natural numbers, then $m^2 + n^2 - 5$ is equal to

Sol. **20**

$$\int_{\frac{1}{3}}^{3} |\log_e x| dx$$

$$= \int_{\frac{1}{3}}^{1} (-\ln x) dx + \int_{1}^{3} (\ln x) dx$$

$$-[x \ln x - x]_{\frac{1}{3}}^{1} + [x \ln x - x]_{1}^{3}$$

$$= \frac{4}{3} \ln\left(\frac{9}{e}\right) = \frac{m}{n} \ln\left(\frac{n^2}{e}\right)$$

m = 4 and n = 3

so $m^2 + n^2 - 5 = 16 + 9 - 5 = 20$

90. A triangle is formed by X- axis, Y-axis and the line 3x + 4y = 4y = 60. Then the number of points P(a, b) which lie strictly inside the triangle, where a is an integer and b is a multiple of a, is

Sol. **31**

3x + 4y = 60

x = 1, 4y = 57, y=14.2

x = 1, y = 1, 2, 3,…….14 $\rightarrow$ 14 points

x = 2, 4y = 54, y=13.5

x = 2, y = 2, 4, 6, 8, 10, 12 $\rightarrow$ 6 points

x = 3, y = 3, 6, 9, 12 $\rightarrow$ 4 points

x = 4, y = 4, 8 $\rightarrow$ 2 points

x = 5, y = 5, 10 $\rightarrow$ 2 points

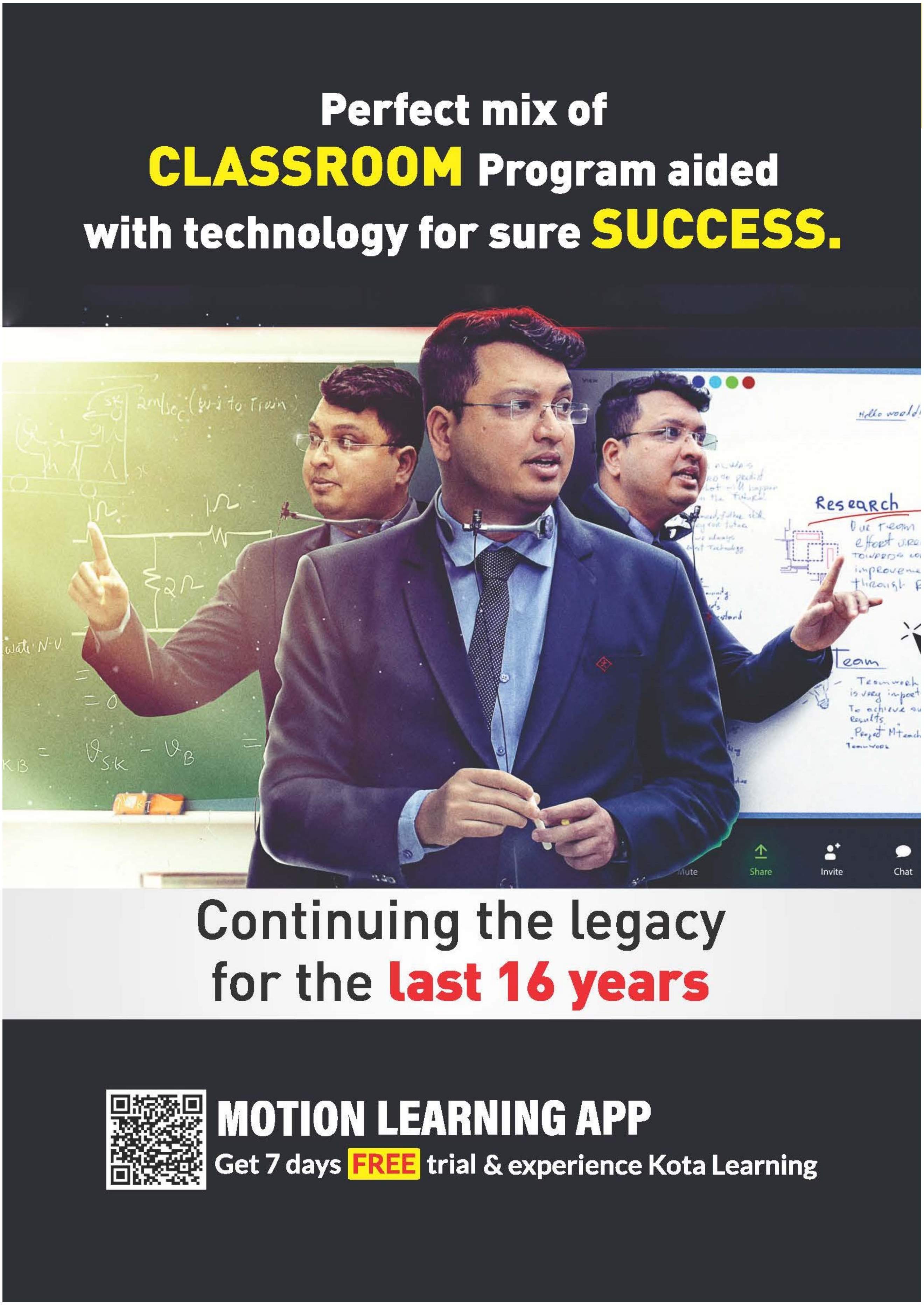
x = 6, y = 6 $\rightarrow$ 1 points

x = 7, y = 7 $\rightarrow$ 1 points

x = 8, y = 8 $\rightarrow$ 1 points

x = 9, 4y = 23, y=5.7 × no point

Total points = 14 + 6 + 4 + 2 + 2 + 1 + 1 + 1 = 31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Target: JEE/NEET 2024
Enthuse & प्रयास Batch
Class 11th to 12th Moving
Target: JEE/NEET 2024
Dropper & प्रयास Batch
Class 12th to 13th Moving
Target: PRE FOUNDATION
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