# JEE MAIN 2023
## Paper with Solution

**MATHS | 29th Jan 2023 _ Shift-2**

# Umeed Rank Ki Ho Ya Selection Ki, JEET NISCHIT HAI!

## Most Promising RANKS
Produced by MOTION Faculties

### NEET / AIIMS

**AIR-1 to 10**
25 Times

**AIR-11 to 50**
83 Times

**AIR-51 to 100**
81 Times

### JEE MAIN+ADVANCED

**AIR-1 to 10**
8 Times

**AIR-11 to 50**
32 Times

**AIR-51 to 100**
36 Times

## Nation's Best SELECTION
Percentage (%) Ratio

**Student Qualified in NEET**

(2022)
4837/5356 = **90.31%**
(2021)
3276/3411 = **93.12%**

**Student Qualified in JEE ADVANCED**

(2022)
1756/4818 = **36.45%**
(2021)
1256/2994 = **41.95%**

**Student Qualified in JEE MAIN**

(2022)
4818/6653 = **72.41%**
(2021)
2994/4087 = **73.25%**

NITIN VIIJAY (NV Sir)
Founder & CEO

## SECTION - A

**61.** The statement $B \Rightarrow ((\sim A) \vee B)$ is equivalent to :

(1) $A \Rightarrow (A \Leftrightarrow B)$
(2) $A \Rightarrow ((\sim A) \Rightarrow B)$
(3) $B \Rightarrow (A \Rightarrow B)$
(4) $B \Rightarrow ((\sim A) \Rightarrow B)$

**Sol.** **1, 3 or 4**

$B \Rightarrow (\sim A)\, VB$

| A | B | ~ A | ~ AVB | B ⇒ (~A) VB |
|---|---|---|---|---|
| T | T | F | T | T |
| T | F | F | F | T |
| F | T | T | T | T |
| F | F | T | T | T |

| A ⇒ B | ~A ⇒ B | B ⇒ A ⇒ B | A ⇒ ((~A) ⇒ B) | B ⇒ ((~A) ⇒ B) |
|---|---|---|---|---|
| T | T | T | T | T |
| F | T | T | T | T |
| T | T | T | T | T |
| T | F | T | T | T |

**62.** The value of the integral $\int_1^2 \left(\frac{t^4+1}{t^6+1}\right) dt$ is

(1) $\tan^{-1} 2 - \frac{1}{3}\tan^{-1} 8 + \frac{\pi}{3}$
(2) $\tan^{-1}\frac{1}{2} + \frac{1}{3}\tan^{-1} 8 - \frac{\pi}{3}$
(3) $\tan^{-1}\frac{1}{2} - \frac{1}{3}\tan^{-1} 8 + \frac{\pi}{3}$
(4) $\tan^{-1} 2 + \frac{1}{3}\tan^{-1} 8 - \frac{\pi}{3}$

**Sol.**

$$I = \int_1^2 \left(\frac{t^4+1}{t^6+1}\right) dt$$

$$\Rightarrow \int \frac{t^4+1-t^2+t^2}{(t^2+1)(t^4-t^2+1)} dt$$

$$\Rightarrow \int \frac{(t^4-t^2+1)+t^2}{(t^2+1)(t^4-t^2+1)} dt$$

$$\Rightarrow \int_1^2 \left[\frac{t^4-t^2+1}{(t^2+1)(t^4-t^2+1)} + \frac{t^2}{t^6+1}\right] dt$$

$$\Rightarrow \int_1^2 \frac{1}{t^2+1} dt + \frac{1}{3}\int_1^2 \frac{3t^2}{(t^3)^2+1} dt$$

$$\Rightarrow \left[\tan^{-1} t + \frac{1}{3}\tan^{-1}(t^3)\right]_1^2$$

$$\Rightarrow \tan^{-1} 2 + \frac{1}{3}\tan^{-1}(8) - \tan^{-1}(1) - \frac{1}{3}\tan^{-1}(1)$$

$$\Rightarrow \tan^{-1} 2 + \frac{1}{3}\tan^{-1} 8 - \frac{\pi}{4} - \frac{\pi}{4}.\frac{1}{3}$$

$\Rightarrow \tan^{-1}2+\frac{1}{3}\tan^{-1}8-\frac{3\pi+\pi}{12}$

$\Rightarrow \tan^{-1}2+\frac{1}{3}\tan^{-1}8-\frac{\pi}{3}$

**63.** The set of all values of $\lambda$ for which the equation $\cos^2 2x - 2\sin^4 x - 2\cos^2 x = \lambda$ has a real solution x, is

(1) $[-2,-1]$ (2) $\left[-1,-\frac{1}{2}\right]$ (3) $\left[-\frac{3}{2},-1\right]$ (4) $\left[-2,-\frac{3}{2}\right]$

**Sol.**

$\cos^2 2x - 2\left(\frac{1-\cos 2x}{2}\right)^2 - (1+\cos 2x) = \lambda$

$\Rightarrow \cos^2 2x - 2\left(\frac{1-\cos^2 2x - 2\cos 2x}{4}\right) - 1 - \cos 2x = \lambda$

Let cos2x = t

$\Rightarrow 2t^2 - 1 - t^2 + 2t - 2 - 2t = 2\lambda$

$\Rightarrow t^2 - 3 = 2\lambda$ $\because 0 \le t^2 \le 1$

$\Rightarrow t^2 = 2\lambda + 3$

$0 \le 2\lambda + 3 \le 1$

$-3 \le 2\lambda \le -2$

$\frac{-3}{2} \le \lambda \le -1$

**64.** Let R be a relation defined on $\mathbb{N}$ as a R b if $2a + 3b$ is a multiple of 5, $a, b \in \mathbb{N}$.
Then R is

(1) an equivalence relation (2)transitive but not symmetric
(3) not reflexive (4) symmetric but not transitive

**Sol.**

Reflexive

Let $a \in N$

a R a $\Rightarrow$ 2a + 3a is a multiple of 5

$\Rightarrow$ 5a which is a multiple of 5

$\Rightarrow$ R is reflexive

Symmetric

Let $a, b \in N$

a R b $\Rightarrow 2a + 3b = 5\lambda_1$ $\lambda_1 \in N$

b R a $\Rightarrow 2b + 3a = 5\lambda_2$ $\lambda_2 \in N$

On Adding

$(2a + 3b) + (2b + 3a) = 5(\lambda_1 + \lambda_2)$

$5a + 5b = 5(\lambda_1 + \lambda_2)$

$\Rightarrow$ Both sides are multiple of 5

$\Rightarrow$ R is symmetric

Transitive
Let a, b, c $\in$ N
a R b $\Rightarrow$ $2a + 3b = 5\lambda_1$ …(1)
b R c $\Rightarrow$ $2a + 3c = 5\lambda_2$ …(2)
$2a + 5b + 3c = 5(\lambda_1 + \lambda_2)$
$\Rightarrow (2a + 3c) = 5(\lambda_1 + \lambda_2 - b)$
2a + 3c is divisible by 5
$\Rightarrow$ a R c is true
$\Rightarrow$ R is transitive
R is Equivalence Relation

**65.** Consider a function $f: \mathbb{N} \to \mathbb{R}$, satisfying
$f(1) + 2f(2) + 3f(3) + \cdots + xf(x) = x(x+1)f(x); x \geq 2$ with $f(1) = 1$.
Then $\frac{1}{f(2022)} + \frac{1}{f(2028)}$ is equal to
(1) 8100 (2) 8400 (3) 8000 (4) 8200

**Sol.** **1**
$f(1) + 2f(2) + 3f(3) + \ldots\ldots + xf(x) = x^2f(x) + xf(x)$
$\Rightarrow f(1) + 2f(2) + 3f(3) + \ldots\ldots + (x-1)\, f(x-1) = x^2\, f(x)$
$x = 2 \quad f(1) = 2^2\, f(2) \Rightarrow f(2) = \frac{1}{4}$
$x = 3 \quad f(1) + 2f(2) = 3^2\, f(3)$
$$\Rightarrow f(3) = \frac{1}{9}\left(1+\frac{2}{4}\right) = \frac{1}{9}\times\frac{3}{2} = \frac{1}{6}$$
$x = 4 \quad f(1) + 2f(2) + 3f(3) = 4^2f(4)$
$$\Rightarrow f(4) = \left(1+\frac{1}{2}+\frac{1}{2}\right).\frac{1}{16} \Rightarrow f(4) = \frac{1}{8}$$
$x = 5 \quad f(1) + 2f(2) + 3f(3) + 4f(4) = 5^2f(5)$
$$f(5) = \left(1+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\right)\frac{1}{25} = \frac{5}{2}.\frac{1}{25} = \frac{1}{10}$$
In general $f(x) = \frac{1}{2x}$
$$\therefore \quad \frac{1}{f(2022)} + \frac{1}{f(2028)}$$
$$\frac{1}{\frac{1}{2\times 2022}} + \frac{1}{\frac{1}{2\times 2028}}$$
$\Rightarrow \quad 2[2022 + 2028]$
$\Rightarrow \quad 2 \times 4050$
$\Rightarrow \quad 8100$

**66.** If $\vec{a} = \hat{i} + 2\hat{k}, \vec{b} = \hat{i} + \hat{j} + \hat{k}, \vec{c} = 7\hat{i} - 3\hat{j} + 4\hat{k}, \vec{r} \times \vec{b} + \vec{b} \times \vec{c} = \vec{0}$ and $\vec{r} \cdot \vec{a} = 0$.
Then $\vec{r} \cdot \vec{c}$ is equal to
(1) 32 (2) 30 (3) 36 (4) 34

**Sol.** **4**

$\vec{r}\times\vec{b}+\vec{b}\times\vec{c}=0$

$\Rightarrow\ \vec{r}\times\vec{b}-\vec{c}\times\vec{b}=0$

$\Rightarrow\ (\vec{r}-\vec{c})\times\vec{b}=0$

$\vec{r}-\vec{c}\ ||\ \vec{b}$

$\vec{r}-\vec{c}=\lambda\vec{b}$

$\vec{r}=\lambda\vec{b}+\vec{c}$

$=\lambda(i+j+k)+(7i-3j+4k)$

$= i(\lambda + 7) + j(\lambda - 3) + k(\lambda + 4)$

$\vec{r}.\vec{a}=0$

$\Rightarrow (7 + \lambda) + 2(\lambda + 4) = 0$

$\Rightarrow 3\lambda = -15 \quad \Rightarrow \lambda = -5$

$\therefore\ \vec{r}=2i-8j-k$

$\vec{r}.\vec{c} = (2i - 8j - k) . (7i - 3j + 4k)$

$= 14 + 24 - 4 = 34$

**67.** The shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$ and $\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}$

(1) $5\sqrt{3}$ (2) $2\sqrt{3}$ (3) $3\sqrt{3}$ (4) $4\sqrt{3}$

**Sol.** **4**

$L_1=\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}=\lambda$

$L_2\ \frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}=\mu$

$S.D.=\left|\frac{(\vec{b}-\vec{a}).(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$ $\qquad \vec{a}=i-8j+4k$

$\vec{b}=i+2j+6k$

$\vec{b}_1\times\vec{b}_2=\begin{vmatrix} i & j & k \\ 2 & -7 & 5 \\ 2 & 1 & -3 \end{vmatrix}$

$= i(21\text{-}5) - j(-6 -10) + k(2 + 14)$

$= 16i + 16j + 16\ k$

$|\vec{b}_1\times\vec{b}_2|=|16(i+j+k)|$

$=16\times\sqrt{3}$

$\vec{b}-\vec{a}=(10j+2k)$

$S.D. = \left|\frac{(10j+2k).16(i+j+k)}{16\sqrt{3}}\right|$

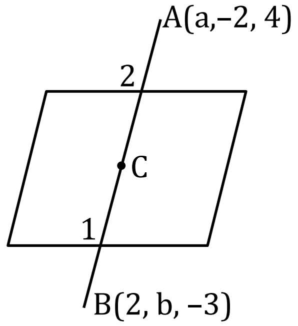
$=\left|\frac{16(10+2)}{16\sqrt{3}}\right|=\frac{12}{\sqrt{3}}\Rightarrow\frac{12}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$

$=\frac{12\sqrt{3}}{3}=4\sqrt{3}$

**68.** The plane $2x - y + z = 4$ intersects the line segment joining the points $A(a, -2, 4)$ and $B(2, b, -3)$ at the point C in the ratio $2:1$ and the distance of the point C from the origin is $\sqrt{5}$. If $ab < 0$ and P is the point $(a - b, b, 2b - a)$ then $CP^2$ is equal to

(1) $\frac{97}{3}$ (2) $\frac{17}{3}$ (3) $\frac{16}{3}$ (4) $\frac{73}{3}$

**Sol.** 2




C divides AB in 2 : 1

$$C\left(\frac{4+a}{3}, \frac{2b-2}{3}, \frac{-6+4}{3}\right)$$

$$C\left(\frac{a+4}{3}, \frac{2b-2}{3}, \frac{-2}{3}\right)$$

C lies in the plane

$$\therefore\ 2\left(\frac{a+4}{3}\right) - \left(\frac{2b-2}{3}\right) + \left(\frac{-2}{3}\right) = 4$$

$\Rightarrow 2a - 2b = 4$

$a - b = 2$

$\because OC = \sqrt{5}$

$OC^2 = 5$

$$C\left(\frac{b+6}{3}, \frac{2b-2}{3}, \frac{-2}{3}\right)$$

$$\Rightarrow \left(\frac{b+6}{3}\right)^2 + \left(\frac{2b-2}{3}\right)^2 + \left(\frac{-2}{3}\right)^2 = 5$$

$\Rightarrow 5b^2 + 4b - 1 = 0$

$\Rightarrow 5b^2 + 5b - b - 1 = 0$

$\Rightarrow 5b(b + 1) - 1(b + 1) = 0$

$b = -1\ \&\ \frac{1}{5}$

$a = 1$

$\because ab < 0$

$\therefore\ a = 1, b = -1$

Now,

$$C\left(\frac{5}{3}, \frac{-4}{3}, \frac{-2}{3}\right)$$

$P(a - b, b, 2b - a)$

$(2, -1, -3)$

$$CP^2 = \left(\frac{5}{3} - 2\right)^2 + \left(\frac{-4}{3} + 1\right)^2 + \left(\frac{-2}{3} + 3\right)^2 = \frac{17}{3}$$

**69.** The value of the integral $\int_{1/2}^{2} \frac{\tan^{-1} x}{x} dx$ is equal to

(1) $\frac{\pi}{2}\log_e 2$ (2) $\pi\log_e 2$ (3) $\frac{1}{2}\log_e 2$ (4) $\frac{\pi}{4}\log_e 2$

**Sol.**

Let $x = \frac{1}{t}$

$dx = -\frac{1}{t^2}dt$

$$I = \int_{2}^{1/2} \frac{\tan^{-1}\left(\frac{1}{t}\right)}{\frac{1}{t}} \times -\frac{1}{t^2}dt$$

$$\Rightarrow \int_{1/2}^{2} \frac{\cot^{-1}(t)}{t}\,dt$$

$$2I = \int_{1/2}^{2} \frac{\tan^{-1}x + \cot^{-1}x}{x}\,dx$$

$$\Rightarrow \int_{1/2}^{2} \frac{\pi/2}{x}\,dx$$

$$\Rightarrow \frac{\pi}{2}\left[\ell n\, x\right]_{1/2}^{2}$$

$$\Rightarrow \frac{\pi}{2}\left(\ell n\, 2 - \ell n \frac{1}{2}\right)$$

$$\Rightarrow \frac{\pi}{2}\left(\ell n\, 2 + \ell n\, 2\right)$$

$$2I = \pi \ell n 2$$

$$\Rightarrow I = \frac{\pi}{2}\ell n\, 2$$

**70.** The letters of the word OUGHT are written in all possible ways and these words are arranged as in a dictionary, in a series. Then the serial number of the word TOUGH is

(1) 84 (2) 79 (3) 89 (4) 86

**Sol.** **3**

G H O T U

The words start from G $|\underline{4}$

The words start from H $|\underline{4}$ $24\times3$

The words start from O $|\underline{4}$

T

Start form TG $|\underline{3}$

TH $|\underline{3}$ $6\times2$

TO

TOG $|\underline{2}$ $2\times2$

TOH $|\underline{2}$

TOU

TOUG $|\underline{1}$ 1

89

**71.** The set of all values of $t \in \mathbb{R}$, for which the matrix

$$\begin{bmatrix} e^t & e^{-t}(\sin t - 2\cos t) & e^{-t}(-2\sin t - \cos t) \\ e^t & e^{-t}(2\sin t + \cos t) & e^{-t}(\sin t - 2\cos t) \\ e^t & e^{-t}\cos t & e^{-t}\sin t \end{bmatrix}$$ is invertible, is

(1) $\mathbb{R}$ (2) $\left\{k\pi + \frac{\pi}{4}, k \in \mathbb{Z}\right\}$ (3) $\{k\pi, k \in \mathbb{Z}\}$ (4) $\left\{(2k+1)\frac{\pi}{2}, k \in \mathbb{Z}\right\}$

**Sol.** **(1)**

$$|A| = \begin{vmatrix} e^t & e^{-t}(s-2c) & e^{-t}(-2s-c) \\ e^t & e^{-t}(2s+c) & e^{-t}(s-2c) \\ e^t & e^{-t}c & e^{-t}s \end{vmatrix}$$

$$\Rightarrow \;= e^t.e^{-t}.e^{-t} \begin{vmatrix} 1 & s-2c & -2s-c \\ 1 & 2s+c & s-2c \\ 1 & c & s \end{vmatrix}$$

$R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$$= e^t \begin{vmatrix} 0 & -s-3c & -3s-c \\ 0 & 2s & -2c \\ 1 & c & s \end{vmatrix}$$

$\Rightarrow e^{-t}[1(2sc + 6c^2 + 6s^2 + 2sc)]$

$\Rightarrow e^{-t}[4sc + 6(c^2 + s^2)] \quad = e^{-t}(6 + 2\sin 2t)$

$\because 2\sin 2t \in [-2, 2]$

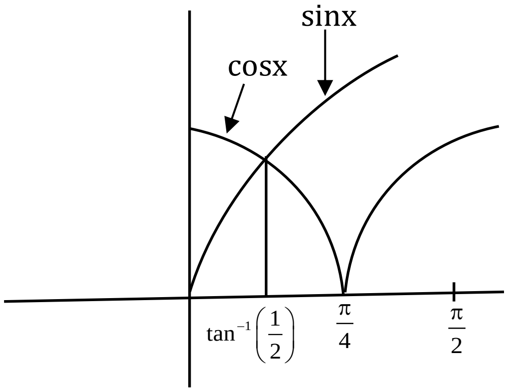
$\therefore e^{-t}(6 + 2\sin 2t) \neq 0 \quad \forall t \in R$

**72.** The area of the region $A = \left\{(x, y): |\cos x - \sin x| \le y \le \sin x, 0 \le x \le \frac{\pi}{2}\right\}$ is

(1) $\sqrt{5} + 2\sqrt{2} - 4.5$ (2) $1 - \frac{3}{\sqrt{2}} + \frac{4}{\sqrt{5}}$ (3) $\frac{3}{\sqrt{5}} - \frac{3}{\sqrt{2}} + 1$ (4) $\sqrt{5} - 2\sqrt{2} + 1$

**Sol.** **(4)**

$|\cos x - \sin x|$




A = area under the curve

y = sinx & above the curve |cosx – sinx|

$$A = \int_0^{\pi/2} (\sin x - |\cos x - \sin x|)\,dx$$

When 0 to $\frac{\pi}{4}$

$|\cos x - \sin x| = \cos x - \sin x$

$\sin x = \cos x - \sin x$

$2\sin x = \cos x$

$\tan x = \frac{1}{2}$

$x = \tan^{-1}\left(\frac{1}{2}\right)$

when $\frac{\pi}{4}$ to $\frac{\pi}{2}$

$|\cos x - \sin x| = \sin x - \cos x$

$\sin x = \sin x - \cos x$

$\cos x = 0$

$x = \frac{\pi}{2}$

$$A = \int_{\tan^{-1}(1/2)}^{\pi/4} \{\sin x - (\cos x - \sin x)\}\,dx + \int_{\pi/4}^{\pi/2} \{\sin x + (\cos x - \sin x)\}\,dx$$

$$\Rightarrow \int_{\tan^{-1}\left(\frac{1}{2}\right)}^{\pi/4}(2\sin x-\cos x)dx + \int_{\pi/4}^{\pi/2}\cos x\,dx$$

$$\Rightarrow (-2\cos x-\sin x)_{\tan^{-1}\left(\frac{1}{2}\right)}^{\pi/4}+(\sin x)_{\pi/4}^{\pi/2}$$

$$\Rightarrow \left(-2\times\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right)-\left\{-2\cos\left(\tan^{-1}\frac{1}{2}\right)-\sin\left(\tan^{-1}\frac{1}{2}\right)\right\}+\left(1-\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow -\frac{3}{\sqrt{2}}+2\times\frac{2}{\sqrt{5}}+\frac{1}{\sqrt{5}}+1-\frac{1}{\sqrt{2}} = \frac{-4}{\sqrt{2}}+\frac{5}{\sqrt{5}}+1 = -2\sqrt{2}+\sqrt{5}+1$$

**73.** The number of 3 digit numbers, that are divisible by either 3 or 4 but not divisible by 48, is

(1) 507 (2) 432 (3) 472 (4) 400

**Sol.** (2)

Nos div. by 3

102, 105, 108 ……… 999

A.P. a = 102 d = 3 $\ell$ = 999

$$n=\frac{\ell-a}{d}+1=\frac{999-102}{3}+1 = 300$$

Numbers div. by 4

100, 104, 108 ……… 996

A.P. a = 100 d = 4 $\ell$ = 996

$$n=\frac{996-100}{4}+1=\frac{896}{4}+1 = 224 + 1 = 225$$

Numbers div. by 12

108, 120, ……… 996

A.P. a = 108 d = 12 $\ell$ = 996

$$n=\frac{996-108}{12}+1=\frac{888}{12}+1 = 74 + 1 = 75$$

Numbers div. by 48

144, 192, ……… 960

A.P. a = 144 d = 48 $\ell$ = 960

$$n=\frac{996-144}{12}+1=\frac{816}{48}+1 = 17 + 1 = 18$$

$\therefore$No. Div. by 354 but not by 48

300 + 225 – 75 – 18

= 450 – 18 = 432

**74.** If the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z+3}{1}$ and $\frac{x-a}{2}=\frac{y+2}{3}=\frac{z-3}{1}$ intersect at the point $P$, then the distance of the point $P$ from the plane $z=a$ is :

(1) 28 (2) 16 (3) 10 (4) 22

**Sol.** 1

Let $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z+3}{1}=\lambda$

$P(\lambda + 1, 2\lambda + 2, \lambda - 3)$

& $\frac{x-a}{2}=\frac{y+2}{3}=\frac{z-3}{1}=\mu$

$P(2\mu + a, 3\mu - 2, \mu + 3)$

$\lambda + 1 = 2\mu + a$
$22 + 1 = 2 \times 16 + a$
$a = 23 - 32$
$a = -9$

$2\lambda + 2 = 3\mu - 2$
$2(\mu + 6) + 2 = 3\mu - 2$
$2\mu + 12 = 3\mu - 4$
$\mu = 16$
$\therefore \lambda = 22$

$\lambda - 3 = \mu + 3$
$\lambda = \mu + 6$

$\therefore P(23, 46, 19)$

Plane is $z = a$
$z = -9$

The distance of p from $z = -9$ is $19 - (-9) = 28$

**75.** Let $y = y(x)$ be the solution of the differential equation $x\log_e x\frac{dy}{dx} + y = x^2\log_e x, (x > 1)$. If $y(2) = 2$, then $y(e)$ is equal to

(1) $\frac{1+e^2}{2}$ (2) $\frac{4+e^2}{4}$ (3) $\frac{2+e^2}{2}$ (3) $\frac{1+e^2}{4}$

**Sol.** 2

D.E. $\frac{dy}{dx}+\frac{1}{x\log x}y = x$

Linear diff. $eq^n$ $\frac{dy}{dx}+py = Q$

$If = e^{\int Pdx}$

$= e^{\int \frac{1}{x\log x}dx}$ $\log x = t$

$\frac{1}{x}dx = dt$

$= e^{\int \frac{1}{t}dt} = e^{\ell nt} = t$

I.F. $= \ell nx$

Solution of DE.

$y. If = \int q(If)\,dx + c$

$y.\ell nx = \int x.(\ell n x)dx + c$

$= \ell nx . \frac{x^2}{2}\ell nx - \frac{x^2}{4} + C$

At $x = 2$, $y = 2$

$2\ell n2 = \frac{4}{2}\ell n 2 - \frac{4}{4} + C \Rightarrow C = 1$

$\therefore \quad y\ell nx = \frac{x^2}{2}\ell n x - \frac{x^2}{4} + 1$

At x = e

$$y(e)\,\ell ne = \frac{e^2}{2}\ell n e - \frac{e^2}{4} + 1$$

$$y(e) = \frac{e^2}{4} + 1$$

**76.** Let $f$ and $g$ be twice differentiable functions on $\mathbb{R}$ such that

$f''(x) = g''(x) + 6x$

$f'(1) = 4g'(1) - 3 = 9$

$f(2) = 3g(2) = 12.$

Then which of the following is NOT true?

(1) There exists $x_0 \in (1,3/2)$ such that $f(x_0) = g(x_0)$

(2) $|f'(x) - g'(x)| < 6 \Rightarrow -1 < x < 1$

(3) If $-1 < x < 2$, then $|f(x) - g(x)| < 8$

(4) $g(-2) - f(-2) = 20$

**Sol.** **3**

Let $F(x) = f(x) - g(x)$

Given $f'(x) = g''(x) + 6x$

$$f'(x) = g'(x) + \frac{6x^2}{2} + c_1$$

$x = 1 \quad f'(1) = g'(1) + 3 \times (1)^2 + c_1$

$9 = 3 + 3 + c_1$

$c_1 = 3$

$\therefore \quad f'(x) = g'(x) + 3x^2 + 3$

$$f(x) = g(x) + \frac{3x^3}{3} + 3x + c_2$$

$x = 2 \quad f(2) = g(2) + (2)^3 + 3(2) + c_2$

$12 = 4 + 8 + 6 + c_2$

$c_2 = -6$

$\therefore \quad f'(x) = g(x) + x^3 + 3x - 6$

$= x^3 + 3x - 6$

Option (1)

$x_0 \in \left(1, \frac{3}{2}\right)$ such that $f(x_0) = 9(x_0)$

$\because \quad F(1) = f(1) - g(1)$ $\qquad F\left(\frac{3}{2}\right) = f\left(\frac{3}{2}\right) - g\left(\frac{3}{2}\right)$

$= 1 + 3 - 6 = -2$ $\qquad = (2)^3 + 3(2) - 6$

$= 8 + 6 - 6 = 8$

$\because \quad F(1)\ F\left(\frac{3}{2}\right) < 0$

$\Rightarrow$ At least one root of $F(x) = 0$ lies in $\left(1, \frac{3}{2}\right)$

$\Rightarrow \quad f(x) - g(x) = 0$

$\Rightarrow \quad f(x) = g(x)$

Option (2)

$|f'(x) - g'(x)| < 6 \Rightarrow -1 < x < 1$

$F'(x) = x^3 + 3x - 6$

$F'(x) = 3x^2 + 3$

$f'(x) - g'(x) = 3x^2 + 3$

$|f'(x) - g'(x)| < 6$

$\Rightarrow$ $3x^2 + 3 < 6$

$\Rightarrow$ $3x^2 <x\ 3$

$x^2 < 1$

$\Rightarrow$ $x \in (-1, 1)$

Option (3)

If $-1 < x < 2$ then $|f(x) - g(x)| < 8$

$F(x) = x^3 + 3x - 6$

$F(-1) = -1 -3 - 6 = -10$ But $|f'(x) - g'(x)| < 10$

$F(2) = (2)^3 + 3(2) - 6 = 8$

Option is not true

Option (4)

$g(-2) - f(-2) = 20$

$F(-2) = f(-2) - g(-2)$

$= (-2)^3 + 3(-2) - 6$

$-8 -6 -6 = -20$

$g(-2) - f(-2) = 20$

**77.** If the tangent at a point $P$ on the parabola $y^2 = 3x$ is parallel to the line $x + 2y = 1$ and the tangents at the points $Q$ and $R$ on the ellipse $\frac{x^2}{4} + \frac{y^2}{1} = 1$ are perpendicular to the line $x - y = 2$, then the area of the triangle PQR is :

(1) $\frac{3}{2}\sqrt{5}$ (2) $3\sqrt{5}$ (3) $\frac{9}{\sqrt{5}}$ (4) $5\sqrt{3}$

**Sol.** **2**

$x + 2y = 1$

$m = -\frac{1}{2}$

$y^2 = 3x$

$T_p : y = -\frac{1}{2}x + \frac{\frac{3}{4}}{-\frac{1}{2}}$

$y = -\frac{x}{2} - \frac{3}{2}$

$2y + x + 3 = 0$ …(1)

$x - y = 2$

$m = 1$

slope of tangent at Q & R is $-1$

$E : \frac{x^2}{4} + \frac{y^2}{1} = 1$

$y = -x \pm \sqrt{(-1)^2 4 + 1}$

$y = -x \pm \sqrt{5}$

$x + y = \sqrt{5}$ …(2) $x + y = -\sqrt{5}$ …(3)

Point P :

T = O

$yy_1 = \frac{3}{2}(x+x_1)$

$3x - 2yy_1 + 3x_1 = 0$

Comparison with (1)

$\frac{3}{1} = \frac{-2y_1}{2} = \frac{3x_1}{3}$

$y_1 = -3, \quad x_1 = 3$

Point Q :

$\frac{xx_2}{4} + \frac{yy_2}{1} = 1$

$xx_2 + 4yy_2 - 4 = 0$

$\frac{x_2}{1} = \frac{4y_2}{1} = \frac{-4}{-\sqrt{5}}$

$x_2 = \frac{4}{\sqrt{5}} - y_2 = \frac{1}{\sqrt{5}}$

Point R:

$\frac{x_2}{1} = \frac{4y_2}{1} = \frac{-4}{\sqrt{5}}$

$x_2 = \frac{-4}{\sqrt{5}}, \ y = \frac{1}{\sqrt{5}}$

Area of ΔPQR

$$= \frac{1}{2}\begin{vmatrix} 3 & -3 & 1 \\ \frac{4}{\sqrt{5}} & \frac{1}{\sqrt{5}} & 1 \\ -\frac{4}{\sqrt{5}} & -\frac{1}{\sqrt{5}} & 1 \end{vmatrix}$$

$$\Rightarrow \frac{1}{2}\left[3\left(\frac{1}{\sqrt{5}} + \frac{1}{\sqrt{5}}\right) + 3\left(\frac{4}{\sqrt{5}} + \frac{4}{\sqrt{5}}\right) + 1\left(-\frac{4}{5} + \frac{4}{5}\right)\right]$$

$$\Rightarrow \frac{1}{2}\left[\frac{6}{\sqrt{5}} + \frac{24}{\sqrt{5}}\right]$$

$$\Rightarrow \frac{1}{2} \times \frac{30}{\sqrt{5}} = \frac{5\times 3}{\sqrt{5}} = 3\sqrt{5}$$

**78.** Let $\vec{a} = 4\hat{\imath} + 3\hat{\jmath}$ and $\vec{b} = 3\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$. If $\vec{c}$ is a vector such that $\vec{c}\cdot(\vec{a}\times\vec{b}) + 25 = 0, \vec{c}\cdot(\hat{\imath} + \hat{\jmath} + \hat{k}) = 4$, and projection of $\vec{c}$ on $\vec{a}$ is 1 , then the projection of $\vec{c}$ on $\vec{b}$ equals

(1) $\frac{1}{5}$ (2) $\frac{5}{\sqrt{2}}$ (3) $\frac{3}{\sqrt{2}}$ (4) $\frac{1}{\sqrt{2}}$

**Sol.** **(2)**

Let $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

$\vec{c}\ .\ \vec{a}\times\vec{b}$

$$\begin{vmatrix} c_1 & c_2 & c_3 \\ 4 & 3 & 0 \\ 3 & -4 & 5 \end{vmatrix} = -25$$

$\Rightarrow c_1(15-0) - c_2(20-0) + c_3(-16-9) = -25$

$\Rightarrow 15c_1 - 20c_2 - 25c_3 = -25$

$\Rightarrow 3c_1 - 4c_2 - 5c_3 = -5$ …(2)

$\vec{c}\cdot(i+j+k) = 4$

$c_1 + c_2 + c_3 = 4$ …(i)

Proj. of $\vec{c}$ on $\vec{a} = \frac{\vec{a}.\vec{c}}{|\vec{a}|} = 1$

$\Rightarrow \frac{(4\hat{i}+3\hat{j})(c_1\hat{i}+c_2\hat{j}+c_3\hat{k})}{\sqrt{16+9}} = 1$

$\Rightarrow 4c_1 + 3c_2 = 5$

$\Rightarrow 4c_1 = 5 - 3c_2$

$\Rightarrow c_1 = \frac{5-3c_2}{4}$ ...(3)

Eq$^n$. (1) & (3)

$\frac{5-3c_2}{4} + c_2 + c_3 = 4$

$5 - 3c_2 + 4c_2 + 4c_3 = 16$

$c_2 + 4c_3 = 11$ ...(4)

Eq$^n$. (2) & (3)

$3\left(\frac{5-3c_2}{4}\right) - 4c_2 - 5c_3 = -5$

$15 - 9c2 - 16c2 - 20c_3 = -20$

$-25c_2 - 20c_3 = -35$ ...(5)

Eq$^n$. (4) & (5)

$-25c_2 - 20c_3 = -35$

$-25(11 - 4c_3) - 20c_3 = -35$

$5(11 - 4c_3) + 4c_3 = 7$

$55 - 20c_3 + 4c_3 = 7$

$-16c_3 = -48$

$c_3 = 3$

$c_2 = 11 - 4c_3$

$c_2 = 11 - 4 \times 3$

$= 11 - 12$

$c_2 = -1$

$c_1 = \frac{5-3c_2}{4}$

$= \frac{5-3(-1)}{4}$

$c_1 = 2$

$\vec{c} = 2\hat{i} - \hat{j} + 3\hat{k}$

Projection of $\vec{c}$ on $\vec{b}$ = $\left|\frac{\vec{c}.\vec{b}}{|\vec{b}|}\right|$

$\Rightarrow \left|\frac{(2\hat{i}-\hat{j}+3\hat{k}).(3\hat{i}-4\hat{j}+5\hat{k})}{\sqrt{9+16+25}}\right|$

$\Rightarrow \left|\frac{6+4+15}{5\sqrt{2}}\right| = \frac{25}{5\sqrt{2}} = \frac{5}{\sqrt{2}}$

**79.** Let $S = \{w_1, w_2, \ldots \ldots\}$ be the sample space associated to a random experiment. Let $P(w_n) = \frac{P(w_{n-1})}{2}, n \geq 2$. Let $A = \{2k + 3l : \boldsymbol{k}, l \in \mathbb{N}\}$ and $B = \{w_n : n \in A\}$. Then $P(B)$ is equal to

(1) $\frac{3}{64}$ (2) $\frac{1}{16}$ (3) $\frac{1}{32}$ (4) $\frac{3}{32}$

**Sol.** **1**

A = {5, 7, 8, 9, 10, 11 ......}

$P(W_1) + P(W_2) + P(W_3) + \ldots\ldots P(W_n) = 1$

$P(W_1) + \frac{P(W_1)}{2} + \frac{P(W_2)}{2^2} + \ldots\ldots = 1$

$\Rightarrow P(W_1).\left(\frac{1}{1-1/2}\right) = 1$

$\boxed{P(W_1)=\frac{1}{2}}$ $\qquad P(W_n)=\frac{1}{2}\cdot\left(\frac{1}{2}\right)^{n-1}=\frac{1}{2^n}$

$\because B = \{W_n : n \in A\}$

$= \{W_5, W_7, W_8, \ldots\ldots.\}$

$P(B) = P(W_5) + P(W_7) + P(W_8) + P(W_9) + P(W_{10}) + P(W_{11})$

$$=\frac{1}{2^5}+\frac{1}{2^7}+\frac{1}{2^8}+\ldots..$$

$$=\frac{1}{32}+\frac{\frac{1}{2^7}}{1-\frac{1}{2}}$$

$$=\frac{1}{32}+\frac{1}{2^7}\times 2$$

$$=\frac{1}{32}+\frac{1}{64}=\frac{2+1}{64}=\frac{3}{64}$$

**80.** Let $K$ be the sum of the coefficients of the odd powers of $x$ in the expansion of $(1+x)^{99}$. Let $a$ be the middle term in the expansion of $\left(2+\frac{1}{\sqrt{2}}\right)^{200}$. If $\frac{{}^{200}C_{99}\,K}{a}=\frac{2^l\,m}{n}$, where $m$ and $n$ are odd numbers, then the ordered pair $(l, n)$ is equal to

(1) (50,51) (2) (50,101) (3) (51,99) (4) (51,101)

**Sol.** **(2)**

$$K=\frac{(1+1)^{99}}{2}=2^{98}$$

$$a={}^{200}C_{100}\,2^{100}\times\frac{1}{\left(\sqrt{2}\right)^{100}}$$

$$a={}^{200}C_{100}\,2^{50}$$

$$\frac{{}^{200}C_{99}.K}{a}=\frac{{}^{200}C_{99}.2^{98}}{{}^{200}C_{100}.2^{50}}$$

$$\because \frac{{}^{200}C_{99}}{{}^{200}C_{100}}=\frac{|200}{|99|101}\times\frac{|100|100}{|200}$$

$$=\frac{100}{101}$$

$$\therefore \frac{{}^{200}C_{99}K}{a}=\frac{100}{101}\times 2^{48}$$

$$=\frac{25\times 2^{50}}{101}$$

$\ell = 50$ , $n = 101$

$(\ell, n) = (50, 101)$

## Section B

**81.** The total number of 4-digit numbers whose greatest common divisor with 54 is 2 , is

**Sol.** **3000**

$54 = 2 \times 3^3$

4 digit even numbers are $\underset{4500}{\boxed{\overset{9}{\ }}\boxed{\overset{10}{\ }}\boxed{\overset{10}{\ }}\boxed{\overset{5}{\ }}}$

4 digit even numbers which are multiple of 3

Are the numbers which are multiple of 6

$= \frac{9000}{6} = 1500$

$\therefore$ The no. which has GCD with 54 us 2 is 4500 – 1500 = 3000

**82.** Let $a_1 = b_1 = 1$ and $a_n = a_{n-1} + (n-1), b_n = b_{n-1} + a_{n-1}, \forall n \geq 2$. If $S = \sum_{n=1}^{10} \frac{b_n}{2^n}$ and $T = \sum_{n=1}^{8} \frac{n}{2^{n-1}}$, then $2^7(2S - T)$ is equal to

**Sol.** **461**

$$T = \frac{1}{2^0} + \frac{2}{2^1} + \frac{3}{2^2} + \frac{4}{2^3} + \ldots\ldots\ldots\ldots \frac{8}{2^7} \quad \ldots\ldots(1)$$

$$S = \frac{b_1}{2} + \frac{b_2}{2^2} + \frac{b_3}{2^3} + \frac{b_4}{2^4} + \ldots\ldots\ldots\ldots \frac{b_{10}}{2^{10}}$$

$$\frac{S}{2} = \frac{b_1}{2^2} + \frac{b_2}{2^3} + \frac{b_3}{2^4} + \ldots\ldots\ldots\ldots \frac{b_9}{2^{10}} + \frac{b_{10}}{2^{11}} \text{ (Subtract)}$$

---

$$\frac{S}{2} = \frac{b_1}{2} + \left(\frac{b_2 - b_1}{2^2}\right) + \left(\frac{b_3 - b_2}{2^3}\right) + \left(\frac{b_4 - b_3}{2^4}\right) + \ldots\ldots + \left(\frac{b_{10} - b_9}{2^{10}}\right) - \frac{b_{10}}{2^{11}}$$

$$\frac{S}{2} = \frac{b_1}{2} + \frac{a_1}{2^2} + \frac{a_2}{2^3} + \frac{a_3}{2^4} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots \frac{a_9}{2^{10}} - \frac{b_{10}}{2^{11}}$$

$$\Rightarrow S = b_1 + \frac{a_1}{2} + \frac{a_2}{2^2} + \frac{a_3}{2^3} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots \frac{a_9}{2^{10}} - \frac{b_{10}}{2^{10}}$$

$$S = \left(b_1 - \frac{b_{10}}{2^{10}}\right) + \left(\frac{a_1}{2} + \frac{a_2}{2^2} + \ldots\ldots\ldots \frac{a_9}{2^9}\right)$$

$$\Rightarrow \frac{S}{2} = \left(\frac{b_1}{2} - \frac{b_{10}}{2^{11}}\right) + \left(\frac{a_1}{2^2} + \frac{a_2}{2^3} + \ldots\ldots\ldots \frac{a_8}{2^9} + \frac{a_9}{2^{10}}\right) \text{ (Subtract)}$$

---

$$\frac{S}{2} = \frac{b_1}{2} - \frac{b_{10}}{2^{11}} + \left(\frac{a_1}{2} - \frac{a_9}{2^{10}}\right) + \left(\frac{1}{2^2} + \frac{2}{2^3} + \ldots\ldots\ldots\ldots\ldots \frac{8}{2^9}\right)$$

$$= \frac{b_1}{2} + \frac{a_1}{2} - \left(\frac{b_{10} + 2a_9}{2^{11}}\right) + \frac{T}{4} \qquad \text{from } \ldots\ldots\ldots(1)$$

Now $2s = 2(a_1 + b_1) - \left(\frac{b_{10} + 2a_9}{2^9}\right) + T$

$2S - T = 2(a_1 + b_1) - \left(\frac{b_{10} + 2a_9}{2^9}\right)$

$$\boxed{2^7(2S-T)=2^8(a_1+b_1)-\frac{b_{10}+2a_9}{4}} \qquad \ldots\ldots\ldots\ldots\ldots (2)$$

$\because a_n - a_{n-1} = n-1$

$a_2 - a_1 = 1$

$a_3 - a_2 = 2$

$a_4 - a_3 = 3$

……

……

$a_9 - a_8 = 8$

$a_9 - a_1 = 1 + 2 + 3 + \ldots\ldots\ldots\ldots + 8$

$\boxed{a_9 = 36 + 1 = 37}$

and $b_n = b_{n-1} = a_{n-1}$

$b_{10} - b_1 = a_1 + a_2 + a_3 + \ldots\ldots + a_9$

$b_{10} - 1 = 1 + 2 + 4 + 7 + 11 + 16 + 22 + 29 + 37$

$b_{10} - 1 = 29$

$\boxed{b_{10} = 130}$

$$\boxed{\begin{aligned}&2^7(2S-T)=2^8\times(1+1)-\frac{130+2\times 37}{4}\\&=2^9-\frac{102}{2}\\&=512-51=461\end{aligned}}$$

**83.** A triangle is formed by the tangents at the point $(2,2)$ on the curves $y^2 = 2x$ and $x^2 + y^2 = 4x$, and the line $x + y + 2 = 0$. If $r$ is the radius of its circumcircle, then $r^2$ is equal to

**Sol.** **10**

Tangent at $y^2 = 2x$

$T: 2y = 2\left(\frac{x+2}{2}\right)$

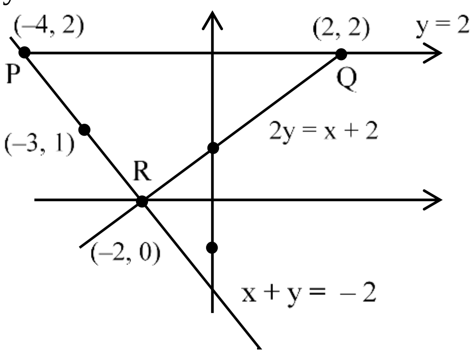
$2y = x + 2$

Tangent at $x^2 + y^2 = 4x$

$2x + 2y = \frac{4\times(x+2)}{2}$

$2x + 2y = 2x + 4$

$y = 2$



$M_{PR} = -1$
Slope of $\perp^{r\ell}$ Bisector = 1
$y - 1 = 1(x + 3)$
$y = x + 3 + 1$
$y = x + 4$
$\perp^{r\ell}$ Bisector of PQ
$x = -1$
$\therefore$ Centre is
$y = -1 + 4 = 3$
$(-1, 3)$
Radius: $r = \sqrt{(-1+4)^2 + (3-2)^2}$
$= \sqrt{9+1}$
$= \sqrt{10}$
$r^2 = 10$

**84.** Let $\alpha_1, \alpha_2, \ldots, \alpha_7$ be the roots of the equation $x^7 + 3x^5 - 13x^3 - 15x = 0$ and $|\alpha_1| \geq |\alpha_2| \geq \cdots \geq |\alpha_7|$. Then $\alpha_1\alpha_2 - \alpha_3\alpha_4 + \alpha_5\alpha_6$ is equal to

**Sol.** **3**

$\alpha_1, \alpha_2, \ldots \alpha_7$
$x^7 + 3x^5 - 13x^3 - 15x = 0$
$x(x^6 - 3x^4 - 13x^2 - 15) = 0$
$|\alpha_1| \geq |\alpha_2| \geq \ldots \geq |\alpha_7|$
$\alpha_1\alpha_2 - \alpha_3\alpha_4 + \alpha_3\alpha_6 = ?$
$\boxed{\alpha_7 = 0}$
$\Rightarrow x(x^6 + 3x^4 - 13x^2 - 15) = 0$
$x = 0$ $x^6 + 3x^4 - 13x^2 - 15 = 0$
$\Rightarrow t^3 + 3t^2 - 13t - 15 = 0$
$\Rightarrow (t - 3)(t^2 + 6t + 5) = 0$
$t = 3, t = -5, -1$
$x = 0, \ x = \pm\sqrt{3}, \ x = \pm\sqrt{5}i, \ x = \pm i$
$\alpha_1 = \sqrt{5}i$
$\alpha_2 = -\sqrt{5}i$
$\alpha_3 = \sqrt{3}$
$\alpha_4 = -\sqrt{3}$
$\alpha_5 = i$
$\alpha_6 = -i$
$\alpha_7 = 0$
$\alpha_1\alpha_2 = 5, \ \alpha_3\alpha_4 = 3, \ \alpha_5\alpha_6 = 1$
$\alpha_1\alpha_2 - \alpha_3\alpha_4 + \alpha_5\alpha_6$
$\Rightarrow 5 - 3 + 1 = 3$

**85.** Let $X = \{11,12,13,\ldots,40,41\}$ and $Y = \{61,62,63,\ldots,90,91\}$ be the two sets of observations. If $\bar{x}$ and $\bar{y}$ are their respective means and $\sigma^2$ is the variance of all the observations in $X \cup Y$, then $|\bar{x}+\bar{y}-\sigma^2|$ is equal to

**Sol.** **603**

$$\bar{x} = \frac{11+12+\ldots\ldots+41}{31} \qquad \bar{y} = \frac{61+62+63+\ldots..+91}{31}$$

$$= \frac{\frac{31}{2}(11+41)}{31} = \frac{52}{2} = 26 \qquad = \frac{\frac{31}{2}(61+91)}{31} = \frac{152}{2} = 76$$

$$\sigma^2 = \frac{\sum x_i^2 + \sum y_i^2}{31+31} - \bar{x}^2$$

$$= \frac{\left(\sum_{n=1}^{41} n^2 - \sum_{n=1}^{10} n^2\right) + \left(\sum_{n=1}^{91} n^2 - \sum_{n=1}^{60} n^2\right)}{62} - \left(\frac{31\times 26 + 76\times 31}{62}\right)^2$$

$$\Rightarrow \frac{\frac{41\times42\times83}{6} - \frac{10\times11\times21}{6} + \frac{91\times92\times183}{6} - \frac{60\times61\times121}{6}}{62} - (51)^2$$

$$\Rightarrow \frac{7(41\times83-55)+61(91\times46-1210)}{62}$$

$$\Rightarrow \frac{7(3403-55)+61(4186-1210)}{62}$$

$$\Rightarrow \frac{7\times3348+61\times2976}{62} - 2601$$

$\Rightarrow 3306 - 2601 = 705 \Rightarrow \sigma^2 = 705$

$\therefore |\bar{x}+\bar{y}-\sigma^2| = |26 + 76 - 705| = 603$

**86.** If the equation of the normal to the curve $y = \frac{x-a}{(x+b)(x-2)}$ at the point $(1,-3)$ is $x - 4y = 13$, then the value of $a + b$ is equal to

**Sol.** **– 6**

(1, –3) is on the curve

$$\therefore -3 = \frac{1-a}{(1+b)(1-2)} \Rightarrow -3 = \frac{1-a}{(-1)(1+b)}$$

$\Rightarrow 3 + 3b = 1 - a \Rightarrow a + 3b = -2$

$a = -2 - 3b$

$\ell ny = \ell n(x - a) - \ell n(x + b) - \ell n(x - 2)$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x-a} - \frac{1}{x+b} - \frac{1}{x-2}$$

$$\left.\frac{dy}{dx}\right|_{(1,-3)} = -3\left(\frac{1}{1-a} - \frac{1}{1+b} - \frac{1}{1-2}\right) = -4$$

$$\Rightarrow \left(\frac{1}{1+2+3b} - \frac{1}{1+b} + 1\right) = \frac{4}{12} = \frac{1}{3}$$

$\Rightarrow \dfrac{1}{3(b+1)} - \dfrac{1}{b+1} = \dfrac{1}{3} - 1$

$\Rightarrow \dfrac{1-3}{3(b+1)} = -\dfrac{2}{3}$

$\Rightarrow$ b + 1 = 3

b = 2

a = – 2– 3b
a = –2 – 3 × 2
a = – 2 – 6 = – 8

a + b
$\Rightarrow$ –8 + 2
$\Rightarrow$ – 6

**87.** Let $A$ be a symmetric matrix such that $|A| = 2$ and $\begin{bmatrix} 2 & 1 \\ 3 & \frac{3}{2} \end{bmatrix} A - \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$.

If the sum of the diagonal elements of $A$ is $s$, then $\frac{\beta s}{\alpha^2}$ is equal to

**Sol.** 5

A be a symmetric matrix such that |A| = 2 and $\begin{bmatrix} 2 & 1 \\ 3 & 3/2 \end{bmatrix} A = \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$

$A = \begin{bmatrix} a & b \\ b & d \end{bmatrix}$ $\quad$ $|A| = ad - b^2 = 2$

$\begin{bmatrix} 2 & 1 \\ 3 & 3/2 \end{bmatrix}\begin{bmatrix} a & b \\ b & d \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$

$\begin{bmatrix} 2a+b & 2b+d \\ 3a+3/2\,b & 3b+3/2\,d \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$

2a + b = 1
b = 1 – 2a

2b + d = 2
d = 2 – 2b
= 2 – 2(1 – 2a)
= 2 – 2 +4a

$ad - b^2 = 2$

$a.4a - (1-2a)^2 = 2$

$\Rightarrow 4a^2 - 1 - 4a^2 + 4a = 2$

4a = 3

$a = \dfrac{3}{4}$

$b = 1 - 2 \times \dfrac{3}{4}$

$= \dfrac{-1}{2}$

$d = 4 \times \dfrac{3}{4} = 3$

Now $\alpha = 3a + \dfrac{3}{2}b$

$= \dfrac{9}{4} + \dfrac{3}{2}\cdot\left(\dfrac{-1}{2}\right)$

$= \dfrac{9-3}{4} = \dfrac{6}{4} = \dfrac{3}{2}$

$\beta = 3b + \dfrac{3}{2}d$

$3 \times \left(\dfrac{-1}{2}\right) + \dfrac{3}{2} \times 3$

$\dfrac{-3+9}{2} = 3$

$$A=\begin{bmatrix} 3/4 & -1/2 \\ -1/2 & 3 \end{bmatrix} \quad s=\frac{3}{4}+3=\frac{15}{4}$$

$$\frac{Bs}{\alpha^2}=\frac{3\times\frac{15}{4}}{\frac{9}{4}}=5$$

**88.** Let $\alpha=8-14i, A=\left\{z\in\mathbb{C}:\frac{\alpha z-\bar{\alpha}\bar{z}}{z^2-(\bar{z})^2-112i}=1\right\}$ and $B=\{z\in\mathbb{C}:|z+3i|=4\}$.

Then $\sum_{z\in A\cap B}(Re\,z-Im\,z)$ is equal to

**Sol.** **14**

$$\alpha=8-14i \quad A=\left\{z\in c:\frac{\alpha z-\overline{\alpha z}}{z^2-\left(\bar{z}\right)^2-112i}=1\right\}$$

$z^2-\bar{z}^2$

$\Rightarrow\left(z+\bar{z}\right)\left(z-\bar{z}\right)\Rightarrow 2x\cdot 2yi=4xyi$

$\alpha z=(8-14i)(x+iy)$

$= 8x-8iy+14xi+14y$

$=(8x+14y)+i(8y-14x)$

$\overline{\alpha z}=(8+14i)(x-iy)$

$=8x-8iy+14ix+14y$

$=(8x+14y)+i(14x-8y)$

$$\frac{\alpha z-\overline{\alpha z}}{z^2-\bar{z}^2-112i}=1$$

$$\Rightarrow\frac{(16y-28x)i}{4xyi-112i}=1$$

$\Rightarrow 4y-7x=xy-28$

$\Rightarrow 4y-7x-xy+28=0$

$\Rightarrow y(4-x)-7(x-4)=0$

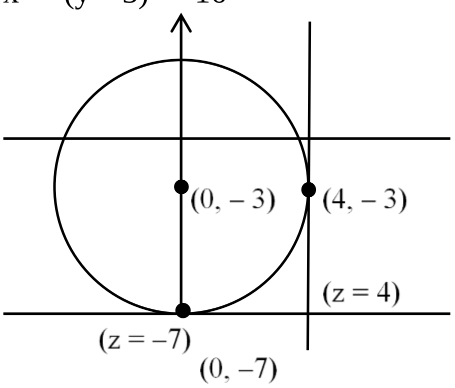
$(x-4)(-y-7)=0$

$x=4 \quad y=-7$

$z=4$ or $z=-7i$

$B\Rightarrow|z+3i|=4$

$x^2+(y-3)^2=16$




$\sum Re(z)-Im(z)$

$= (0 + 4) - (-7 - 3)$
$= 4 + 10 = 14$

**89.** A circle with centre (2,3) and radius 4 intersects the line $x + y = 3$ at the points $P$ and $Q$. If the tangents at $P$ and $Q$ intersect at the point $S(\alpha, \beta)$, then $4\alpha - 7\beta$ is equal to

**Sol.** **11**

$(x-2)^2 + (y-3)^2 = 16$
$x^2 + y^2 - 4x - 6y - 3 = 0$
let the chord of contact w.r.t. to the point $S(\alpha,\beta)$ is
$T = 0$
$\alpha x + \beta y - 2(x + \alpha) - 3(y + \beta) - 3 = 0$
$x(\alpha - 2) + y(\beta - 3) - 2\alpha - 3\beta - 3 = 0$
Comparison with $x + y = 3$

$$\frac{\alpha-2}{1} = \frac{\beta-3}{1} = \frac{-2\alpha-3\beta-3}{-3}$$

$\alpha - 2 = \beta - 3$ $\quad$ $-3\beta + 9 = -2\alpha - 3\beta - 3$
$\alpha - \beta = -1$ $\quad$ $2\alpha = -3-9$
$\beta = \alpha + 1$ $\quad$ $\alpha = \frac{-12}{2}$
$= -6 + 1 = -5$ $\quad$ $\alpha = -6$
$4\alpha - 7\beta$
$4(-6) - 7(-5)$
$\Rightarrow -24 + 35 = 11$

**90.** Let $\{a_k\}$ and $\{b_k\}, k \in \mathbb{N}$, be two G.P.s with common ratios $r_1$ and $r_2$ respectively such that $a_1 = b_1 = 4$ and $r_1 < r_2$. Let $c_k = a_k + b_k, k \in \mathbb{N}$. If $c_2 = 5$ and $c_3 = \frac{13}{4}$ then $\sum_{k=1}^{\infty} c_k - (12a_6 + 8b_4)$ is equal to

**Sol.** **9**

$a_1 = 4$ $\quad$ GP $4, 4r_1, 4r_1^2$ - -
$b_1 = 4$ $\quad$ GP $4, 4r_2, 4r_2^2$ - -
$C_2 = a_2 + b_2$ $\quad$ $C_3 = a_3 + b_3$
$5 = 4r_1 + 4r_2$ $\quad$ $\frac{13}{4} = 4r_1^2 + 4r_2^2$
$\frac{5}{4} = r_1 + r_2 \ldots(1)$ $\quad$ $r_1^2 + r_2^2 = \frac{13}{16} \ldots (2)$

$$\frac{25}{16} = r_1^2 + r_2^2 + 2r_1r_2$$

$$\frac{25}{16} = \frac{13}{16} + 2r_1r_2$$

$$\Rightarrow r_1r_2 = \frac{12}{16\times2} = \frac{3}{8}$$

Now $r_1 + \frac{3}{8r_1} = \frac{5}{4}$

$8r_1^2 + 3 = 10\, r_1$

$\Rightarrow 8r_1^2 - 10r_1 + 3 = 0$

$r_1 = \frac{3}{4}$, $r_1 = \frac{1}{2}$

$r_2 = \frac{1}{2}$ $r_2 = \frac{3}{4}$

$\because r_1 < r_2$

$\therefore \begin{matrix} r_1 = \frac{1}{2} \\ r_2 = \frac{3}{4} \end{matrix}$

Now $C_k = a_k + b_k$

$\sum_{k=1}^{\infty} C_k = \frac{4}{1-r_1} + \frac{4}{1-r_2}$

$= \frac{4}{1-\frac{1}{2}} + \frac{4}{1-\frac{3}{4}}$

$= 8 + 16 = 24$

$\sum_{k=1}^{\infty} C_k - (12a_6 + 8b_4) \Rightarrow 24 - \left\{12 \times 4\left(\frac{1}{2}\right)^5 + 8 \times 4\left(\frac{3}{4}\right)^3\right\}$

$= 24 - \left(12 \times \frac{1}{8} + 8 \times \frac{27}{16}\right)$

$= 24 - \left\{\frac{3}{2} + \frac{27}{2}\right\}$

$= 24 - 15$

$= 9$


मोशन है, तो
भरोसा है।
#RankBhiSelectionBhi
ADMISSION ANNOUNCEMENT
Session 2023-24 (English & हिन्दी Medium)
Target: JEE/NEET 2025
Nurture & प्रयास Batch
Class 10th to 11th Moving
Target: JEE/NEET 2024
Enthuse & प्रयास Batch
Class 11th to 12th Moving
Target: JEE/NEET 2024
Dropper & प्रयास Batch
Class 12th to 13th Moving
Target: PRE FOUNDATION
SIP, Evening & Tapasya Batch
Class 6th to 10th Students
MOTION