JEE MAIN 2023
Paper with Solution

MATHS | 30^{th} Jan 2023 _ Shift-1

Umeed Rank Ki Ho Ya Selection Ki, JEET NISCHIT HAI!

Most Promising RANKS
Produced by MOTION Faculties

NEET / AIIMS

AIR-1 to 10
25 Times

AIR-11 to 50
83 Times

AIR-51 to 100
81 Times

JEE MAIN+ADVANCED

AIR-1 to 10
8 Times

AIR-11 to 50
32 Times

AIR-51 to 100
36 Times

Nation's Best SELECTION
Percentage (%) Ratio

Student Qualified in NEET

(2022)
4837/5356 = **90.31%**
(2021)
3276/3411 = **93.12%**

Student Qualified in JEE ADVANCED

(2022)
1756/4818 = **36.45%**
(2021)
1256/2994 = **41.95%**

Student Qualified in JEE MAIN

(2022)
4818/6653 = **72.41%**
(2021)
2994/4087 = **73.25%**

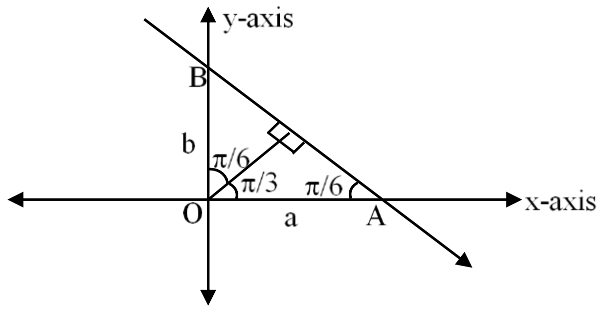
NITIN VIJAY (NV Sir)
Founder & CEO

SECTION - A

61. A straight line cuts off the intercepts $OA = a$ and $OB = b$ on the positive directions of x-axis and y axis respectively. If the perpendicular from origin O to this line makes an angle of $\frac{\pi}{6}$ with positive direction of y-axis and the area of $\triangle OAB$ is $\frac{98}{3}\sqrt{3}$, then $a^2 - b^2$ is equal to:

(1) $\frac{392}{3}$ (2) $\frac{196}{3}$ (3) 98 (4) 196

Sol. **1**



In ΔAOB

$$\tan\frac{\pi}{6} = \frac{OB}{OA} = \frac{b}{a}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{b}{a}$$

$$\Rightarrow \boxed{a = \sqrt{3}b}$$

$\because$ area of triangle $\Delta OAB = \frac{1}{2}\times ab = \frac{98}{3}\times\sqrt{3}$

$$\Rightarrow \frac{\sqrt{3}b^2}{2} = \frac{98}{\sqrt{3}}$$

$$\Rightarrow b^2 = \frac{98}{3}\times 2$$

$$\Rightarrow \boxed{b = \sqrt{\frac{196}{3}}}$$

$$\boxed{a = \sqrt{196}}$$

$$a^2 - b^2 = 196 - \frac{196}{3} = \frac{588-196}{3}$$

$$\Rightarrow \boxed{a^2 - b^2 = \frac{392}{3}}$$

62. The minimum number of elements that must be added to the relation R={(a, b), (b, c)} on the set {a, b, c} so that is becomes symmetric and transitive is :

(1) 3 (2) 4 (3) 5 (4) 7

Sol. **4**

$R=\{(a,b),(b,c)\}$

For symmetric relation (b, a), (c, b) must be added in R

For transitive relation (a, c), (a, a), (b, b), (c, c), (c, a) must be added in R

So, minimum number of element = 7

63. If an unbiased die, marked with $-2,-1,0,1,2,3$ on its faces, is thrown five times, then the probability that the product of the outcomes is positive, is :

(1) $\frac{881}{2592}$ (2) $\frac{27}{288}$ (3) $\frac{440}{2592}$ (4) $\frac{521}{2592}$

Sol. **4**

Unbiased die. Marked with –2, –1, 0, 1, 2, 3

Product of outcomes is positive if

All time get positive number, 3 time positive and 2 time negative, 1 time positive and 4 time negative.

P (Product of the outcomes is positive) = $\underbrace{{}^5C_5\left(\frac{3}{6}\right)^5}_{\text{All positive}}+\underbrace{{}^5C_3\left(\frac{3}{6}\right)^3\left(\frac{2}{6}\right)^2}_{3\text{positive},2\text{negative}}+\underbrace{{}^5C_1\left(\frac{3}{6}\right)\left(\frac{2}{6}\right)^4}_{1\text{positive},4\text{negative}}$

$$=\frac{3^5}{6^5}+\frac{10\times3^3\times2^2}{6^5}+\frac{5\times3\times2^4}{6^5}$$

$$=\frac{1563}{6^5}=\frac{521}{2592}$$

64. If $\vec{a},\vec{b},\vec{c}$ are three non-zero vectors and $\hat{n}$ is a unit vector perpendicular to $\vec{c}$ such that $\vec{a}=\alpha\vec{b}-\hat{n}$, $(\alpha\neq0)$ and $\vec{b}\cdot\vec{c}=12$, then $|\vec{c}\times(\vec{a}\times\vec{b})|$ is equal to :

(1) 9 (2) 15 (3) 6 (4) 12

Sol. **4**

$\vec{a}=\alpha\vec{b}-\hat{n},\vec{b}.\vec{c}=12$

$\vec{c}\times(\vec{a}\times\vec{b})=(\vec{c}.\vec{b})\vec{a}-(\vec{c}.\vec{a})\vec{b}$

$\vec{c}\times(\vec{a}\times\vec{b})=12\vec{a}-(\vec{c}.\vec{a})\vec{b}$ ....(1)

$\because\ \vec{a}=\alpha\vec{b}-n$

$\vec{c}.\vec{a}=\alpha\vec{c}.\vec{b}-\vec{c}.n$

$\boxed{\vec{c}.\vec{a}=12\alpha}$ ....(2)

Equation (2) put in equation (1)

$\vec{c}\times(\vec{a}\times\vec{b})=12\vec{a}-12\alpha\vec{b}$

$\left|\vec{c}\times(\vec{a}\times\vec{b})\right|=12\left|\vec{a}-\alpha\vec{b}\right|$ $\left[\because\vec{a}-\alpha\vec{b}=-n\text{ then }|\vec{a}-\alpha\vec{b}|=1\right]$

$\Rightarrow\boxed{\left|\vec{c}\times(\vec{a}\times\vec{b})\right|=12}$

65. Among the statements :

(S1) $((p \vee q) \Rightarrow r) \Leftrightarrow (p \Rightarrow r)$

(S2) $((p \vee q) \Rightarrow r) \Leftrightarrow ((p \Rightarrow r) \vee (q \Rightarrow r))$

(1) only (S2) is a tautology (2) only (S1) is a tautology

(3) neither (S1) nor (S2) is a tautology (4) both (S1) and (S2) are tautologies

Sol. **3**

$S_1 : ((p \vee q) \Rightarrow r) \Leftrightarrow (p \Rightarrow r)$

p	q	r	$(p \vee q) \Rightarrow r$ $(\sim p \wedge \sim q) \vee r$	$\sim p \vee r$	$((p \vee q) \Rightarrow r) \Leftrightarrow (p \Rightarrow r)$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	F	T	T	T
F	T	F	F	T	F
F	F	T	T	T	T

S_1 is not a tautology

$S_2 = ((p \vee q) \Rightarrow r) \Leftrightarrow ((p \Rightarrow r) \vee (q \Rightarrow r))$

p	q	r	$(p \vee q) \Rightarrow r$	$(p \Rightarrow r) \vee (q \Rightarrow r)$	$((p \vee q) \Rightarrow r) \Leftrightarrow ((p \Rightarrow r) \vee (q \Rightarrow r))$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	F	F	T	T	T
F	T	F	F	T	F
F	F	T	T	T	T

S_2 is not a tautology

So, neither S_1 nor S_2 is a tautology.

66. If $P(h, k)$ be a point on the parabola $x = 4y^2$, which is nearest to the point $Q(0,33)$, then the distance of P from the directrix of the parabola $y^2 = 4(x + y)$ is equal to :

(1) 2 (2) 6 (3) 8 (4) 4

Sol. **2**

Equation of normal of the parabola $x = 4y^2$

At a point $P\left(\frac{t^2}{16}, \frac{2t}{16}\right)$ is

$$y + tx = \frac{2t}{16} + \frac{1}{16}t^3$$

$\because$ Normal pass through $Q(0,33)$ then

$33=\frac{t}{8}+\frac{t^3}{16}$

$\Rightarrow t^3+2t-528=0$

$\Rightarrow (t-8)(t^2+8+166)=0$

$\Rightarrow t=8$

Point P is (4, 1)

Given parabola is $y^2=4(x+y)$

$y^2-4y=4x$

$(y-2)^2=4(x+1)$

directrix is x + 1 = – 1

$\boxed{x=-2}$

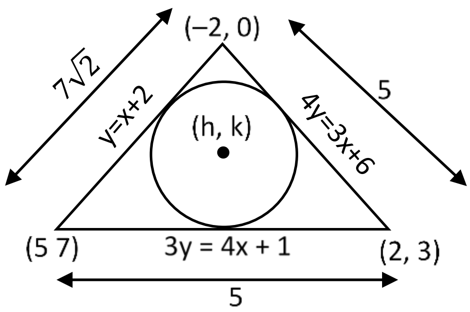
Distance of P(4, 1) from the directrix x = –2 is 6.

67. Let $y=x+2, 4y=3x+6$ and $3y=4x+1$ be three tangent lines to the circle $(x-h)^2+(y-k)^2=r^2$.

Then $h+k$ is equal to :

(1) $5(1+\sqrt{2})$ (2) $5\sqrt{2}$ (3) 6 (4) 5

Sol. **4**



In centre of triangle is (h, k)

$=\left(\frac{5(-2)+2\times7\sqrt{2}+5\times5}{5+5+7\sqrt{2}},\frac{3\times(7\sqrt{2})+0\times5+7\times5}{5+5+7\sqrt{2}}\right)$

$=\left(\frac{14\sqrt{2}+15}{10+7\sqrt{2}},\frac{21\sqrt{2}+35}{10+7\sqrt{2}}\right)$

So, $h+k=\frac{14\sqrt{2}+15}{10+7\sqrt{2}}+\frac{21\sqrt{2}+35}{10+7\sqrt{2}}$

$h+k=\frac{35\sqrt{2}+50}{7\sqrt{2}+10}=\frac{5(7\sqrt{2}+10)}{7\sqrt{2}+10}=5$

$\Rightarrow \boxed{h+k=5}$

68. The number of points on the curve $y = 54x^5 - 135x^4 - 70x^3 + 180x^2 + 210x$ at which the normal lines are parallel to $x + 90y + 2 = 0$ is :

(1) 4 (2) 2 (3) 0 (4) 3

Sol. **1**

Given curve is $y = 54x^5 - 135x^4 - 70x^3 + 180x^2 + 210x$

$$\frac{dy}{dx} = 270x^4 - 540x^3 - 210x^2 + 360x + 210$$

$\because$ Normal is parallel to x + 90y + 2 = 0

Then tangent is $\perp^r$ to x + 90 y + 2 = 0

Then $(270x^4 - 540x^3 - 210x^2 + 360x + 210)\left(\frac{-1}{90}\right) = -1$

$$270x^4 - 540x^3 - 210x^2 + 360x + 120 = 0$$
$$\Rightarrow 9x^4 - 18x^3 - 7x^2 + 12x + 4 = 0$$
$$\Rightarrow (x-1)(x-2)(3x+1)(3x+2) = 0$$
$$\Rightarrow x = 1, 2, -\frac{1}{3}, -\frac{2}{3}$$

Number of points are 4

69. If $a_n = \frac{-2}{4n^2-16n+15}$, then $a_1 + a_2 + \cdots \ldots + a_{25}$ is equal to:

(1) $\frac{52}{147}$ (2) $\frac{49}{138}$ (3) $\frac{50}{141}$ (4) $\frac{51}{144}$

Sol. **3**

given that $a_n = \dfrac{-2}{4n^2 - 16n + 15}$

$$a_1 + a_2 + a_3 + \ldots . a_{25} = \sum_{n=1}^{25} \frac{-2}{(2n-3)(2n-5)}$$
$$= \sum_{n=1}^{25} \frac{(2n-5)-(2n-3)}{(2n-3)(2n-5)}$$
$$= \sum_{n=1}^{25} \left(\frac{1}{2n-3} - \frac{1}{(2n-5)}\right)$$
$$= \frac{1}{-1} - \frac{1}{-3}$$
$$+ \frac{1}{1} - \frac{1}{-1}$$
$$+ \frac{1}{3} - \frac{1}{1}$$
$$\vdots \qquad \vdots$$
$$\frac{1}{47} - \frac{1}{45}$$
$$= \frac{1}{47} + \frac{1}{3}$$
$$= \frac{3+47}{141} = \frac{50}{141}$$

70. If $\tan 15^\circ + \frac{1}{\tan 75^\circ} + \frac{1}{\tan 105^\circ} + \tan 195^\circ = 2a$, then the value of $\left(a + \frac{1}{a}\right)$ is :

(1) 2 (2) $4 - 2\sqrt{3}$ (3) $5 - \frac{3}{2}\sqrt{3}$ (4) 4

Sol. 4

$$\tan 15^\circ + \frac{1}{\tan 75^\circ} + \frac{1}{\tan 105^\circ} + \tan 195^\circ = 2a$$

$$\Rightarrow \tan 15^\circ + \frac{1}{\cot 15^\circ} - \frac{1}{\cot 15^\circ} + \tan 15^\circ = 2a$$

$$\Rightarrow \tan 15^\circ + \tan 15^\circ - \tan 15^\circ + \tan 15^\circ = 2a$$

$$\Rightarrow 2\tan 15^\circ = 2a$$

$$\Rightarrow a = \tan 15^\circ$$

$$a + \frac{1}{a} = \tan 15^\circ + \frac{1}{\tan 15^\circ}$$

$$= \tan 15^\circ + \cot 15^\circ$$

$$= 2 - \sqrt{3} + 2 + \sqrt{3}$$

$$\Rightarrow a + \frac{1}{a} = 4$$

71. If the solution of the equation $\log_{\cos x} \cot x + 4\log_{\sin x} \tan x = 1, x \in \left(0, \frac{\pi}{2}\right)$, is $\sin^{-1}\left(\frac{\alpha + \sqrt{\beta}}{2}\right)$, where α and β are integers, then $\alpha + \beta$ is equal to :

(1) 5 (2) 6 (3) 4 (4) 3

Sol. 3

$$\log_{\cos x} \cot x + 4\log_{\sin x} \tan x = 1, x \in \left(0, \frac{\pi}{2}\right)$$

$$\Rightarrow \log_{\cos x} \frac{\cos x}{\sin x} + 4\log_{\sin x} \frac{\sin x}{\cos x} = 1$$

$$\Rightarrow 1 - \log_{\cos x} \sin x + 4 - 4\log_{\sin x} \cos x = 1$$

$$\Rightarrow 4 = \log_{\cos x} \sin x + 4\log_{\sin x} \cos x$$

Let $\log_{\cos x} \sin x = t$

$$\Rightarrow 4 = t + \frac{4}{t}$$

$$\Rightarrow t^2 - 4t + 4 = 0$$

$$\Rightarrow (t-2)^2 = 0$$

$$\Rightarrow t = 2$$

$$\Rightarrow \log_{\cos x} \sin x = 2$$

$$\Rightarrow \sin x = \cos^2 x$$

$$\Rightarrow \sin x = 1 - \sin^2 x$$

$$\Rightarrow \sin^2 x + \sin x - 1 = 0$$

$\Rightarrow \ \sin x = \dfrac{-1 \pm \sqrt{1+4}}{2}$

$\Rightarrow \ \sin x = \dfrac{-1+\sqrt{5}}{2}, \dfrac{-1-\sqrt{5}}{2} \quad \because \ x \in \left(0, \dfrac{\pi}{2}\right)$ then $\dfrac{-1-\sqrt{5}}{2}$ not possible

$\Rightarrow \ x = \sin^{-1}\left(\dfrac{-1+\sqrt{5}}{2}\right)$

$\because \ \alpha = -1, \beta = 5$ then

$\boxed{\alpha + \beta = 4}$

72. Let the system of linear equations

$x + y + kz = 2$

$2x + 3y - z = 1$

$3x + 4y + 2z = k$

have infinitely many solutions. Then the system

$(k + 1)x + (2k - 1)y = 7$

$(2k + 1)x + (k + 5)y = 10$

has:

(1) infinitely many solutions (2) unique solution satisfying $x - y = 1$

(3) unique solution satisfying $x + y = 1$ (4) no solution

Sol. **3**

$x + y + kz = 2$

$2x + 3y - z = 1$

$3x + 4y + 2z = k$

Have Infinitely many solution then

$$\begin{vmatrix} 1 & 1 & k \\ 2 & 3 & -1 \\ 3 & 4 & 2 \end{vmatrix} = 0$$

$1(10) - 1(7) + k(8-9) = 0$

$\Rightarrow 10 - 7 - k = 0$

$\Rightarrow \boxed{k = 3}$

For $k = 3$

$4x + 5y = 7$

$7x + 8y = 10$

has unique solution and solution is $(-2, 3)$.

Hence solution is unique and satisfying $x + y = 1$

73. The line l_1 passes through the point $(2,6,2)$ and is perpendicular to the plane $2x + y - 2z = 10$. Then the shortest distance between the line l_1 and the line $\frac{x+1}{2} = \frac{y+4}{-3} = \frac{z}{2}$ is :

(1) $\frac{13}{3}$ (2) $\frac{19}{3}$ (3) 7 (4) 9

Sol. **9**

equation of l_1 is $\frac{x-2}{2} = \frac{y-6}{1} = \frac{z-2}{-2}$

Let l_2 is $\frac{x+1}{2} = \frac{y+4}{-3} = \frac{z}{2}$

Point on l_1 is a = (2, 6, 2), direction $\vec{p} = <2,1,-2>$

Point on l_2 is b = (–1, –4, 0) direction $\vec{q} = <2,-3,2>$

Shortest distance between l_1 and l_2 = $\left|\frac{(\vec{a}-\vec{b}).(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right|$

$$\because \quad \vec{p}\times\vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 2 & 1 & -2 \\ 2 & -3 & 2 \end{vmatrix} = \hat{i}(-4) - \hat{j}(8) + k(-8)$$

$$= \left|\frac{\langle 3,10,2\rangle\cdot\langle -4,-8,-8\rangle}{\sqrt{16+64+64}}\right|$$

$$= \left|\frac{-12-80-16}{\sqrt{144}}\right|$$

$$= \frac{108}{12}$$

= 9

Shortest distance between the lines is 9.

74. Let $A = \begin{pmatrix} m & n \\ p & q \end{pmatrix}$, $d = |A| \neq 0$ and $|A - d(\text{Adj}A)| = 0$. Then

(1) $1 + d^2 = m^2 + q^2$ (2) $1 + d^2 = (m + q)^2$

(3) $(1 + d)^2 = m^2 + q^2$ (4) $(1 + d)^2 = (m + q)^2$

Sol. **4**

$A = \begin{bmatrix} m & n \\ p & q \end{bmatrix}$, d =|A| = mq – np

$$A - d(\text{Adj. }A) = \begin{bmatrix} m & n \\ p & q \end{bmatrix} - d\begin{bmatrix} q & -n \\ -p & m \end{bmatrix}$$

$$= \begin{bmatrix} m - dq & n + dn \\ p + pd & q - dm \end{bmatrix}$$

$|A - d(\text{Adj }A)| = (m - dq)(q - dm) - (n + dn)(p + pd) = 0$

$\Rightarrow mq - m^2d - dq^2 + d^2qm = np(1+d)^2$

$\Rightarrow (mq - m^2d - dq^2 + d^2qm) = (mq - d)(1 + d)^2$

$\Rightarrow mq - m^2d - dq^2 + d^2qm = mq + mqd^2 + 2mqd - d(1+d)^2$

$\Rightarrow d(1+d)^2 = m^2d + dq^2 + 2mqd$

$\Rightarrow \boxed{(1+d)^2 = (m+q)^2}$

75. If [t] denotes the greatest integer $\le$ t, then the value of $\frac{3(e-1)}{e}\int_1^2 x^2 e^{[x]+[x^3]}dx$ is :

(1) $e^8 - 1$ (2) $e^7 - 1$ (3) $e^8 - e$ (4) $e^9 - e$

Sol. **3**

$$\frac{3(e-1)^2}{e}\int_1^2 x^2 e^{[x]+[x^3]}\,dx$$

Let $I = \int_1^2 x^2 e^{[x]+[x^3]}\,dx$

$I = \int_1^2 x^2 e^{1+[x^3]}\,dx\,dx$

$\Rightarrow I = e\int_1^2 x^2 e^{[x^3]}\,dx$

Let $x^3 = t$

$3x^2\,dx = dt$

$I = \frac{e}{3}\int_1^8 e^{[t]}\,dt$

$\Rightarrow I = \frac{e}{3}\left[\int_1^2 e\,dt + \int_2^3 e^2dt + \int_3^4 e^3dt + \ldots.. + \int_7^8 e^7dt\right]$

$\Rightarrow I = \frac{e}{3}\left[e + e^2 + e^3 + \ldots.. + e^7\right]$

$\Rightarrow I = \frac{e}{3}\left[\frac{e(e^7-1)}{e-1}\right]$

Therefore $\frac{3(e-1)}{e}\int_1^2 x^2 e^{[x]+[x^3]}\,dx = \frac{3(e-1)}{e}\times\frac{e^2}{3}\frac{(e^7-1)}{e-1}$

$\Rightarrow \frac{3(e-1)}{e}\int_1^2 x^2 e^{[x]+[x^3]}dx = e^8 - e$

76. Let a unit vector $\widehat{OP}$ make angles α, β, γ with the positive directions of the co-ordinate axes OX, OY, OZ respectively, where $\beta \in \left(0, \frac{\pi}{2}\right)$. If $\widehat{OP}$ is perpendicular to the plane through points (1,2,3), (2,3,4) and (1,5,7), then which one of the following is true ?

(1) $\alpha \in \left(0,\frac{\pi}{2}\right)$ and $\gamma \in \left(0,\frac{\pi}{2}\right)$

(2) $\alpha \in \left(0,\frac{\pi}{2}\right)$ and $\gamma \in \left(\frac{\pi}{2},\pi\right)$

(3) $\alpha \in \left(\frac{\pi}{2},\pi\right)$ and $\gamma \in \left(\frac{\pi}{2},\pi\right)$

(4) $\alpha \in \left(\frac{\pi}{2},\pi\right)$ and $\gamma \in \left(0,\frac{\pi}{2}\right)$

Sol. **3**

$\because$ $\overrightarrow{OP}$ makes angle α, β, γ with positive directions of the co-ordinate axes then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

Point on planes are a(1, 2, 3), b(2, 3, 4) and c(1, 5, 7).

$\because$ $\overrightarrow{ab} = <1, 1, 1>$

$\overrightarrow{ac} = <0, 3, 4>$

normal vector of plane = $\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 0 & 3 & 4 \end{bmatrix}$

$= \hat{i}(1) - \hat{j}(4) + \hat{k}(3)$

$= <1, -4, 3>$

direction cosine of normal is $= \left\langle \pm\frac{1}{\sqrt{26}}, \pm\frac{4}{\sqrt{26}}, \pm\frac{3}{\sqrt{26}} \right\rangle$

then direction cosine of $\overrightarrow{op}$ is $\left\langle -\frac{1}{\sqrt{26}}, \frac{4}{\sqrt{26}}, -\frac{3}{\sqrt{26}} \right\rangle$

$\left(\because \beta \in \left(0, \frac{\pi}{2}\right)\right)$

Hence $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\gamma \in \left(\frac{\pi}{2}, \pi\right)$

77. The coefficient of x^{301} in $(1+x)^{500} + x(1+x)^{499} + x^2(1+x)^{498} + \cdots \ldots x^{500}$ is :

(1) ${}^{500}C_{300}$ (2) ${}^{501}C_{200}$ (3) ${}^{501}C_{302}$ (4) ${}^{500}C_{301}$

Sol. **2**

$x^0(1+x)^{500} + x(1+x)^{499} + x^2(1+x)^{498} + \ldots + x^{500}$

$$= (1+x)^{500} \frac{\left[\left(\frac{x}{1+x}\right)^{501} - 1\right]}{\frac{x}{1+x} - 1}$$

$$= \frac{(1+x)^{500}(x^{501} - (1+x)^{501})}{(1+x)^{501}\left(\frac{-1}{x+x}\right)}$$

$= (1+x)^{501} - x^{501}$

Coefficient of x^{301} in above expression is ${}^{501}C_{301}$ or ${}^{501}C_{200}$.

78. Let the solution curve $y = y(x)$ of the differential equation

$\frac{dy}{dx} - \frac{3x^5 \tan^{-1}(x^3)}{(1+x^6)^{\frac{3}{2}}} y = 2x \exp\left\{\frac{x^3 - \tan^{-1} x^3}{\sqrt{(1+x^6)}}\right\}$ pass through the origin. Then $y(1)$ is equal to :

(1) $\exp\left(\frac{4+\pi}{4\sqrt{2}}\right)$ (2) $\exp\left(\frac{1-\pi}{4\sqrt{2}}\right)$ (3) $\exp\left(\frac{\pi-4}{4\sqrt{2}}\right)$ (4) $\exp\left(\frac{4-\pi}{4\sqrt{2}}\right)$

Sol. **4**

$$\left(\frac{dy}{dx}\right) - \frac{3x^5 \tan^{-1}(x^3)}{(1+x^6)^{3/2}} y = 2x \exp\left(\frac{x^3 - \tan^{-1} x^3}{\sqrt{(1+x^6)}}\right)$$

above equation is linear differential equation.

$$\text{I.F.} = e^{\int \frac{-3x^5 \tan^{-1}(x^3)}{(1+x^6)^{3/2}} dx}$$

$$= e^{-\int \frac{3x^2 \cdot x^3 \tan^{-1}(x^3)}{(1+x^6)^{3/2}} dx}$$

Let $\tan^{-1}(x^3) = t$ then

$$\frac{3x^2 \cdot dx}{1+x^6} = dt$$

$$= e^{-\int \frac{t \tan t}{\sqrt{1+\tan^2 t}} dt}$$

$$= e^{-\int \frac{t \tan t}{\sec t} dt}$$

$$= e^{-\int t \sin t \, dt}$$

$$= e^{-[-t\cos t + \sin t]}$$

$$= e^{t\cos t - \sin t}$$

$$\text{I.F.} = e^{\frac{\tan^{-1} x^3}{\sqrt{1+x^6}} - \frac{x^3}{\sqrt{1+x^6}}}$$

Solution is

$$y\left(e^{\frac{\tan^{-1} x^3}{\sqrt{1+x^6}} - \frac{x^3}{\sqrt{1+x^6}}}\right) = \int 2x\, e^{\left(\frac{x^3 - \tan^{-1} x^3}{\sqrt{1+x^6}}\right)} \cdot e^{\frac{\tan^{-1} x^3 - x^3}{\sqrt{1+x^6}}} dx$$

$$y\left(e^{\frac{\tan^{-1} x^3 - x^3}{\sqrt{1+x^6}}}\right) = \int 2x\, dx = x^2 + c$$

above eq. is passing through (0, 0) then $c = 0$

$$y = x^2\, e^{\frac{x^3 - \tan^{-1} x^3}{\sqrt{1+x^6}}}$$

Put $x = 1$ then

$$y(1) = e^{\frac{1-\frac{\pi}{4}}{\sqrt{2}}} = e^{\frac{4-\pi}{4\sqrt{2}}}$$

$$\Rightarrow y(1) = \exp\left(\frac{4-\pi}{4\sqrt{2}}\right)$$

79. If the coefficient of x^{15} in the expansion of $\left(ax^3 + \frac{1}{bx^{1/3}}\right)^{15}$ is equal to the coefficient of x^{-15} in the expansion of $\left(ax^{1/3} - \frac{1}{bx^3}\right)^{15}$, where a and b are positive real numbers, then for each such ordered pair (a, b) :

(1) $ab = 3$ (2) $ab = 1$ (3) $a = b$ (4) $a = 3b$

Sol. **2**

$$\left(ax^3 + \frac{1}{bx^{1/3}}\right)^{15}$$

general term is $T_{r+1} = {}^{15}C_r\,(ax^3)^{15-r}\left(\frac{1}{bx^{1/3}}\right)^r$

$\Rightarrow T_{r+1} = {}^{15}C_r\,\frac{a^{15-r}}{b^r}\,x^{45-3r-\frac{r}{3}}$

For coefficient of $x^{15} \Rightarrow 45 - 3r - \frac{r}{3} = 15$

$$30 = \frac{10r}{3}$$

$$r = 9$$

Coefficient of x^{15} is $= {}^{15}C_9\,a^6\,b^{-9}$ … (1)

$\because$ general term of $\left(ax^{1/3} - \frac{1}{bx^3}\right)^{15}$ is

$T_{r+1} = {}^{15}C_r\,(ax^{1/3})^{15-r}\left(\frac{-1}{bx^3}\right)^r$

For coefficient of $x^{-15} \Rightarrow \frac{15-r}{3} - 3r = -15$

$\Rightarrow 15 - r - 9r = -45$

$\Rightarrow 60 = 10r$

$\boxed{r = 6}$

Coefficient of x^{-15} is $= {}^{15}C_6\,a^9\,b^{-b}$ … (2)

$\because$ both coefficient are equal then

${}^{15}C_9\,a^6\,b^{-9} = {}^{15}C_6\,a^9\,b^{-6}$

$\Rightarrow a^6\,b^{-9} = a^9\,b^{-6}$

$\Rightarrow a^3\,b^3 = 1$

$\Rightarrow \boxed{ab = 1}$

80. Suppose $f: \mathbb{R} \to (0, \infty)$ be a differentiable function such that $5f(x + y) = f(x) \cdot f(y), \forall x, y \in \mathbb{R}$. If $f(3) = 320$, then $\sum_{n=0}^{5} f(n)$ is equal to :

(1) 6875 (2) 6525 (3) 6825 (4) 6575

Sol. **3**

$f : \mathbb{R} \to (0, \infty)$

$5\, f(x+y) = f(x) \cdot f(y)$

Put x = 3, y = 0 then

$5\, f(3) = f(3)\, f(0)$

$\Rightarrow \boxed{f(0) = 5}$

Put x = 1, y = 1 then $5\, f(2) = f^2(1)$

Put x = 1, y = 2 then $5\, f(3) = f(1)\, f(2)$

$$5 \times 320 = \frac{f^3(1)}{5} = f(1) = 20$$

$$\Rightarrow f(2) = 80$$

Put x = 2, y = 2 then $5\, f(4) = f(2)\, f(2)$

$$f(4) = \frac{80 \times 80}{5} = 1280$$

Put x = 2, y = 3 then $5\, f(5) = f(2) \cdot f(2)$

$$f(5) = \frac{80 \times 320}{5} = 5120$$

$$\sum_{n=0}^{5} F(n) = f(0) + f(1) + \ldots + f(5)$$

$$= 5 + 20 + 80 + 320 + 1280 + 5120$$

$$= 5(1 + 2^2 + 2^4 + 2^6 + 2^8 + 2^{10})$$

$$= 6825$$

Section B

81. Let $z = 1 + i$ and $z_1 = \frac{1+i\bar{z}}{\bar{z}(1-z)+\frac{1}{z}}$. Then $\frac{12}{\pi}\arg(z_1)$ is equal to

Sol. **9**

$z = 1 + i$, $\bar{z} = 1 - i$, $i\,\bar{z} = 1 + i$

$$z_1 = \frac{1 + i\bar{z}}{\bar{z}(1 - z) + \frac{1}{z}}$$

$$z_1 = \frac{i + z}{\bar{z} - z\bar{z}) + \frac{1}{z}}$$

$$z_1 = \frac{i + 2}{1 - i - 2 + \frac{1 - i}{2}}$$

$$z_1 = \frac{i + 2}{-\frac{1}{2} - \frac{3i}{2}}$$

$z_1 = \frac{-2(i+2)}{(1+3i)} \times \frac{(1-3i)}{(1-3i)}$

$z_1 = \frac{-2(5-5i)}{10}$

$z_1 = -1+i$

$\arg.(z_1) = \pi - \tan^{-1}\left(\frac{1}{1}\right) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

$\therefore \arg(z_1) = \frac{12}{\pi} \times \frac{3\pi}{4} = 9$

82. If $\lambda_1 < \lambda_2$ are two values of λ such that the angle between the planes $P_1: \vec{r}\cdot(3\hat{i} - 5\hat{j} + \hat{k}) = 7$ and $P_2: \vec{r}\cdot(\lambda\hat{i} + \hat{j} - 3\hat{k}) = 9$ is $\sin^{-1}\left(\frac{2\sqrt{6}}{5}\right)$, then the square of the length of perpendicular from the point $(38\lambda_1, 10\lambda_2, 2)$ to the plane P_1 is

Sol. **315s**

Plane $P_1 : \vec{r}.(3\hat{i} - 5\hat{j} + \hat{k}) = 7$

$P_2 : \vec{r}.(\lambda\hat{i} + \hat{j} - 3k) = 9$

angle between plane is same as angle between their normal.

angle between normal θ then

$\text{Cos}\,\theta = \frac{\langle 3,-5,1\rangle \cdot \langle \lambda, 1, -3\rangle}{\sqrt{9+25+1}\sqrt{\lambda^2+1+9}}$

$\text{Cos}\,\theta = \frac{3\lambda - 5 - 3}{\sqrt{35}\sqrt{\lambda^2+10}}$ ... (1)

$\because \quad \theta = \sin^{-1}\left(\frac{2\sqrt{6}}{5}\right)$ then

$\sin\theta = \frac{2\sqrt{6}}{5}$

$\cos\theta = \frac{1}{5}$

from equation (1)

$\frac{3\lambda - 8}{\sqrt{35}\sqrt{\lambda^2+10}} = \frac{1}{5}$

$\Rightarrow \frac{(3\lambda-8)^2}{35(\lambda^2+10)} = \frac{1}{25}$

$\Rightarrow 5(3\lambda - 8)^2 = 7(\lambda^2 + 10)$

$\Rightarrow 5(9\lambda^2 - 48\lambda + 64) = 7\lambda^2 + 70$

$\Rightarrow 38\lambda^2 - 240\lambda + 250 = 0$

$\Rightarrow 19\lambda^2 - 120\lambda + 125 = 0$

$\Rightarrow \lambda = 5, \frac{25}{19}$

$\lambda_1 = \frac{25}{19}, \lambda_2 = 5$

Point (38 λ_1, 10 λ_2, 2) ≡ (50, 50, 2)

distance of (50, 50, 2) from plane P_1 is

$$d = \left|\frac{3\times 50 - 5\times 50 + 2 - 7}{\sqrt{9+25+1}}\right|$$

$$d = \left|\frac{150 - 250 + 2 - 7}{\sqrt{35}}\right|$$

$$d = \left|\frac{105}{\sqrt{35}}\right|$$

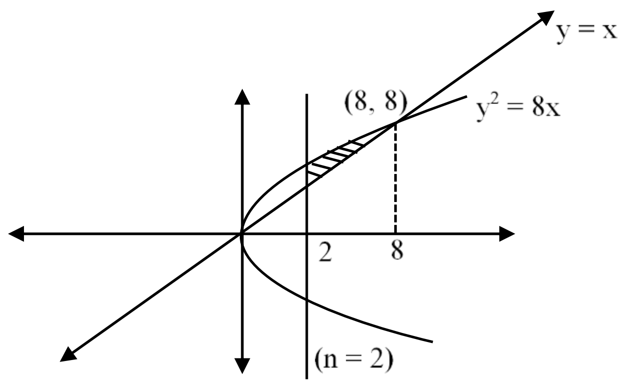
$$d = 3\sqrt{35}$$

$\boxed{d^2 = 315}$

83. Let α be the area of the larger region bounded by the curve $y^2 = 8x$ and the lines $y = x$ and $x = 2$, which lies in the first quadrant. Then the value of 3α is equal to

Sol. **22**

$$\text{area }(\alpha) = \int_2^8 (2\sqrt{2}\sqrt{x} - x)dx$$



$$= \left[2\sqrt{2}\cdot\frac{2}{3}x^{3/2} - \frac{x^2}{2}\right]_2^8$$

$$= \frac{4\sqrt{2}}{3}[8\times 2\sqrt{2} - 2\sqrt{2}] - 30$$

$$= \frac{28\times 4}{3} - 30$$

$$= \frac{112}{3} - 30$$

$$\alpha = \frac{22}{3}$$

$\boxed{3\alpha = 22}$

84. Let $\sum_{n=0}^{\infty} \frac{n^3((2n)!)+(2n-1)(n!)}{(n!)((2n)!)} = ae + \frac{b}{e} + c$, where $a, b, c \in \mathbb{Z}$ and $e = \sum_{n=0}^{\infty} \frac{1}{n!}$ Then $a^2 - b + c$ is equal to

Sol. **26**

Let $\sum_{n=0}^{\infty} \frac{n^3((2n)!) + (2n-1)n!}{(n!)((2n)!)}$

$= \sum_{n=0}^{\infty} \frac{n^3(2n)!}{n!(2n)!} + \frac{(2n-1)n!}{n!(2n)!}$

$= \qquad S_1 \qquad + \qquad S_2$

Let $S_1 = \sum_{n=0}^{\infty} \frac{n^3(2n)!}{n!(2n)!} = \sum_{n=0}^{\infty} \frac{n^3}{n!} = \sum_{n=1}^{\infty} \frac{n^2}{(n-1)!}$

$= \sum_{n=1}^{\infty} \frac{n^2 - 1 + 1}{(n-1)!}$

$= \sum_{n=2}^{\infty} \frac{(n+1)}{(n-2)!} + \sum_{n=1}^{\infty} \frac{1}{(n-1)!}$

$= \sum_{n=2}^{\infty} \frac{(n-2)+3}{(n-2)!} + \sum_{n=1}^{\infty} \frac{1}{(n-1)!}$

$= \sum_{n=3}^{\infty} \frac{1}{(n-3)!} + 3\sum_{n=2}^{\infty} \frac{1}{(n-2)!} + \sum_{n=1}^{\infty} \frac{1}{(n-1)!}$

$S_1 = e + 3e + e = 5e$

$\because \quad S_2 = \sum_{n=0}^{\infty} \frac{(2n-1)n!}{n!(2n)!}$

$= \sum_{n=0}^{\infty} \frac{2n-1}{(2n)!}$

$= \sum_{n=1}^{\infty} \frac{1}{(2n-1)!} - \sum_{n=0}^{\infty} \frac{1}{(2n)!}$

$= \left(\frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \ldots.\right) - \left(1 + \frac{1}{2!} + \frac{1}{4!} + \ldots..\right)$

$= -1 + \frac{1}{1!} - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \frac{1}{5!} - \ldots.$

$= -\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} + \ldots.\right)$

$= -e^{-1}$

$S_1 + S_2 = 5e - \frac{1}{e} = ae + \frac{b}{e} + c$

Compare both side

$a = 5, b = -1, c = 0$

$a^2 - b + c = 25 + 1 + 0 = 26$

85. If the equation of the plane passing through the point (1,1,2) and perpendicular to the line $x - 3y + 2z - 1 = 0 = 4x - y + z$ is $Ax + By + Cz = 1$, then $140(C - B + A)$ is equal to

Sol. 15

give line is $x - 3y + 2z - 1 = 0 = 4x - y + z$

Direction of line $\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & -3 & 2 \\ 4 & -1 & 1 \end{vmatrix} = \hat{i}(-1) - \hat{j}(-7) + k(11)$

$\Rightarrow \quad \vec{a} = <-1, 7, 11>$

$\because$ Line is $\perp^r$ to the plane then direction of line is parallel to normal of plane.

$\vec{n} = <-1, 7, 11>$

Equation of plane is

$-1(x-1) + 7(y-1) + 11(z-2) = 0$

$-x + 7y + 11z + 1 - 7 - 22 = 0$

$\Rightarrow \quad -x + 7y + 11z = 28$

$\Rightarrow \quad -\frac{1}{28}x + \frac{7}{28}y + \frac{11}{28}z = 1$

$A = -\frac{1}{28}, B = \frac{7}{28}, C = \frac{11}{28}$

$140(C - B + A) = 140\left(\frac{11}{28} - \frac{7}{28} - \frac{1}{28}\right)$

$= \frac{140 \times 3}{28} = 15$

86. Number of 4-digit numbers (the repeation of digits is allowed) which are made using the digits 1, 2, 3 and 5 , and are divisible by 15 , is equal to

Sol. 21

			5

Last digit must be 5 and sum of digits is divisible by 3 for divisible by 15

Remaining 3 digits	Arrange
(1, 1, 2)	$\frac{3!}{2} = 3$
(1, 3, 3)	$\frac{3!}{2} = 3$
(1, 5, 1)	$\frac{3!}{2} = 3$
(2, 2, 3)	$\frac{3!}{2} = 3$
(2, 3, 5)	3! = 6
(3, 5, 5)	$\frac{3!}{2} = 3$

Total numbers = 21

87. Let $f^1(x) = \frac{3x+2}{2x+3}, x \in \mathbf{R} - \left\{\frac{-3}{2}\right\}$

For $n \geq 2$, define $f^n(x) = f^1$ of $f^{n-1}(x)$

If $f^5(x) = \frac{ax+b}{bx+a}, \gcd(a,b) = 1$, then $a + b$ is equal to

Sol. **3125**

$f^1(x) = \dfrac{3x+2}{2x+3}, x \in R - \left\{-\dfrac{3}{2}\right\}$

$f^2(x) = f^1 0\, f^1(x) = f^1\left(\dfrac{3x+2}{2x+3}\right)$

$= \dfrac{3\left(\dfrac{3x+2)}{2x+3)}\right)+2}{2\left(\dfrac{3x+2}{2x+3}\right)+3}$

$= \dfrac{9x+6+4x+6}{6x+4+6x+9}$

$f^2(x) = \dfrac{13x+12}{12x+13}$

$f^3(x) = f^1 \text{ o } f^2(x)$

$= f^1\left(\dfrac{13x+12}{12x+13}\right)$

$= \dfrac{3\left(\dfrac{13x+12}{12x+13}\right)+2}{2\left(\dfrac{13x+12}{12x+13}\right)+3}$

$= \dfrac{39x+36+24x+26}{26x+24+36x+39}$

$f^3(x) = \dfrac{63x+62}{62x+63}$

$f^4(x) = f^1\left(\dfrac{63x+62}{62x+63}\right)$

$= \dfrac{3\left(\dfrac{63x+62}{62x+63}\right)+2}{2\left(\dfrac{63x}{62x}+\dfrac{62}{63}\right)+2}$

$f^4(x) = \dfrac{313x+312}{312x+313}$

$f^5(x) = f^1\left(\dfrac{313x+312}{312x+313}\right)$

$$= \frac{3\left(\frac{313x+312}{312x+313}\right)+2}{2\left(\frac{313x+312}{312x+313}\right)+3}$$

$$f^5(x) = \frac{1563x+1562}{1562x+1563}$$

$$\because a = 1563, b = 1562$$

$$\boxed{a+b=3125}$$

88. The mean and variance of 7 observations are 8 and 16 respectively. If one observation 14 is omitted and a and b are respectively mean and variance of remaining 6 observation, then $a + 3b - 5$ is equal to

Sol. **37**

Mean of 7 observations = $\frac{\sum_{i=1}^{7} x_i}{7}$

$\Rightarrow \quad \sum_{i=1}^{7} x_i = 7 \times 8 = 56$

Variance = $\frac{\Sigma x_i^2}{n} - (\bar{x})^2$

$\Sigma x_i^2 = 7(16+64) = 560$

If 14 is removed then

Mean = $a = \frac{\sum_{i=1}^{7} x_i - 14}{6} \Rightarrow 6a = 56 - 14$

$\Rightarrow a = 7$

Variance = $b = \frac{\sum_{i=1}^{7} x_i^2 - (14)^2}{6} - 49$

$\Rightarrow \quad 6b = 560 - 196 - 294$

$\Rightarrow \quad 6b = 70$

$\Rightarrow \quad 3b = 35$

$\therefore \quad a + 3b - 5 = 7 + 35 - 5 = 37$

89. Let $S = \{1,2,3,4,5,6\}$. Then the number of one-one functions $f: S \to P(S)$, where $P(S)$ denote the power set of S, such that $f(n) \subset f(m)$ where $n < m$ is

Sol. **3240**

Case – I

f(6) = S i.e. 1 option

f(5) = any 5 element subset A of S i.e. ${}^6C_5 = 6$ options

f(4) = any 4 element subset B of A i.e. ${}^5C_4 = 5$ options

f(3) = any 3 element subset C of B i.e. ${}^4C_3 = 4$ options

f(2) = any 2 element subset D of C i.e. ${}^3C_2 = 3$ options

f(1) = any 1 element subset E of D or empty subset i.e. 3 options

Total function = 6×5×4×3×2×3=1080

Case – II

f(6) = S

f(5) = any 4 element subset A of S i.e. ${}^6C_4 = 15$ options

f(4) = any 3 element subset B of A i.e. ${}^4C_3 = 4$ options

f(3) = any 2 element subset C of B i.e. ${}^3C_2 = 2$ options

f(2) = any 1 element subset D of C i.e. ${}^2C_1 = 2$ options

f(1) = empty subset i.e. 1 options

Total function = 15×4×3×2×1=360

Case – III

f(6) = S

f(5) = any 5 element subset A of S i.e. ${}^6C_5 = 6$ options

f(4) = any 3 element subset B of A i.e. ${}^5C_3 = 10$ options

f(3) = any 2 element subset C of B i.e. ${}^3C_2 = 3$ options

f(2) = any 1 element subset D of C i.e. ${}^2C_1 = 2$ options

f(1) = empty subset i.e. 1 options

Total function = 6×10×3×2×1=360

Case – IV

f(6) = S

f(5) = any 5 element subset A of S i.e. ${}^6C_5 = 6$ options

f(4) = any 4 element subset B of A i.e. ${}^5C_4 = 5$ options

f(3) = any 2 element subset C of B i.e. ${}^4C_2 = 6$ options

f(2) = any 1 element subset D of C i.e. ${}^2C_1 = 2$ options

f(1) = empty subset i.e. 1 options

Total function = 6×5×6×2×1=360

Case – V

f(6) = S

f(5) = any 5 element subset A of S i.e. $^6C_5 = 6$ options

f(4) = any 4 element subset B of A i.e. $^5C_4 = 5$ options

f(3) = any 3 element subset C of B i.e. $^4C_3 = 4$ options

f(2) = any 1 element subset D of C i.e. $^3C_1 = 3$ options

f(1) = empty subset i.e. 1 options

Total function = 6×5×4×3×1=360

Case – VI

f(6) = any 5 element subset A of S i.e. $^6C_5 = 6$ options

f(5) = any 4 element subset B of A i.e. $^5C_4 = 5$ options

f(4) = any 3 element subset C of B i.e. $^4C_3 = 4$ options

f(3) = any 2 element subset D of C i.e. $^3C_2 = 3$ options

f(2) = any 1 element subset E of D i.e. $^2C_1 = 2$ options

f(1) = empty subset i.e. 1 options

Total function = 6×5×4×3×2×1=720

Total number of such functions = 1080 + (4× 360) +720 = **3240**

90. $\lim\limits_{x\to 0}\frac{48}{x^4}\int_0^x \frac{t^3}{t^6+1}$ dt is equal to

Sol. **12**

$$\lim_{x\to 0}\frac{48}{x^4}\int_0^x \frac{t^3}{t^6+1} = \left(\frac{0}{0}\right) \text{ form}$$

Using L Hopital Rule

$$= \lim_{x\to 0}\frac{48\times\frac{x^3}{x^6+1}}{4x^3}$$

$$= \lim_{x\to 0}\frac{48}{4(x^6+1)}$$

$$= \frac{48}{4}$$

= 12


मोशन है, तो
भरोसा है।
#RankBhiSelectionBhi
ADMISSION ANNOUNCEMENT
Session 2023-24 (English & हिन्दी Medium)
Target: JEE/NEET 2025
Nurture & प्रयास Batch
Class 10th to 11th Moving
Target: JEE/NEET 2024
Enthuse & प्रयास Batch
Class 11th to 12th Moving
Target: JEE/NEET 2024
Dropper & प्रयास Batch
Class 12th to 13th Moving
Target: PRE FOUNDATION
SIP, Evening & Tapasya Batch
Class 6th to 10th Students
Motion