JEE MAIN 2023
Paper with Solution

MATHS | 31ST Jan 2023 _ Shift-2

Umeed Rank Ki Ho Ya Selection Ki,
JEET NISCHIT HAI!

Most Promising RANKS
Produced by MOTION Faculties

NEET / AIIMS

AIR-1 to 10
25 Times

AIR-11 to 50
83 Times

AIR-51 to 100
81 Times

JEE MAIN+ADVANCED

AIR-1 to 10
8 Times

AIR-11 to 50
32 Times

AIR-51 to 100
36 Times

Nation's Best SELECTION
Percentage (%) Ratio

Student Qualified in NEET

(2022)
4837/5356 = **90.31%**
(2021)
3276/3411 = **93.12%**

Student Qualified in JEE ADVANCED

(2022)
1756/4818 = **36.45%**
(2021)
1256/2994 = **41.95%**

Student Qualified in JEE MAIN

(2022)
4818/6653 = **72.41%**
(2021)
2994/4087 = **73.25%**

NITIN VIIJAY (NV Sir)
Founder & CEO

Section A

61. The equation $e^{4x}+8e^{3x}+13e^{2x}-8e^{x}+1=0, x\in\mathbb{R}$ has :

(1) four solutions two of which are negative

(2) two solutions and only one of them is negative

(3) two solutions and both are negative

(4) no solution

Sol. **3**

$e^{4x}+8e^{3x}+13e^{2x}+13e^{2x}-8e^{x}+1=0, x \in R$

Let $e^x = t > 0$ & $x = \ln t$

$t^4+8t^3+13t^2-8t+1=0$

Dividing by t^2,

$t^2+8t+13-\frac{8}{t}+\frac{1}{t^2}=0$

$t^2+\frac{1}{t^2}+8\left(t-\frac{1}{t}\right)+13=0$

Let $t-\frac{1}{t}=u \Rightarrow t^2+\frac{1}{t^2}-2\ u^2$

$\Rightarrow t^2+\frac{1}{t^2}=u^2+2$

$u^2+2+8u+13=0$

$(u+3)(u+5)=0$

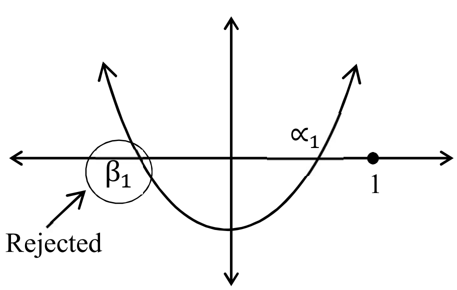
$u=-3$ & $u=-5$

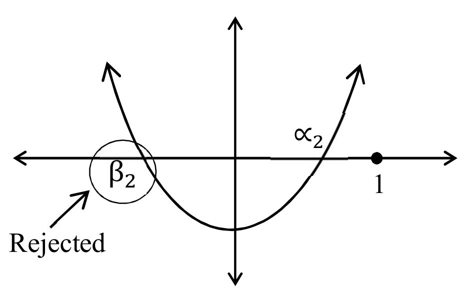
$t-\frac{1}{t}=-3$

$t^2+3t-1=0$

$t-\frac{1}{t}=-5$

$t^2+5t-1=0$





$0<\alpha_1<1$

$0<\alpha_2<1$

$\Rightarrow x_1 = \ell n\ \alpha_1 < 0$ $\Rightarrow x_2 = \ln \alpha_2 < 0$

62. Among the relations

$S = \left\{(a,b): a, b \in \mathbb{R} - \{0\}, 2 + \frac{a}{b} > 0\right\}$ and $T = \{(a,b): a, b \in \mathbb{R}, a^2 - b^2 \in \mathbb{Z}\}$,

(1) neither S nor T is transitive (2) S is transitive but T is not

(3) T is symmetric but S is not (4) both S and T are symmetric

Sol. **3**

$S = \{(a,b) \mid a, b \in R - \{0\}, 2 + \frac{a}{b} > 0\}$ &

$T = \left\{(a,b) \mid a, b \in R, a^2 - b^2 \in Z\right\}$

For S, $2 + \frac{a}{b} > 0 \Rightarrow \frac{a}{b} > -2$

Let $(-1,2) \in S\left(\therefore -\frac{1}{2} > -2\right)$

$\&(2,-1) \notin s\left(\therefore \frac{2}{-1} \text{ not greater than } -2\right)$

So, S is not symmetric

For T,

If $(a,b) \in T \Rightarrow a^2 - b^2 \in Z$

$\Rightarrow -\left(a^2 - b^2\right) \in Z$

$\Rightarrow b^2 - a^2 \in Z$

$\Rightarrow (b,a) \in T$

So, T is symmetric

63. Let $\alpha > 0$. If $\int_0^{\alpha} \frac{x}{\sqrt{x+\alpha} - \sqrt{x}} dx = \frac{16 + 20\sqrt{2}}{15}$, then α is equal to :

(1) 4 (2) $2\sqrt{2}$ (3) $\sqrt{2}$ (4) 2

Sol. **3**

$\alpha > 0$

$$I = \int_0^{\alpha} \frac{x}{\sqrt{x+2} - \sqrt{x}} dx = \frac{16 + 2\sqrt{2}}{15}$$

$$I = \int_0^{\alpha} \frac{x\left(\sqrt{x+\alpha} + \sqrt{x}\right)}{\alpha} dx$$

$$= \frac{1}{\alpha}\left[\int_0^{\alpha} x\sqrt{x+\alpha}\sqrt{x+\alpha}\, dx + \int_0^{\alpha} x^{3/2} dx\right]$$

$$I_1=\int_0^{\alpha}(x+\alpha-\alpha)\sqrt{x+\alpha}\,dx$$

$$=\int_0^{\alpha}(x+\alpha)^{3/2}-\alpha\int_0^{\alpha}(x+\alpha)^{1/2}dx$$

$$=\frac{2}{5}\left[(x+\alpha)^{5/2}\right]^{\alpha}-\frac{\alpha(2)}{3}\left[(x+\alpha)^{3/2}\right]_0^{\alpha}$$

$$=\frac{2}{5}\left[(2\alpha)^{5/2}-\alpha^{5/2}\right]-\frac{2\alpha}{3}\left[(2\alpha)^{3/2}-\alpha^{3/2}\right]$$

$$=\frac{2}{5}(2\alpha)^{5/2}-\frac{2}{5}\alpha^{5/2}-\frac{(2\alpha)^{5/2}}{3}+\frac{2\alpha^{5/2}}{3}$$

$$=(2\alpha)^{5/2}\left[\frac{2}{5}-\frac{1}{3}\right]+2\alpha^{5/2}\left[\frac{1}{3}-\frac{1}{5}\right]$$

$$=(2\alpha)^{5/2}\left[\frac{1}{15}\right]+2\alpha^{5/2}\left[\frac{2}{15}\right]$$

$$=\frac{(2\alpha)^{5/2}}{15}+\frac{4\alpha^{5/2}}{15}$$

$$=\frac{4\alpha^{5/2}}{15}\left[\sqrt{2}+1\right]$$

$$I_2=\int_0^{\alpha}x^{3/2}dx=\frac{2}{5}\left[x^{5/2}\right]_0^{\alpha}=\frac{2}{5}\alpha^{5/2}$$

$$I=\frac{1}{\alpha}(I_1+I_2)$$

$$I=\frac{1}{\alpha}\left[\frac{4\alpha^{5/2}(\sqrt{2}+1)}{15}+\frac{2}{5}\alpha^{5/2}\right]$$

$$=\frac{2\alpha^{5/2}}{15\alpha}\left[2(\sqrt{2}+1)+3\right]$$

$$=\frac{2}{15}\alpha^{3/2}\left[2\sqrt{2}+5\right]$$

$$\frac{16+20\sqrt{2}}{15}=\frac{2}{15}\alpha^{3/2}\left[2\sqrt{2}+5\right]$$

$$\alpha^{3/2}=2\sqrt{2}$$

$$\alpha^3=8$$

$$\alpha=2$$

64. The complex number $z = \frac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$ is equal to :

(1) $\sqrt{2}i\left(\cos\frac{5\pi}{12} - i\sin\frac{5\pi}{12}\right)$ (2) $\sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$

(3) $\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$ (4) $\cos\frac{\pi}{12} - i\sin\frac{\pi}{12}$

Sol. **3**

$$Z = \frac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$$

$$i - 1 = \sqrt{2}\left(\frac{-1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right) = \sqrt{2}\cdot e^{i\cdot\frac{3\pi}{4}}$$

$$z = \frac{\sqrt{2}\cdot e^{i\cdot\frac{3\pi}{4}}}{e^{i\cdot\frac{3\pi}{4}}}$$

$$= \sqrt{2}\cdot e^{i\left(\frac{3\pi}{4} - \frac{\pi}{3}\right)}$$

$$= \sqrt{2}\, e^{\frac{5\pi}{12}i}$$

$$= \sqrt{2}\left(\cos\left(\frac{5\pi}{12}\right) + i\sin\left(\frac{5\pi}{12}\right)\right)$$

65. Let $y = y(x)$ be the solution of the differential equation $(3y^2 - 5x^2)y\, dx + 2x(x^2 - y^2)dy = 0$ such that $y(1) = 1$. Then $|(y(2))^3 - 12y(2)|$ is equal to :

(1) $16\sqrt{2}$ (2) $32\sqrt{2}$ (3) 32 (4) 64

Sol. **2**

$$\left(3y^2 - 5x^2\right)ydx + 2x\left(x^2 - y^2\right)dy = 0$$

$$2x\left(x^2 - y^2\right)dy = \left(5x^2 - 3y^2\right)y\,dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{\left(5x^2 - 3y^2\right)y}{2x\left(x^2 - y^2\right)}$$

Let y = tx

$$\frac{dy}{dx} = t + x\frac{dt}{dx}$$

$$t + x\frac{dt}{dx} = \frac{\left(5 - 3t^2\right)t}{2\left(1 - t^2\right)}$$

$$x\frac{dt}{dx} = t\left[\frac{5 - 3t^2 - 2 + 2t^2}{2\left(1 - t^2\right)}\right]$$

$$=\frac{t}{2}\left[\frac{3-t^2}{1-t^2}\right]$$

$$\frac{1}{3}\int\frac{3\left(1-t^2\right)dt}{3t-t^3}=\int\frac{dx}{2x}$$

$$\frac{1}{3}\ell n\left|3t-t^3\right|=\frac{1}{2}\ln|x|+c$$

$$2\ell n\left|3t-t^3\right|=3\ln|x|+6c$$

$$\left(3t-t^3\right)^2=x^3\lambda$$

$$\left(\frac{3y}{x}-\frac{y^3}{x^3}\right)^2=x^3\lambda$$

$$\left(3yx^2-y^3\right)^2=x^9\lambda$$

$$x=1\Rightarrow y=1$$

$$(3-1)^2=1\times\lambda$$

$$\lambda=4$$

$$\left(3yx^2-y^3\right)^2=4x^9$$

Let x =2

$$\left(3y(2)\times4-\left(y(2)^3\right)^2\right)=4(2)^9$$

Taking square root both the sides,

$$\left|(y(2))^3-12y(2)\right|=2(2)^{9/2}$$

$$=2(2)^4\sqrt{2}=32\sqrt{2}$$

66. $\lim\limits_{x\to\infty}\frac{(\sqrt{3x+1}+\sqrt{3x-1})^6+(\sqrt{3x+1}-\sqrt{3x-1})^6}{\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6}x^3$

(1) does not exist (2) is equal to 27 (3) is equal to $\frac{27}{2}$ (4) is equal to 9

Sol. **2**

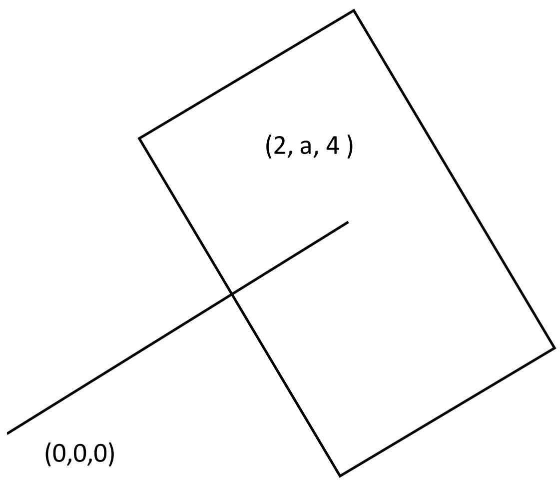
$$\lim_{x\to\infty}\frac{\left(\sqrt{3x+1}+\sqrt{3x-1}\right)^6+\left(\sqrt{3x+1}-\sqrt{3x-1}\right)^6}{\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6}$$

Taking height power common

$$\lim_{x\to\infty}\frac{x^6\left[\left(\sqrt{3+\frac{1}{x}}+\sqrt{3-\frac{1}{x}}\right)^6+\left(\sqrt{3+\frac{1}{x}}-\sqrt{3-\frac{1}{x}}\right)^6\right]}{x^6\left[\left(1+\sqrt{1-\frac{1}{x^2}}\right)^6+\left(1-\sqrt{1-\frac{1}{x^2}}\right)^6\right]}$$

$$=\frac{\left(\sqrt{3}+\sqrt{3}\right)^6+\left(\sqrt{3}-\sqrt{3}\right)^6}{(1+1)^6+(1-1)^6}$$

$$=\frac{\left(2\sqrt{3}\right)^6}{2^6}=\left(\sqrt{3}\right)^6=27$$



67. The foot of perpendicular from the origin O to a plane P which meets the co-ordinate axes at the points A, B, C is $(2, a, 4), a \in N$. If the volume of the tetrahedron OABC is 144unit^3, then which of the following points is NOT on P ?

(1) (0,6,3) (2) (0,4,4) (3) (2,2,4) (4) (3,0,4)

Sol. **4**

$\vec{n}=(2,a,4)$

Plane is

$2x+ay+4z=4+a^2+16$

$=20+a^2$

$A\left(\frac{20+a^2}{2},0,0\right)$

$B\left(0,\frac{20+a^2}{a},0\right)$

$C\left(0,0,\frac{20+a^2}{4}\right)$

$\frac{1}{6}\times\frac{\left(20+a^2\right)^3}{8a}=144=2^4\times3^2$

$\left(20+a^2\right)^3=2^8 3^3 a$

$20+a^2=\left(4a\right)^{\frac{1}{3}}\left(12\right)$

a = 2 satisfies above equation

So, 2x + 2y + 4z = 24

X + Y + 2z = 12

(A) (0, 6, 3)

(B) (0, 4, 4)

(C) (2,2,4)

(D) (3,0,4)

68. Let $(a, b) \subset (0,2\pi)$ be the largest interval for which $\sin^{-1}(\sin\theta) - \cos^{-1}(\sin\theta) > 0, \theta \in (0,2\pi)$, holds. If $\alpha x^2 + \beta x + \sin^{-1}(x^2 - 6x + 10) + \cos^{-1}(x^2 - 6x + 10) = 0$ and $\alpha - \beta = b - a$, then α is equal to :

(1) $\frac{\pi}{16}$ (2) $\frac{\pi}{48}$ (3) $\frac{\pi}{12}$ (4) $\frac{\pi}{8}$

Sol. **3**

$x^2-6x+10=\left(x-3\right)^2+1\geq1$

So, x = 3 is the only element in the Domain

So, $\alpha x^2+\beta x+\sin^{-1}\left(x^2-6x+10\right)+\cos^{-1}\left(x^2-6x+10\right)=0$

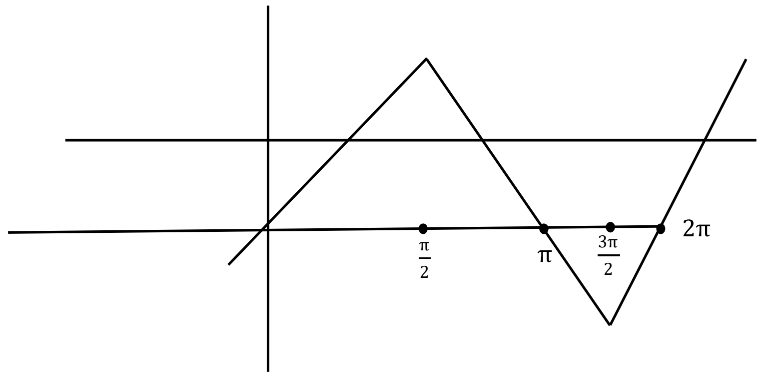
$9\alpha+3\beta+\frac{\pi}{2}=0$

$\text{Sin}^{-1}\left(\sin\theta\right)-\cos^{-1}\left(\sin\theta\right)>0$

$\sin^{-1}\left(\sin\theta\right)-\left(\frac{\pi}{2}-\sin^{-1}\left(\sin\theta\right)\right)>0$

$2\sin^{-1}\left(\sin\theta\right)>\frac{\pi}{2}$

$\sin^{-1}\left(\sin\theta\right)>\frac{\pi}{4}$



So, $\theta \in \left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

$\alpha - \beta = \frac{3\pi}{4} - \frac{\pi}{4} = \frac{\pi}{2}$ ……(1)

& $9\alpha + 3\beta = \frac{-\pi}{2}$

$3\alpha + \beta = \frac{-\pi}{6}$ ……(2)

Adding (1) & (2)

$4\alpha = \frac{\pi}{2} - \frac{\pi}{6} = \frac{2\pi}{6}$

$\alpha = \frac{\pi}{12}$

69. Let the mean and standard deviation of marks of class A of 100 students be respectively 40 and α (>0), and the mean and standard deviation of marks of class B of n students be respectively 55 and $30-\alpha$. If the mean and variance of the marks of the combined class of $100+\mathrm{n}$ studants are respectively 50 and 350 , then the sum of variances of classes A and B is :

(1) 650 (2) 450 (3) 900 (4) 500

Sol. **4**

$m_A = 40$ S.d$_A = \alpha > 0$ $n_A = 100$

$m_B = 55$ S.d$_B = 30 - \alpha$ $n_B = n$

$m_{AVB} = 50$ Variance$_{AVB} = 350$ $n_{AVB} = 100 + n$

$A = \{x_1, \ldots\ldots, x_{100}\}$ $B = \{y_1, \ldots, y_n\}$

$\sum x_i = 4000$

$\sum y_i = 55n$

$\sum (x_i + y_i) = 50(100 + n)$

$4000 + 55n = 5000 + 50n$

Using formula of standard deviation

$5n = 1000 \ n = 200$

$\alpha^2 = \frac{\sum x_i^2}{100} - (40)^2$ | $(30-\alpha)^2 \frac{\sum y_i^2}{200} - (55)^2$

$\sum x_i^2 = 100(1000 + \alpha^2)$

$\sum y_i^2 = 200((55)^2 + (30-\alpha)^2)$

$350 = \frac{\sum(x_i^2 + y_i^2)}{300} - (50)^2$

$\sum x_i^2 + \sum y_i^2 = ((50)^2 + 350)300$

$160000 + 100\alpha^2 + 200(55)^2 + 200(30-\alpha)^2$

$(50)^2 300 + 350 \times 300$

$1600 + \alpha^2 + 6050 + 2(30-\alpha)^2 = 7500 + 1050$

$\alpha^2 + 1800 - 120\alpha + 2\alpha^2 - 900 = 0$

$3\alpha^2 - 120\alpha + 900 = 0$

$\alpha^2 - 40\alpha + 300 = 0$

$(\alpha - 10)(\alpha - 30) = 0$

$\alpha = 10 \text{ or } \alpha = 30$

if $\alpha = 10$ VarA = 100 & VarB = 400

$\mathrm{Var}_A + \mathrm{V}_{arB} = 500$

70. The absolute minimum value, of the function $f(x) = |x^2 - x + 1| + [x^2 - x + 1]$, where $[t]$ denotes the greatest integer function, in the interval $[-1,2]$, is:

(1) $\frac{1}{4}$ (2) $\frac{3}{2}$ (3) $\frac{5}{4}$ (4) $\frac{3}{4}$

Sol. **4**

$f(x) = |x^2 - x + 1| + [x^2 - x + 1]$

$x \in [-1, 2]$ Here $x^2 - x + 1 > 0, \forall x \in R$

Minimum value of $x^2 - x + 1$ occurs at $a = \frac{1}{2} \in [-1, 2]$

So, $\mathrm{Min}\, f(x) = f\left(\frac{1}{2}\right)$

$= \frac{3}{4} + \left[\frac{3}{4}\right] = \frac{3}{4}$

71. Let H be the hyperbola, whose foci are $(1 \pm \sqrt{2}, 0)$ and eccentricity is $\sqrt{2}$. Then the length of its latus rectum is

(1) $\frac{3}{2}$ (2) 2 (3) 3 (4) $\frac{5}{2}$

Sol. **2**

$F_1F_2 = 2ae = \left(1+\sqrt{2}\right)-\left(1-\sqrt{2}\right) = 2\sqrt{2}$

$ae = \sqrt{2}$

$e = \sqrt{2}$

$\Rightarrow a = 1 \quad \Rightarrow b = 1 \left(\because e = \sqrt{2}\right)$

L.L.R. $= \frac{2b^2}{a} = \frac{2(1)^2}{1} = 2$

72. Let $a_1, a_2, a_3, \ldots$ be an A.P. If $a_7 = 3$, the product $a_1 a_4$ is minimum and the sum of its first n terms is zero, then $n! - 4a_{n(n+2)}$ is equal to :

(1) 9 (2) $\frac{33}{4}$ (3) $\frac{381}{4}$ (4) 24

Sol. **4**

$a_7 = 3 \; a_1 a_4$ minimum

$a + 6d = 3$

$a(a+3d) \rightarrow$ minimum

$S_n = 0 \quad \Rightarrow \frac{n}{2}\left[na_1 + (n-1)d\right] = 0$

$2a_1 + (n-1)d = 0 \qquad \ldots..(1)$

Let $a(a+3d)$ is minimum

$f(d) = (3-6d)(3-6d+3d)$

$f(d) = (3-6d)(3-3d)$

$= 18d^2 - 27d + 9$ is minimum at $d = \frac{27}{2\times18} = \frac{9\times3}{2\times9\times2} = \frac{3}{4}$

So, $d = \frac{3}{4}$

$a_1 + 6d = 3$

$a_1 = 3 - 6\left(\frac{3}{4}\right) = 3 - \frac{9}{2} = -\frac{3}{2}$

Putting $a_1 = \frac{-3}{2}$ & $d = \frac{3}{4}$ in (1)

$$2\left(\frac{-3}{2}\right)+(n-1)\left(\frac{3}{4}\right)=0$$

$$\frac{3}{4}(n-1)=3$$

$$n-1=4$$

$$n=5$$

$$ni-4a_{n(n+2)}$$

$n=5$ so $n!=5!=120$

& $a_{5(7)}=a_{35}=\frac{-3}{2}+(34)\left(\frac{3}{4}\right)$

$$=\frac{-3}{2}+\frac{51}{2}$$

$$=\frac{48}{2}=24$$

$$5!-4(24)=24$$

73. If a point $P(\alpha,\beta,\gamma)$ satisfying

$$(\alpha\beta\gamma)\begin{pmatrix}2 & 10 & 8\\ 9 & 3 & 8\\ 8 & 4 & 8\end{pmatrix}=(000)$$

lies on the plane $2x+4y+3z=5$, then $6\alpha+9\beta+7\gamma$ is equal to :

(1) -1 (2) $\frac{11}{5}$ (3) $\frac{5}{4}$ (4) 11

Sol. **4**

$2\alpha+9\beta+8\gamma=0$ ....(1)

$10\alpha+3\beta+4\gamma=0$ ....(2)

$8\alpha+8\beta+8\gamma=0$ ....(3)

$$\alpha+\beta+\gamma=0$$

$$\gamma=-\alpha-\beta$$

$$2\alpha+9\beta-8\alpha-8\beta=0$$

$$\beta=6\alpha$$

$$\gamma=-\alpha-6\alpha=-7\alpha$$

$(\alpha,6\alpha,-7\alpha)$ Satisfies the above system of equation

$$2\alpha+4(6\alpha)+3(-7\alpha)=5$$

$$5\alpha=5$$

$$\alpha=1$$

$$\beta=6$$

$\gamma = -7$

$6\alpha + 9\beta + 7\gamma = 6 + 54 - 49 = 11$

74. Let : $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}, \vec{b} = \hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $\vec{c} = 5\hat{\imath} - 3\hat{\jmath} + 3\hat{k}$ be there vectors. If $\vec{r}$ is a vector such that, $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r} \cdot \vec{a} = 0$, then $25|\vec{r}|^2$ is equal to

(1) 560 (2) 449 (3) 339 (4) 336

Sol. **3**

$\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$

$\vec{r} \times \vec{b} - \vec{c} \times \vec{b} = \vec{0}$

$(\vec{r} - \vec{c}) \times \vec{b} = \vec{0}$

$\vec{r} - \vec{c} = \lambda \vec{b}$

$\vec{r} = \lambda \vec{b} + \vec{c} = (\lambda + 5)\hat{i} - (\lambda + 3)\hat{j} + (2\lambda + 3)\hat{k}$

$\vec{r}.\vec{a} = 0$

$1(\lambda + 5) - 2(\lambda + 3) + 3(2\lambda + 3) = 0$

$5\lambda + 8 = 0 \Rightarrow \lambda = \frac{-8}{5}$

$\vec{r} = \frac{17}{5}\hat{i} - \frac{7}{5}\hat{j} + \frac{1}{5}\hat{k}$

$25|\vec{r}|^2 = 17^2 + 7^2 + 1^2 = 339$

75. Let the plane $P: 8x + \alpha_1 y + \alpha_2 z + 12 = 0$ be parallel to the line L: $\frac{x+2}{2} = \frac{y-3}{3} = \frac{z+4}{5}$. If the intercept of P on the y-axis is 1 , then the distance between P and L is :

(1) $\sqrt{\frac{7}{2}}$ (2) $\sqrt{\frac{2}{7}}$ (3) $\frac{6}{\sqrt{14}}$ (4) $\sqrt{14}$

Sol. **4**

$y - \text{intercept} = \frac{-12}{\alpha_1} = 1$

$\alpha_1 = -12$ & $\vec{n} = (8, \alpha_1, \alpha_2)$

$\vec{\ell} = (2, 3, 5)$

$\vec{n}.\vec{\ell} = 0$ ($\therefore$ plane P & line L are parallel)

$16 + 3\alpha_1 + 5\alpha_2 = 0$

$16 - 36 + 5\alpha_2 = 0$

$5\alpha_2 = 20$

$\alpha_2 = 4$

$8x-12y+4z+12=0$

$\Rightarrow 2x-3y+z+3=0$

(–2, 3, – 4) is a point on the line L distance bet^n the point (–2, 3, –4) and the plane P is :-

$$d=\frac{|-4-9-4+3|}{\sqrt{2^2+3^2+1^2}}$$

$$=\frac{14}{\sqrt{14}}=\sqrt{14}$$

76. Let P be the plane, passing through the point $(1,-1,-5)$ and perpendicular to the line joining the points $(4,1,-3)$ and $(2,4,3)$. Then the distance of P from the point $(3,-2,2)$ is

(1) 5 (2) 4 (3) 7 (4) 6

Sol. **1**

Let A(4, 1, –3) & B(2, 4, 3)

$\vec{n}=\overline{AB}=(-2, 3, 6)$

Plane P is :

$-2(x-1)+3(y+1)+6(z+5)=0$

$-2x+2+3y+3+6z+30=0$

$2x-3y-6z=35$

Distance of P from the point (3, –2, 2) is

$$=\frac{|6+6-12-35|}{\sqrt{2^2+3^2+6^2}}$$

$=\frac{35}{7}=5$ **Ans. (1)**

77. The number of values of $r\in\{p, q, \sim p, \sim q\}$ for which $((p\wedge q)\Rightarrow(r\vee q))\wedge((p\wedge r)\Rightarrow q)$ is a tautology, is:

(1) 3 (2) 4 (3) 1 (4) 2

Sol. **4**

$((p\wedge q)\Rightarrow(r\vee q))\wedge((p\wedge r)\Rightarrow q)$

$(p\wedge q)\Rightarrow(r\vee q)$

$\sim(p\wedge q)\vee(r\vee q)$

$\sim p\vee \sim q\vee r\vee q$

& $(p\wedge r)\Rightarrow q$

$\sim(p\wedge r)\vee q$

$\sim p\vee\sim r\vee q$

$(\sim p\vee\sim q\vee r\vee q)\wedge(\sim p\vee\sim r\vee q)$

$\equiv\sim p\vee\sim r\vee q$

If $r=p$

$\sim p\vee\sim p\vee q \rightarrow$ Not tautology

If $r=\sim p$

$\sim p \vee p \vee q \quad \rightarrow \quad$ tautology

If $\quad r = q$

$\sim p \vee \sim q \vee q \quad \rightarrow \quad$ tautology

If $\quad r = \sim q$

$\sim p \vee q \vee q \quad \rightarrow \quad$ Not tautology

Ans. 2 (D)

78. The set of all values of a^2 for which the line $x + y = 0$ bisects two distinct chords drawn from a point $P\left(\frac{1+a}{2}, \frac{1-a}{2}\right)$ on the circle $2x^2 + 2y^2 - (1+a)x - (1-a)y = 0$, is equal to :

(1) $(0,4]$ (2) $(4, \infty)$ (3) $(2,12]$ (4) $(8, \infty)$

Sol. **4**

$$x^2 + y^2 - \left(\frac{1+a}{2}\right)x - \left(\frac{1-a}{2}\right)y = 0$$

$$x\left(x - \left(\frac{1+a}{2}\right)\right) + y\left(y - \left(\frac{1-a}{2}\right)\right) = 0$$

$$y - y_1 = m(x - x_1) \qquad (x_1, y_1) = \left(\frac{1+a}{2}, \frac{1-a}{2}\right)$$

$\&\ x + y = 0$

$-\ x - y_1 = mx - mx_1$

$mx_1 - y_1 = (1 + m)x$

$$x = \frac{mx_1 - y_1}{1+m} \quad \& \quad y = \frac{y_1 - mx_1}{1+m}$$

$$\frac{\frac{y_1}{2} - y}{\frac{x_1}{2} - x} = -\frac{1}{m}$$

$$\frac{\frac{y_1}{2} - \left(\frac{y_1 - mx_1}{1+m}\right)}{\frac{x_1}{2} - \left(\frac{mx_1 - y_1}{1+m}\right)} = -\frac{1}{m}$$

$$= \frac{(1+m)y_1 - 2y_1 + 2mx_1}{(1+m)x_1 - 2mx_1 + 2y_1} = -\frac{1}{m}$$

$$\frac{m(y_1 + 2x_1) - y_1}{-mx_1 + x_1 + 2y_1} = -\frac{1}{m}$$

$m^2(y_1+2x_1) - my_1 = mx_1 - x_1 - 2y_1$

$m^2(y_1+2x_1) - (y_1 + x_2)m + x_1 + 2y_1 = 0$

$D > 0$

$(y_1+x_1)^2 - 4(y_1 + 2x_1)(x_2 + 2y_1) > 0$

$x_1 = \frac{1+a}{2}, y_1 = \frac{1-a}{2}$

$x_1 + y_1 = 1$

$y_1 + 2x_1 = \frac{1-a}{2} + 1 + a = \frac{3}{2} - \frac{a}{2} = \frac{3-a}{2}$

$x_1 + 2y_1 = \frac{1+a}{2} + 1 - a = \frac{3}{2} + \frac{a}{2} = \frac{3+a}{2}$

$1 - 4\left(\frac{3-a}{2}\right)\left(\frac{3+a}{2}\right) > 0$

$1 - (9 - a^2) > 0$

$a^2 - 8 > 0$

$a^2 > 8 \qquad \to (8, \infty)$

Ans. 4

79. If $\phi(x) = \frac{1}{\sqrt{x}}\int_{\frac{\pi}{4}}^{x}\left(4\sqrt{2}\sin t - 3\phi'(t)\right)dt, x > 0$, then $\emptyset'\left(\frac{\pi}{4}\right)$ is equal to :

(1) $\frac{8}{6+\sqrt{\pi}}$ (2) $\frac{4}{6+\sqrt{\pi}}$ (3) $\frac{8}{\sqrt{\pi}}$ (4) $\frac{4}{6-\sqrt{\pi}}$

Sol. **1**

$\sqrt{x}\ \phi(x) = \int_{\frac{\pi}{4}}^{x}(4\sqrt{2}\sin t - 3\phi'(t))dt$

Differentiating w.r.t. x,

$\frac{1}{2\sqrt{x}}\phi(x) + \sqrt{x}\,\phi'(x) = 4\sqrt{2}\sin x - 3\phi'(x)$

$(\sqrt{x}+3)\,\phi'(x) + \frac{1}{2\sqrt{x}}\phi(x) = 4\sqrt{2}\sin x$

$\phi'(x) + \frac{1}{2\sqrt{x}(\sqrt{x}+3)}\ \phi(x) = \frac{4\sqrt{2}\sin x}{\sqrt{x}+3}$

Put $x = \left(\frac{\pi}{4}\right)$

$\phi'\left(\frac{\pi}{4}\right) + 0 = \frac{4\sqrt{2}\times\frac{1}{\sqrt{2}}}{\sqrt{\frac{\pi}{4}}+3} \quad \left(\because \phi\left(\frac{\pi}{4}\right) = 0\right)$

$\phi'\left(\frac{\pi}{4}\right) = \frac{4\times 2}{\sqrt{\pi}+6} = \frac{8}{\sqrt{\pi}+6}$ **Ans. 1**

80. Let $f:\mathbb{R}-\{2,6\} \to \mathbb{R}$ be real valued function defined as $f(x) = \frac{x^2+2x+1}{x^2-8x+12}$. Then range of f is

(1) $\left(-\infty, -\frac{21}{4}\right) \cup (0, \infty)$ (2) $\left(-\infty, -\frac{21}{4}\right] \cup [1, \infty)$

(3) $\left(-\infty, -\frac{21}{4}\right] \cup \left[\frac{21}{4}, \infty\right)$ (4) $\left(-\infty, -\frac{21}{4}\right] \cup [0, \infty)$

Sol. **4**

$y = \dfrac{x^2 + 2x + 1}{x^2 - 8x + 12}$

$x^2y - 8xy + 12y = x^2 + 2x + 1$

$x^2(y-1) - (8y+2)x + 12y - 1 = 0$

$D \geq 0$

$(8y+2)^2 - 4(y-1)(12y - 1) \geq 0$

$4(4y+1)^2 - 4(y-1)(12y - 1) \geq 0$

$16y^2 + 8y+1 - (12y^2 - 13y+1) \geq 0$

$4y^2 + 21y \geq 0$

$4y\left[y + \frac{21}{4}\right] \geq 0$

$\left(-\infty, \frac{-21}{4}\right] \cup [0, \infty)$ **Ans. 4**

Section B

81. Let $A = [a_{ij}], a_{ij} \in Z \cap [0,4], 1 \leq i, j \leq 2$. The number of matrices A such that the sum of all entries is a prime number $p \in (2,13)$ is

Sol. **204**

$A = [a_{ij}]$, $aij \in Z \cap [0, 4]$, so $a_{ij} = \{0,1,2,3,4\}$

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times2}$

$a_{11} + a_{12} + a_{21} + a_{22} = P$ where P is a prime number

$a_{11} + a_{12} + a_{21} + a_{22} = 3, 5, 7, 11$

$(1 + x + x^2 + x^3 + x^4)^4$

$(1 - x^5)^4 (1 - x)^{-4}$

$(1 - 4x^5 + 6x^{10} - 4x^{15} + x^{20}) (1 + {}^4C_1 x + {}^5C_2 x^2 + {}^6C_3 x^3 + {}^7C_4 x^7 \ldots .)$

For 3 ${}^6C_3 = \dfrac{6 \times 5 \times 4}{6} = 20.$

For 5 $(-4) + {}^8C_5 = -4 + 56 = 52.$

For 7 $(-4)\,{}^5C_2 + {}^{10}C_7 = -40 + 120 = 80.$

For 11 ${}^{14}C_{11} + (-4)\,{}^9C_6 + 6({}^4C_1) = 364 - 336 + 24 = 52$

Total number of matrices = 204

$a_{11} + a_{12} + a_{22} + a_{12} = 3$

$0 \quad 0 \quad 0 \quad 3 \quad = \frac{4!}{3!} = 4$

$0 \quad 0 \quad 2 \quad 1 \quad = \frac{4!}{2!} = 12$

$0 \quad 1 \quad 1 \quad 1 \quad = \frac{4!}{3!} = 4$

20

$a_{11} + a_{12} + a_{21} + a_{22} = 5$

$0 \quad 0 \quad 1 \quad 4 \quad = \frac{4!}{2!} = 12$

$0 \quad 0 \quad 2 \quad 3 \quad = \frac{4!}{2!} = 12$

$0 \quad 1 \quad 1 \quad 3 \quad = \frac{4!}{2!} = 12$

$0 \quad 1 \quad 2 \quad 2 \quad = \frac{4!}{2!} = 12$

$1 \quad 1 \quad 1 \quad 2 \quad = \frac{4!}{3!} = 4$

52

$a_{11} + a_{12} + a_{21} + a_{22} = 7$

$0 \quad 0 \quad 3 \quad 4 \quad = \frac{4!}{2!} = 12$

$0 \quad 1 \quad 3 \quad 3 \quad = \frac{4!}{2!} = 12$

$0 \quad 1 \quad 2 \quad 4 \quad = 4! = 24$

$1 \quad 1 \quad 1 \quad 4 \quad = \frac{4!}{3!} = 4$

$0 \quad 2 \quad 2 \quad 3 \quad = \frac{4!}{3!} = 12$

$1 \quad 1 \quad 2 \quad 3 \quad = \frac{4!}{2!} = 12$

$1 \quad 2 \quad 2 \quad 2 \quad = \frac{4!}{3!} = 4$

80

$a_{11} + a_{12} + a_{21} + a_{22} = 11$

$0 \quad 3 \quad 4 \quad 4 \quad = \frac{4!}{2!} = 12$

$1 \quad 2 \quad 4 \quad 4 \quad = \frac{4!}{2!} = 12$

1 3 3 4 $= \frac{4!}{2!} = 12$

2 3 3 4 $= \frac{4!}{3!} = 12$

2 3 3 3 $= \frac{4!}{3!} = 4$

52

total matrix is = 20 + 52 + 80 + 52 = 204.

82. Let A be a $n \times n$ matrix such that $|A| = 2$. If the determinant of the matrix $\mathrm{Adj}(2 \cdot \mathrm{Adj}(2\,A^{-1})) \cdot$ is 2^{84}, then n is equal to

Sol. **84**

$|A| = 2$

$|\mathrm{Adj}\,(2\,\mathrm{Adj}(2A^{-1}))| = 2^{84}$

$|2\,\mathrm{Adj}\,(2\,A^{-1})|^{n-1} = 2^{84}$

$(2^n\,|\mathrm{Adj}\,(2\,A^{-1})|)^{n-1} = 2^{84}$

$(2^n\,|\,2^{n-1}\,\mathrm{Adj}\,(A^{-1})|)^{n-1} = 2^{84}$

$(2^n \times (2^{n-1})^n\,|\mathrm{Adj}\,(A^{-1})|)^{n-1} = 2^{84}$

$(2^n \times 2^{n(n-1)} \times |A^{-1}|)^{n-1} = 2^{84}$

$(2^n \times 2^{n(n-1)} \times \left(\frac{1}{2}\right)^{n-1})^{n-1} = 2^{84}$

$(2^{n+n^2-n-n+1})^{n-1} = 2^{84}$

$(2^{n^2-n+1})^{n-1} = 2^{84}$

$(n^2 - n + 1)(n + 1) = 84$

83. If the constant term in the binomial expansion of $\left(\frac{x^{\frac{5}{2}}}{2} - \frac{4}{x^l}\right)^9$ is -84 and the coefficient of x^{-3l} is $2^\alpha\beta$, where $\beta < 0$ is an odd number, then $|\alpha l - \beta|$ is equal to

Sol. **98**

$$\left(\frac{x^{5/2}}{2} - \frac{4}{x^\ell}\right)^9$$

$$T_{r+1} = {}^9C_r \left(\frac{x^{5/2}}{2}\right)^{9-r} \left(\frac{-4}{x^\ell}\right)^r$$

$$= {}^9C_r \left(\frac{1}{2}\right)^{9-r} (-4)^r \quad x^{\frac{45-5r}{2} - \ell r}$$

For constant term

$= {}^9C_r \left(\frac{1}{2}\right)^{9-r} (-4)^r = -84$

$= {}^9C_r \left(\frac{1}{2}\right)^{9-r} (-1)^r\, 2^{2r} = -84$

$= {}^9C_r\, 2^{r-9}\, 2^{2r}\, (-1)^r = -84$

$= {}^9C_r\, 2^{3r-9}\, (-1)^r = -84$

$\Rightarrow \boxed{r = 3}$

$\frac{45 - 5r}{2} - \ell r = 0$

$\frac{45 - 15}{2} - 3\ell = 0$

$15 = 3\,\ell$

$\boxed{\ell = 5}$

For coefficient of x^{-15} is

$\frac{45 - 5r}{2} - 5r = -15$

$45 - 5r - 10r = -30$

$75 = 15r$

$\boxed{r = 5}$

For coefficient of x^{-15} is ${}^9C_5 \left(\frac{1}{2}\right)^4 (-4)^5$

$\frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} \times \frac{1}{2^4} \times 2^{10} \times (-1)$

$= 9 \times 2 \times 7 \times 2^6 \times (-1)$

$= 2^7(-63) = 2^\alpha \beta$

$\alpha = 7, \beta = -63$

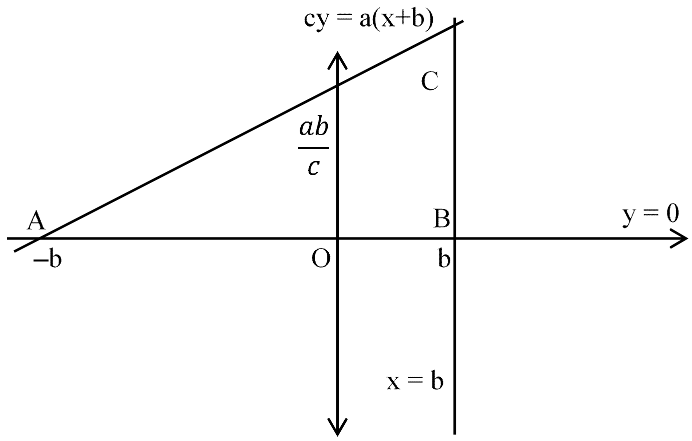
$|\alpha \ell - \beta| = |7(5)+63| = |35 + 63|$

$|\alpha \ell - \beta| = 98.$

84. Let S be the set of all a ∈ N such that the area of the triangle formed by the tangent at the point P(b, c), b, c ∈ ℕ, on the parabola $y^2 = 2ax$ and the lines $x = b, y = 0$ is 16 unit^2, then $\sum_{a \in s} a$ is equal to

Sol. **146**

tangent at P(b, c) my^2 = 2ax is



area $= \left|\frac{1}{2} \times 2b \times \frac{2ba}{c}\right| = 16$

$\frac{2b^2a}{C} = 16$

$\frac{b^2a}{C} = 8$

$\because$ P(b, c) lies on $y^2 = 2ax$

$C^2 = 2ab$

$\Rightarrow \quad \frac{b^4a^2}{c^2} = 64$

$\Rightarrow \quad \frac{b^4a^2}{2ab} = 64$

$\Rightarrow \quad b^3a = 128$

$\Rightarrow \quad a = \frac{128}{b^3}$

a can be 128, 16, 2 then

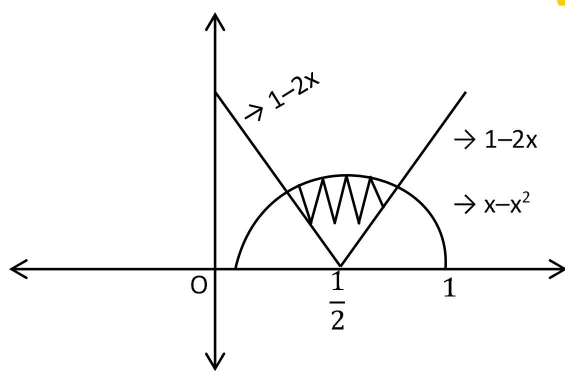
S = {2, 16, 128}

$\sum_{a \in s} a = 146$

85. Let the area of the region $\{(x, y): |2x-1| \le y \le |x^2 - x|, 0 \le x \le 1\}$ be A. Then $(6\text{ A} + 11)^2$ is equal to

Sol. **125**

$|2x-1| \le y \le |x^2 - x|, 0 \le x \le 1$



$$x - x^2 = 1 - 2x$$

$$x^2 - 3x + 1 = 0$$

$$x = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2}$$

$$x = \frac{3-\sqrt{5}}{2} \quad \text{as } 0 < x < \frac{1}{2}$$

$$\text{Area} = 2\int_{\frac{3-\sqrt{5}}{2}}^{1/2} \left[\left(x - x^2\right) - \left(1-2x\right)\right]dx$$

$$2\int_{\frac{3-\sqrt{5}}{2}}^{1/2} \left[3x - x^2 - 1\right]dx$$

$$= 2\left[\frac{3x^2}{2} - \frac{x^3}{3} - x\right]_{\frac{3-\sqrt{5}}{2}}^{1/2}$$

For $x = \dfrac{3-\sqrt{5}}{2}$,

$$x^2 = 3x - 1$$

$$x^3 = 3x^2 - x$$

$$= 3(3x-1) - x$$

$$= 8x - 3$$

$$\frac{3}{2}x^2 - \frac{1}{3}x^3 - x = \frac{3}{2}(3x-1) - \frac{1}{3}(8x-3) - x$$

$$= \frac{9x-3}{2} - \frac{(8x-3)}{3} - x$$

$$= \frac{27x - 9 - (16x - 6)}{6} - x$$

$$= \frac{11x-3}{6} - x$$

$= \dfrac{5x-3}{6}$

For $x = \dfrac{1}{2}$,

$\dfrac{3}{2}x^2 - \dfrac{1}{3}x^3 - x = \dfrac{3}{2}\left(\dfrac{1}{4}\right) - \dfrac{1}{3}\left(\dfrac{1}{8}\right) - \dfrac{1}{2}$

$= \dfrac{9-1-12}{24}$

$= \dfrac{-4}{24} = -\dfrac{1}{6}$

$$\text{Area} = 2\left[-\frac{1}{6} - \left(\frac{\frac{5\left(3-\sqrt{5}\right)}{2} - 3}{6}\right)\right]$$

$$= 2\left[-\frac{1}{6} - \left(\frac{15 - 5\sqrt{5} - 6}{12}\right)\right]$$

$$= 2\left[-\frac{1}{6} - \left(\frac{9 - 5\sqrt{5}}{12}\right)\right]$$

$$= 2\left[\frac{-2 - 9 + 5\sqrt{5}}{12}\right]$$

$$= \frac{5\sqrt{5} - 11}{6}$$

$$(6A + 11)^2 = \left(5\sqrt{5}\right)^2 = 125$$

86. The coefficient of x^{-6}, in the expansion of $\left(\frac{4x}{5} + \frac{5}{2x^2}\right)^9$, is

Sol. **5040**

$\left(\dfrac{4x}{5} \times \dfrac{5}{2x^2}\right)^9$

General term is

$T_{r+1} = {}^9C_r \left(\dfrac{4n}{r}\right)^{9-r} \left(\dfrac{5}{2n^2}\right)^r$

$= {}^9C_r \left(\dfrac{4}{5}\right)^{9-r} \left(\dfrac{5}{2}\right)^r x^{9-r-2r}$

For coefficient of term x^{-6} $\quad 9 - r - 2r = -6$

$15 = 3r$

$\boxed{r = 5}$

Coefficient of term x^{-6} $\quad {}^9C_5\left(\frac{4}{5}\right)^4\left(\frac{5}{2}\right)^5$

$= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} \times 5 \times 2^3$

$= 9 \times 2 \times 7 \times 5 \times 8$

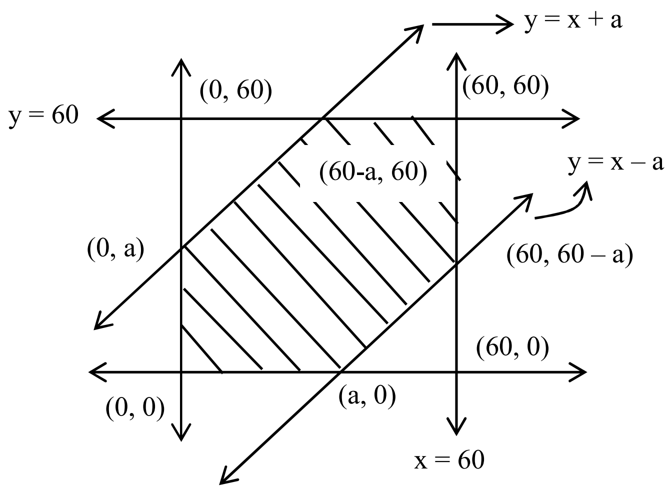
$= 5040$

87. Let A be the event that the absolute difference between two randomly choosen real numbers in the sample space [0,60] is less than or equal to a . If $P(A) = \frac{11}{36}$, then a is equal to

Sol. **10**

$|x - y| < a \rightarrow -a < x - y \quad \& \quad x - y < a$

$x, y \in [0, 60]$



$$P(A) = \frac{\text{Shaded area}}{\text{Total area}} = \frac{(60)^2 - \left[\frac{1}{2}(60-a)^2 + \frac{1}{2} \times (60-a)^2\right]}{(60)^2}$$

$$P(A) = \frac{(60)^2 - (60-a)^2}{(60)^2}$$

$$\frac{11}{36} = \frac{120a - a^2}{3600}$$

$1100 = 120a - a^2$

$a^2 - 120a + 1100 = 0$

$a^2 - 110a - 10a + 1100 = 0$

$a(a - 110) - 10(a - 110) = 0 =$

$(a - 10)(a - 110) = 0$

Ans. $\boxed{a = 10}$

$(\because$ for $a = 110$, $P(A) = 1)$

88. If ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 11:21$, then $n^2 + n + 15$ is equal to :

Sol. **45**

${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 11 : 21$

$$\frac{(2n+1)!}{(n+2)!} \times \frac{(n-1)!}{(2n-1)!} = \frac{11}{21}$$

$$\Rightarrow \frac{(2n+1)(2n)}{(n+2)(n+1)n} = \frac{11}{21}$$

$$\Rightarrow \frac{2(2n+1)}{(n+2)(n+1)} = \frac{11}{21}$$

$\Rightarrow 42(2n+1) = 11(n^2+3n+2)$

$\Rightarrow 84n + 42 = 11n^2+33n+22$

$\Rightarrow 11n^2 - 51n - 20 = 0$

$\Rightarrow n = 5$

$n^2 + n + 15 = 25 + 5 + 15 = 45$

89. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $|\vec{a}| = \sqrt{31}, 4|\vec{b}| = |\vec{c}| = 2$ and $2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a})$. If the angle between $\vec{b}$ and $\vec{c}$ is $\frac{2\pi}{3}$, then $\left(\frac{\vec{a}\times\vec{c}}{\vec{a}\cdot\vec{b}}\right)^2$ is equal to

Sol. **3**

$|\vec{a}| = \sqrt{31} \qquad 4|\vec{b}| = |\vec{c}| = 2$

$2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a})$

$$\vec{b} \wedge \vec{c} = \frac{2\pi}{3}$$

$\vec{a} \times 2\vec{b} = 3\vec{c} \times \vec{a} = -\vec{a} \times 3\vec{c}$

$\vec{a} \times (2\vec{b} + 3\vec{c}) = \vec{o}$

$\vec{a} = \lambda(2\vec{b} + 3\vec{c})$

$|\vec{a}|^2 = \lambda^2 |2\vec{b} + 3\vec{c}|^2$

$$|\vec{a}|^2 = \lambda^2\left(4|\vec{b}|^2 + 9|\vec{c}|^2 + 12|\vec{b}||\vec{c}|\cos\theta\right)$$

$$31 = \lambda^2 (1 + 9(2)^2 + 12|\vec{b}||\vec{c}| \cos\frac{2\pi}{3})$$

$$31 = \lambda^2 (1 + 36 - 6 \times \frac{1}{2} \times 2)$$

$31 = \lambda^2 (31)$

$\boxed{\lambda^2 = 1}$

$\Rightarrow \lambda = \pm 1$

$\vec{a} = \pm\left(2\vec{b} + 3\vec{c}\right)$

$$\vec{a}\times\vec{c} = \pm\left(2\vec{b}+3\vec{c}\right)\times\vec{c}$$

$$= \pm 2\left(\vec{b}\times\vec{c}\right)$$

$$\left|\vec{a}\times\vec{c}\right|^2 = 4\left|\vec{b}\times\vec{c}\right|^2 = 3$$

$$\vec{a}\cdot\vec{b} = \mp 1$$

$$\left(\frac{\left|\vec{a}\times\vec{c}\right|}{\vec{a}\cdot\vec{b}}\right)^2 = 3$$

90. The sum $1^2 - 2\cdot 3^2 + 3\cdot 5^2 - 4\cdot 7^2 + 5\cdot 9^2 - \cdots + 15\cdot 29^2$ is

Sol. **6952**

$1^2 - 2.3^2 + 3.5^2 - 4.7^2 + 5.9^2 - \ldots + 15.29^2$

$S = \underbrace{15.29^2 - 14.27^2} + \ldots. + \underbrace{3.5^2 - 2.3^2} + 1^2$

$(n+1)(2n+1)^2 - n(2n-1)^2$

$n(4n^2+4n+1) + 4n^2+4n+1 - n(4n^2-4n+1)$

$= 12n^2 + 4n + 1$

$S = [\Sigma\ 12n^2+4n+1 \text{ for } n = 2, 4, 6, 8, 10, 12, 14] + 1$

$$S_1 = \sum_{k=1}^{7} 12(2k)^2 + 4(2k) + 1$$

$$= \sum_{k=1}^{7} [48k^2 + 8k + 1]$$

$$= 48\sum_{k=1}^{7} k^2 + 8\sum_{k=1}^{7} k + \sum_{k=1}^{7} 1$$

$$= \frac{48(7)(8)(15)}{6} + \frac{8(7)(8)}{2} + 7 = 6951$$

$S = \boxed{6952}$


मोशन है, तो
भरोसा है।
#RankBhiSelectionBhi
ADMISSION ANNOUNCEMENT
Session 2023-24 (English & हिन्दी Medium)
Target: JEE/NEET 2025
Nurture & प्रयास Batch
Class 10th to 11th Moving
Target: JEE/NEET 2024
Enthuse & प्रयास Batch
Class 11th to 12th Moving
Target: JEE/NEET 2024
Dropper & प्रयास Batch
Class 12th to 13th Moving
Target: PRE FOUNDATION
SIP, Evening & Tapasya Batch
Class 6th to 10th Students
MOTION