JEE MAIN 2023
Paper with Solution

MATHS | 31st Jan 2023 _ Shift-1

Umeed Rank Ki Ho Ya Selection Ki, JEET NISCHIT HAI!

Most Promising RANKS
Produced by MOTION Faculties

NEET / AIIMS

AIR-1 to 10
25 Times

AIR-11 to 50
83 Times

AIR-51 to 100
81 Times

JEE MAIN+ADVANCED

AIR-1 to 10
8 Times

AIR-11 to 50
32 Times

AIR-51 to 100
36 Times

Nation's Best SELECTION
Percentage (%) Ratio

Student Qualified in NEET

(2022)
4837/5356 = **90.31%**
(2021)
3276/3411 = **93.12%**

Student Qualified in JEE ADVANCED

(2022)
1756/4818 = **36.45%**
(2021)
1256/2994 = **41.95%**

Student Qualified in JEE MAIN

(2022)
4818/6653 = **72.41%**
(2021)
2994/4087 = **73.25%**

NITIN VIIJAY (NV Sir)
Founder & CEO

SECTION - A

61. If the maximum distance of normal to the ellipse $\frac{x^2}{4}+\frac{y^2}{b^2}=1, b<2$, from the origin is 1 , then the eccentricity of the ellipse is :

(1) $\frac{1}{2}$ (2) $\frac{\sqrt{3}}{4}$ (3) $\frac{\sqrt{3}}{2}$ (4) $\frac{1}{\sqrt{2}}$

Sol.

Normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at point (a cosθ, b sinθ) is $\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2$

Its distance from origin is

$$d=\frac{|a^2-b^2|}{\sqrt{a^2\sec^2\theta+b^2\operatorname{cosec}^2\theta}}$$

$$d=\frac{|a^2-b^2|}{\sqrt{a^2+b^2+2ab+(a\ \tan\theta-b\ \cot\theta)^2}}$$

$$d\frac{|(a-b)(a+b)|}{\sqrt{a^2+b^2+2ab+(a\ \tan\theta-b\ \tan\theta)^2}}$$

$$d_{max}=\frac{|(a-b)(a+b)|}{a+b}=|a-b|$$

$\because d_{max} = 1$

$|2 - b| = 1$

$2 - b = 1$ [$\because$ b < 2]

$\boxed{b=1}$

Eccentricity = $\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{1}{4}}=\frac{\sqrt{3}}{2}$

$\Rightarrow \boxed{e=\frac{\sqrt{3}}{2}}$

62. Let a differentiable function f satisfy $f(x)+\int_3^x \frac{f(t)}{t}dt=\sqrt{x+1}, x\geq 3$. Then $12f(8)$ is equal to :

(1) 34 (2) 1 (3) 17 (4) 19

Sol.

$$f(x)+\int_3^x \frac{f(t)}{t}dt=\sqrt{x+1},\ x\geq 3$$

Differentiate both side w.r.t. x

$$f^1(x)+\frac{f(x)}{x}=\frac{1}{2\sqrt{x+1}}$$

Above eqn. is linear differential equation

I.f. = $e^{\int \frac{1}{x} dx} = e^{\ln x} = x$

Solution is

$$f(x) \cdot x = \int \frac{x}{2\sqrt{x+1}} dx + C$$

$$f(x) \cdot x = \frac{1}{2} \int \left(\frac{x+1}{\sqrt{x+1}} - \frac{1}{\sqrt{x+1}} \right) dx + C$$

$$f(x) \cdot x = \frac{1}{2} \int \left(\sqrt{x+1} - \frac{1}{\sqrt{x+1}} \right) dx + C$$

$$f(x) \cdot x = \frac{1}{2} \left[\frac{2}{3}(x+1)^{3/2} - 2\sqrt{x+1} \right] + C$$

$\because$ f(3) = 2

than

$$2.3 = \frac{1}{2} \left[\frac{2}{3} \times 8 - 2 \times 2 \right] + C$$

$$6 = \frac{1}{2} \left[\frac{16}{3} - 4 \right] + C$$

$$6 = \frac{2}{3} + C$$

$$\boxed{C = \frac{16}{3}}$$

$$f(x).\ x = \frac{1}{2} \left[\frac{2}{3}(x+1)^{3/2} - 2\sqrt{x+1} \right] + \frac{16}{3}$$

Put x = 8

$$f(8) \cdot 8 = \frac{1}{2} \left[\frac{2}{3} \times 27 - 2 \times 3 \right] + \frac{16}{3}$$

$$f(8) \cdot 8 = \frac{1}{2}[12] + \frac{16}{3}$$

$$f(8) \cdot 8 = 6 + \frac{16}{3} = \frac{34}{3}$$

$$\boxed{12f(8) = 17}$$

63. For all $z \in C$ on the curve $C_1: |z| = 4$, let the locus of the point $z + \frac{1}{z}$ be the curve C_2. Then :

(1) the curve C_1 lies inside C_2
(2) the curve C_2 lies inside C_1
(3) the curves C_1 and C_2 intersect at 4 points
(4) the curves C_1 and C_2 intersect at 2 points

Sol.

$C_1 : |z| = 4$ then $z\bar{z} = 16$

$z + \frac{1}{z} = z + \frac{\bar{z}}{16}$

$= x + iy + \frac{x - iy}{16}$

$z + \frac{1}{z} = \frac{17x}{16} + i\frac{15y}{16}$

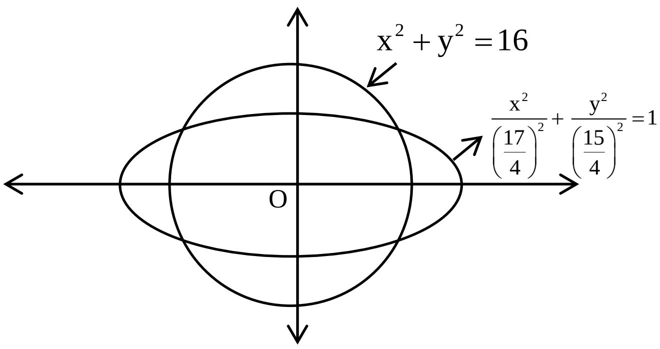
Let $X = \frac{17x}{16}, \quad Y = \frac{15}{16}y$

$\frac{X}{\left(\frac{17}{16}\right)} = x, \quad \frac{Y}{\left(\frac{15}{16}\right)} = y$

$\because x^2 + y^2 = 16$

$\frac{X^2}{\left(\frac{17}{16}\right)^2} + \frac{Y^2}{\left(\frac{15}{16}\right)^2} = 16$

$\Rightarrow C_2 : \frac{x^2}{\left(\frac{17}{4}\right)^2} + \frac{y^2}{\left(\frac{15}{4}\right)^2} = 1$ (Ellipse)



Curve C_1 and C_2 intersect at 4 point.

64. $y = f(x) = \sin^3\left(\frac{\pi}{3}\left(\cos\left(\frac{\pi}{3\sqrt{2}}(-4x^3 + 5x^2 + 1)^{\frac{3}{2}}\right)\right)\right)$. Then, at $x = 1$,

(1) $\sqrt{2}y' - 3\pi^2 y = 0$ (2) $y' + 3\pi^2 y = 0$ (3) $2y' + 3\pi^2 y = 0$ (4) $2y' + \sqrt{3}\pi^2 y = 0$

Sol.

$$y = f(x) = \sin^3\left(\frac{\pi}{3}\left(\cos\left(\frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{3/2}\right)\right)\right)$$

Let $g(x) = \frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{3/2}$

$g(1) = \frac{2\pi}{3}$

$$y = \sin^3\left(\frac{\pi}{3}\cos(g(x))\right)$$

Differentiate w.r.t. x

$$y' = 3\sin^2\left(\frac{\pi}{3}\cos(g(x))\right)\times\cos\left(\frac{\pi}{3}\cos(g(x))\right)\times\frac{\pi}{3}\left(-\sin g(x)\right)g'(x)$$

$$\because g^1(x) = \frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{1/2}\left(-12x^2+10x\right)$$

$$g^1(1) = \frac{\pi}{2\sqrt{2}}\left(\sqrt{2}\right)(-2) = -\pi$$

$$y^1(1) = \frac{3}{4}\times\frac{\sqrt{3}}{2}\times\frac{\pi}{3}\left(\frac{-\sqrt{3}}{2}\right)(-\pi) = \frac{3\pi^2}{16}$$

$$\boxed{y^1(1) = \frac{3\pi^2}{16}}$$

$$y(1) = \sin^3\left(\frac{\pi}{3}\cos\frac{2\pi}{3}\right) = \frac{-1}{8}$$

$$\boxed{2y^1(1) + 3\pi^2 y(1) = 0}$$

65. A wire of length 20 m is to be cut into two pieces. A piece of length l_1 is bent to make a square of area A_1 and the other piece of length l_2 is made into a circle of area A_2. If $2A_1 + 3A_2$ is minimum then $(\pi l_1): l_2$ is equal to :

(1) 1:6 (2) 6:1 (3) 3:1 (4) 4:1

Sol.

Total length of wire = 20 m

area of square (A_1) = $\left(\frac{\ell_1}{4}\right)^2$

area of circle (A_2) = $\pi\left(\frac{\ell_2}{2\pi}\right)^2$

Let S = $2A_1 + 3A_2$

$$S = \frac{\ell_1^2}{8} + \frac{3\ell_2^2}{4\pi}$$

$\because\ \ell_1 + \ell_2 = 20$ then

$$1 + \frac{d\ell_2}{d\ell_1} = 0$$

$$\frac{d\ell_2}{d\ell_1} = -1$$

$$\frac{ds}{d\ell_1} = \frac{\ell_1}{4} + \frac{6\ell_2}{4\pi}\cdot\frac{d\ell_2}{d\ell_1} = 0$$

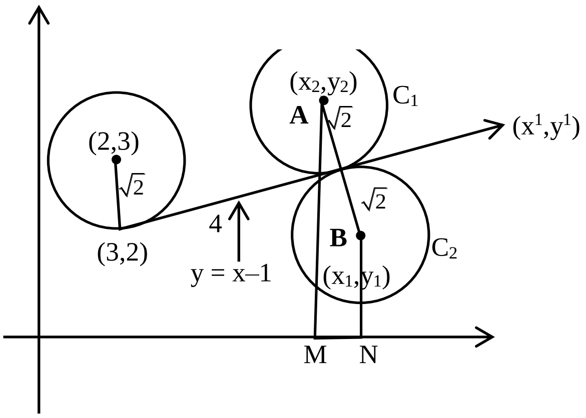
$$= \frac{\ell_1}{4} = \frac{6\ell_2}{4\pi}$$

$$= \frac{\pi\ell_1}{\ell_2} = \frac{6}{1}$$

66. Let a circle C_1 be obtained on rolling the circle $x^2 + y^2 - 4x - 6y + 11 = 0$ upwards 4 units on the tangent T to it at the point $(3,2)$. Let C_2 be the image of C_1 in T. Let A and B be the centers of circles C_1 and C_2 respectively, and M and N be respectively the feet of perpendiculars drawn from A and B on the x-axis. Then the area of the trapezium AMNB is :

(1) $4(1+\sqrt{2})$ (2) $3+2\sqrt{2}$ (3) $2(1+\sqrt{2})$ (4) $2(2+\sqrt{2})$

Sol.



(x', y') point lies on line y = x – 1 have distance 4 unit from (3, 2).

$x' = \frac{4}{\sqrt{2}} + 3 \quad = 2\sqrt{2} + 3$

$y' = \frac{4}{\sqrt{2}} + 2 = 2\sqrt{2} + 2$

Slope of line AB is –1. i.e. = $\tan\theta = -1$ then $\sin\theta = \frac{1}{\sqrt{2}}$, $\cos\theta = -\frac{1}{\sqrt{2}}$

for point A and B

$$x = \pm\sqrt{2}\left(\frac{-1}{\sqrt{2}}\right)+\left(2\sqrt{2}+3\right)$$

$$y = \pm\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)+\left(2\sqrt{2}+2\right)$$

for point A we take +ve sign

$$(x_2, y_2) = \left(2\sqrt{2}+2,\ 2\sqrt{2}+3\right)$$

for point B we take –ve sign

$$(x_1, y_1) = \left(2\sqrt{2}+4,\ 2\sqrt{2}+1\right)$$

$MN = |x_2 - x_1| = 2$

$AM + BN = 2\sqrt{2}+3+2\sqrt{2}+1 = 4+4\sqrt{2}$

area of trapezium $= \frac{1}{2}\times 2\times\left(4+4\sqrt{2}\right)$

$$= 4\left(1+\sqrt{2}\right)$$

67. A bag contains 6 balls. Two balls are drawn from it at random and both are found to be black. The probability that the bag contains at least 5 black balls is

(1) $\frac{3}{7}$ (2) $\frac{5}{7}$ (3) $\frac{5}{6}$ (4) $\frac{2}{7}$

Sol. $\text{Probability} = \frac{{}^{3}C_2 + {}^{6}C_2}{{}^{2}C_2 + {}^{3}C_2 + {}^{4}C_2 + {}^{5}C_2 + {}^{6}C_2}$

$$= \frac{10+15}{1+3+6+10+15}$$

$$= \frac{5}{7}$$

68. Let $y = f(x)$ represent a parabola with focus $\left(-\frac{1}{2}, 0\right)$ and directrix $y = -\frac{1}{2}$.

Then $S = \left\{x \in \mathbb{R}: \tan^{-1}(\sqrt{f(x)}) + \sin^{-1}(\sqrt{f(x)+1}) = \frac{\pi}{2}\right\}$:

(1) contains exactly two elements (2) contains exactly one element

(3) is an empty set (4) is an infinite set

Sol. equation of parabola which have focus $\left(-\frac{1}{2},\ 0\right)$ and directrix $y = -\frac{1}{2}$ is

$$\left(x+\frac{1}{2}\right)^2 = \left(y+\frac{1}{4}\right)$$

$y = f(x) = (x^2 + x)$

$\because\ S = \left\{x \in R : \tan^{-1}\left(\left(\sqrt{f(x)}\right) + \sin^{-1}\left(\sqrt{f(x)+1}\right) = \frac{\pi}{2}\right)\right\}$

$\tan^{-1}\left(\sqrt{f(x)}\right) + \sin^{-1}\left(\sqrt{f(x)+1}\right) = \frac{\pi}{2}$

$f(x) \geq 0$ & $\sqrt{f(x)+1}$ can not greater then 1, so f(x) must be 0

i.e. f(x) = 0

$\Rightarrow\ x^2 + x = 0$

$x(x+1) = 0$

$x = 0,\ x = -1$

S contain 2 element.

69. Let $\vec{a} = 2\hat{\imath} + \hat{\jmath} + \hat{k}$, and $\vec{b}$ and $\vec{c}$ be two nonzero vectors such that $|\vec{a} + \vec{b} + \vec{c}| = |\vec{a} + \vec{b} - \vec{c}|$ and $\vec{b} \cdot \vec{c} = 0$. Consider the following two statements:

(A) $|\vec{a} + \lambda\vec{c}| \geq |\vec{a}|$ for all $\lambda \in \mathbb{R}$.

(B) $\vec{a}$ and $\vec{c}$ are always parallel.

Then.

(1) both (A) and (B) are correct
(2) only (A) is correct
(3) neither (A) nor (B) is correct
(4) only (B) is correct

Sol.

$|\vec{a} + \vec{b} + \vec{c}| = |\vec{a} + \vec{b} - \vec{c}|,\ \vec{b}.\vec{c} = 0$

$|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a} + \vec{b} - \vec{c}|^2$

$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{a}.\vec{b} + 2\vec{b}.\vec{c} + 2\vec{a}.\vec{c}$

$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{a}.\vec{b} - 2\vec{b}.\vec{c} - 2\vec{a}.\vec{c}$

$2\vec{a}.\vec{b} + 2\vec{b}.\vec{c} + 2\vec{a}.\vec{c} = 2\vec{a}.\vec{b} - 2\vec{b}.\vec{c} - 2\vec{a}.\vec{c}$

$\vec{a}.\vec{b} + \vec{a}.\vec{c} = \vec{a}.\vec{b} - \vec{a}.\vec{c}$

$\vec{a}.\vec{c} = 0$ (B is incorrect)

$|\vec{a} + \lambda\vec{c}|^2 \geq |\vec{a}|^2$

$|\vec{a}|^2 + \lambda^2|\vec{c}|^2 + 2\lambda\vec{a}\cdot\vec{c} \geq |\vec{a}|^2$

$= \lambda^2 c^2 \geq 0$

True $\forall \lambda \in R$ (A is correct)

70. The value of $\int_{\pi/3}^{\pi/2} \frac{(2+3\sin x)}{\sin x(1+\cos x)}dx$ is equal to

(1) $\frac{10}{3}-\sqrt{3}-\log_e \sqrt{3}$ (2) $\frac{7}{2}-\sqrt{3}-\log_e \sqrt{3}$

(3) $-2+3\sqrt{3}+\log_e \sqrt{3}$ (4) $\frac{10}{3}-\sqrt{3}+\log_e \sqrt{3}$

Sol. **(4)**

$$\int_{\pi/3}^{\pi/2} \frac{(2+3\sin x)}{\sin x(1+\cos x)}dx$$

$$= \int_{\pi/3}^{\pi/2} \frac{2}{\sin x+\sin x\cos x}dx + \int_{\pi/3}^{\pi/2} \frac{3}{1+\cos x}dx$$

$= I_1 + I_2$

$$I_1 = \int_{\pi/3}^{\pi/2} \frac{2dx}{\sin x(1+\cos x)} = 2\int_{\pi/3}^{\pi/2} \frac{\left(1+\tan^2\frac{x}{2}\right)dx}{2\tan\frac{x}{2}\times\left(1+\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}$$

$$= 2\int_{\pi/3}^{\pi/2} \frac{\left(1+\tan^2\frac{x}{2}\right)\left(1+\tan^2\frac{x}{2}\right)dx}{2\tan\frac{x}{2}\times 2}$$

$$= 2\int_{\pi/3}^{\pi/2} \frac{\sec^2\frac{x}{2}\left(1+\tan^2\frac{x}{2}\right)}{4\tan\frac{x}{2}}dx$$

Let, $\tan\frac{x}{2} = t$ then $\sec^2\frac{x}{2}\times\frac{1}{2}dx = dt$

$$= 2\int_{1/\sqrt{3}}^{1} \frac{1+t^2}{2t}dt$$

$$= \left[\ell nt+\frac{t^2}{2}\right]_{1/\sqrt{3}}^{1}$$

$$= \left[\frac{1}{2}-\ell n\frac{1}{\sqrt{3}}-\frac{1}{6}\right]$$

$$I_1 = \left[\ell n\sqrt{3}+\frac{1}{3}\right]$$

$$I_2 = 3\int_{\pi/3}^{\pi/2} \frac{dx}{1+\cos x} = 3\int_{\pi/3}^{\pi/2} \frac{1-\cos x}{\sin^2 x}dx$$

$I_2 = 3[\operatorname{cosec} x - \cot x]_{\frac{\pi}{3}}^{\frac{\pi}{2}} = 3 - \sqrt{3}$

$I_1 + I_2 = \ln\sqrt{3} + \frac{1}{3} + 3 - \sqrt{3}$

$= \frac{10}{3} + \ln\sqrt{3} - \sqrt{3}$

71. Let the shortest distance between the lines $L: \frac{x-5}{-2} = \frac{y-\lambda}{0} = \frac{z+\lambda}{1}, \lambda \geq 0$ and $L_1: x+1 = y-1 = 4-z$ be $2\sqrt{6}$. If (α, β, γ) lies on L, then which of the following is NOT possible ?

(1) $\alpha - 2\gamma = 19$ (2) $2\alpha + \gamma = 7$ (3) $2\alpha - \gamma = 9$ (4) $\alpha + 2\gamma = 24$

Sol. **(4)**

Let $\vec{b}_1 = <-2, 0, 1> \quad \vec{a}_1 = (5, \lambda, -\lambda)$

$\vec{b}_2 = <1, 1, -1 \quad > \vec{a}_2 = (-1, 1, 4)$

Normal vector of both line is $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix}$

$\hat{i}(-1) - \hat{j}(1) + \hat{k}(-2)$

$\vec{b}_1 \times \vec{b}_2 = <-1, -1, -2>$

$\vec{a}_1 - \vec{a}_2 = <6, \lambda - 1, -\lambda - 4>$

Shortest distance d = $\left| \frac{(\vec{a}_2 - \vec{a}_1) \times (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$

$2\sqrt{6} = \left| \frac{<6, \lambda - 1, -\lambda - 4> \times <-1, -1, -2>}{\sqrt{(1)^2 + (1)^2 + (2)^2}} \right|$

$12 = |-6 - \lambda + 1 + 2\lambda + 8|$

$|\lambda + 3| = 12$

$\lambda = 9, -15$

$\lambda = 9 (\because \lambda \geq 0)$

$\because$ (α, β, γ) lies on line L then

$\frac{\alpha - 5}{-2} = \frac{\beta - 9}{0} = \frac{\gamma + 9}{1} = K$

$\alpha = 5 - 2K$, $\beta = 9K$, $\gamma = -9 + K$

$\alpha + 2\gamma, = 5 - 2K - 18 + 2K = -13 \neq 24$

Therefore $\alpha + 2\gamma = 24$ is not possible.

72. For the system of linear equations

$x + y + z = 6$

$\alpha x + \beta y + 7z = 3$

$x + 2y + 3z = 14$

which of the following is NOT true ?

(1) If $\alpha = \beta$ and $\alpha \neq 7$, then the system has a unique solution

(2) If $\alpha = \beta = 7$, then the system has no solution

(3) For every point $(\alpha, \beta) \neq (7,7)$ on the line $x - 2y + 7 = 0$, the system has infinitely many solutions

(4) There is a unique point (α, β) on the line $x + 2y + 18 = 0$ for which the system has infinitely many solutions

Sol. **(3)**

$x + y + z = 6$ … (1)

$\alpha x + \beta y + 7z = 3$ … (2)

$x + 2y + 3z = 14$ … (3)

equation (3) – equation (1)

$y + 2z = 8$

$y = 8 - 2z$

From (1) $x = -2 + z$

Value of x and y put in equation (2)

$\alpha(-2 + z) + \beta(8 - 2z) + 7z = 3$

$-2\alpha + \alpha z + 8\beta - 2\beta z + 7z = 3$

$(\alpha - 2\beta + 7)z = 2\alpha - 8\beta + 3$

if $\alpha - 2\beta + 7 \neq 0$ then system has unique solution

if $(\alpha - 2\beta + 7 = 0)$ and $2\alpha - 8\beta + 3 \neq 0$ then system has no solution

if $(\alpha - 2\beta + 7 = 0)$ and $2\alpha - 8\beta + 3 = 0$ then system has infinite solution

73. If the domain of the function $f(x) = \frac{[x]}{1+x^2}$, where $[x]$ is greatest integer $\leq x$, is $[2,6)$, then its range is

(1) $\left(\frac{5}{26}, \frac{2}{5}\right]$

(2) $\left(\frac{5}{37}, \frac{2}{5}\right] - \left\{\frac{9}{29}, \frac{27}{109}, \frac{18}{89}, \frac{9}{53}\right\}$

(3) $\left(\frac{5}{37}, \frac{2}{5}\right]$

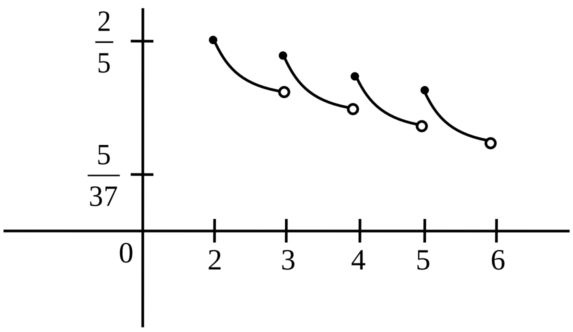
(4) $\left(\frac{5}{26}, \frac{2}{5}\right] - \left\{\frac{9}{29}, \frac{27}{109}, \frac{18}{89}, \frac{9}{53}\right\}$

Sol. **(3)**

$f(x) = \frac{[x]}{1+x^2}$, $x = \in [2, 6]$

$$f(x)=\begin{cases}\dfrac{2}{1+x^2} & x\in[2,3)\\ \dfrac{3}{1+x^2} & x\in[3,4)\\ \dfrac{4}{1+x^2} & x\in[4,5)\\ \dfrac{5}{1+x^2} & x\in[5,6)\end{cases}$$

$\because$ $f(x)$ is $\downarrow$ in $x\in[2, 6)$



range is $\left(\dfrac{5}{37},\dfrac{2}{5}\right]$

74. Let R be a relation on N×N defined by $(a,b)R(c,d)$ if and only if $ad(b-c)=bc(a-d)$. Then R is

(1) transitive but neither reflexive nor symmetric

(2) symmetric but neither reflexive nor transitive

(3) symmetric and transitive but not reflexive

(4) reflexive and symmetric but not transitive

Sol. **(2)**

(a, b) R (c, d) $\Leftrightarrow$ ad(b – c) = bc(a – d)

For reflexive

(a, b) R (a, b)

$\Rightarrow$ ab (b – a) $\neq$ ba(a – b)

R is not reflexive

For symmetric:

(a,b) R(c,d) $\Rightarrow$ ad(b –c) = bc (a –d)

then we check

(c, d) R (a, b) $\Rightarrow$ cb(d – a) = ad(c – b)

$\Rightarrow$ cb(a – d) = ad(b – c)

R is symmetric :

For transitive:

$\because$ (2,3) R (3,2) and (3,2) R (5,30)

But (2,3) is not related to (5,30)

R is not transitive.

75. $(S1)(p \Rightarrow q) \vee (p \wedge (\sim q))$ is a tautology

$(S2)((\sim p) \Rightarrow (\sim q)) \wedge ((\sim p) \vee q)$ is a contradiction.

Then

(1) both (S1) and (S2) are correct
(2) only (S1) is correct
(3) only (S2) is correct
(4) both (S1) and (S2) are wrong

Sol. **(2)**

$S_1 : (P \Rightarrow q) \vee (P \wedge (\sim q))$

P	q	$P \Rightarrow q$	$\sim q$	$P \wedge \sim q$	$(P \Rightarrow q) \vee (P \wedge \sim q)$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	T	F	T

S_1 is a tautology

$S_2 : ((\sim P) \Rightarrow (\sim q)) \wedge ((\sim P) \vee q)$

$\sim P$	$\sim q$	$\sim P \Rightarrow \sim q$	$\sim P \vee q$	$((\sim P) \Rightarrow (\sim q)) \wedge (\sim P) \vee q)$
F	F	T	T	T
F	T	T	F	F
T	F	F	T	F
T	T	T	T	T

S_2 is not a contradiction

76. If the sum and product of four positive consecutive terms of a G.P., are 126 and 1296 , respectively, then the sum of common ratios of all such GPs is

(1) 7
(2) 3
(3) $\frac{9}{2}$
(4) 14

Sol. **(1)**

Four term of G.P. $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$

$$\frac{a}{r^3} + \frac{a}{r} + ar + ar^3 = 126$$

$$\frac{a}{r^3} \cdot \frac{a}{r} \cdot ar \cdot ar^3 = 1296$$

$a^4 = 1296$

$a = 6$

$$\frac{6}{r^3} + \frac{6}{r} + 6r + 6r^3 = 126$$

$$\left(r+\frac{1}{r}\right)+r^3+\frac{1}{r^3}=21$$

$$\left(r+\frac{1}{r}\right)+\left(r+\frac{1}{r}\right)^3-3\left(r+\frac{1}{r}\right)=21$$

Let $r+\frac{1}{r}=t$

$t^3 - 2t = 21$

$\Rightarrow t = 3$

$r+\frac{1}{r}=3$

$r^2 - 3r + 1 = 0$

$r = \frac{3\pm\sqrt{9-4}}{2}$

$r=\frac{3\pm\sqrt{5}}{2}$

Sum of common ratio = $\frac{9}{4}+\frac{5}{4}+\frac{3\sqrt{5}}{2}+\frac{9}{4}+\frac{5}{4}-\frac{3\sqrt{5}}{2}$

= 7

77. Let $A=\begin{pmatrix}1 & 0 & 0\\ 0 & 4 & -1\\ 0 & 12 & -3\end{pmatrix}$. Then the sum of the diagonal elements of the matrix $(A + I)^{11}$ is equal to

(1) 6144 (2) 2050 (3) 4097 (4) 4094

Sol. **(3)**

$$A=\begin{bmatrix}1 & 0 & 0\\ 0 & 4 & -1\\ 0 & 12 & -3\end{bmatrix}$$

$$A^2=\begin{bmatrix}1 & 0 & 0\\ 0 & 4 & -1\\ 0 & 12 & -3\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 4 & -1\\ 0 & 12 & -3\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 4 & -1\\ 0 & 12 & -3\end{bmatrix}$$

$A^3 = A^4 = A^5 \ldots = A$

$(A + I)^{11} = {}^{11}C_0A^{11} + {}^{11}C_1A^{10} + {}^{11}C_2A^9 + \ldots {}^{11}C_{11}I$

$= =\left({}^{11}C_0 + {}^{11}C_1 + {}^{11}C_2 + \ldots {}^{11}C_{10}\right)A + I$

$= (2^{11} - 1)A + I$

$= 2047\,A + I$

Sum of diagonal element = 2047(1 + 4– 3) +3

= 4097

78. The number of real roots of the equation $\sqrt{x^2-4x+3}+\sqrt{x^2-9}=\sqrt{4x^2-14x+6}$, is :

(1) 3 (2) 1 (3) 2 (4) 0

Sol. (2)

$$\sqrt{x^2-4x+3}+\sqrt{x^2-9}=\sqrt{4x^2-14x+6}$$

$$\sqrt{(x-3)(x-1)}+\sqrt{(x-3)(x+3)}=\sqrt{4x^2-12x-2x+6}$$

$$\Rightarrow \sqrt{x-3}\left(\sqrt{x-1}+\sqrt{x+3}\right)=\sqrt{4x(x-3)-2(x-3)}$$

$$\Rightarrow \sqrt{x-3}\left(\sqrt{x-1}+\sqrt{x+3}\right)=\sqrt{(4x-2)(x-3)}$$

$$\Rightarrow \sqrt{x-3}\left(\sqrt{x-1}+\sqrt{x+3}-\sqrt{2(2x-1)}\right)=0$$

$$\sqrt{x-3}=0 \text{ or } \sqrt{x-1}+\sqrt{x+3}-\sqrt{2(2x-1)}=0$$

$x = 3$ or $\sqrt{x-1}+\sqrt{x+3}=\sqrt{2(2x-1)}$

$x-1+x+3+2\sqrt{(x-1)(x+3)}=4x-2$

$$\Rightarrow 2\sqrt{(x-1)(x+3)}=2x-4$$

$$\Rightarrow (x-1)(x+3)=(x-2)^2$$

$$\Rightarrow x^2+2x-3=x^2+4-4x$$

$$\Rightarrow 6x=7$$

$x=\frac{7}{6}$ (not possible)

Number of real root = 1

79. If $\sin^{-1}\frac{\alpha}{17}+\cos^{-1}\frac{4}{5}-\tan^{-1}\frac{77}{36}=0, 0<\alpha<13$, then $\sin^{-1}(\sin\alpha)+\cos^{-1}(\cos\alpha)$ is equal to

(1) 16 (2) 0 (3) π (4) $16-5\pi$

Sol. (3)

$$\sin^{-1}\frac{\alpha}{17}+\cos^{-1}\frac{4}{5}-\tan^{-1}\frac{77}{36}=0,\ 0<\alpha<13$$

$$\sin^{-1}\frac{\alpha}{17}=\tan^{-1}\frac{77}{36}-\tan^{-1}\frac{3}{4}$$

$$=\tan^{-1}\left(\frac{\frac{77}{36}-\frac{3}{4}}{1+\frac{77}{36}\cdot\frac{3}{4}}\right)$$

$\sin^{-1}\frac{\alpha}{17} = \tan^{-1}\left(\frac{8}{15}\right) = \sin^{-1}\left(\frac{8}{17}\right)$

$\frac{\alpha}{17} = \frac{8}{17}$

$\alpha = 8$

$\sin^{-1}(\sin 8) + \cos^{-1}(\cos 8)$

$= 3\pi - 8 + 8 - 2\pi$

$= \pi$

80. Let $\alpha \in (0,1)$ and $\beta = \log_e(1-\alpha)$. Let $P_n(x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots + \frac{x^n}{n}, x \in (0,1)$.

Then the integral $\int_0^\alpha \frac{t^{50}}{1-t}dt$ is equal to

(1) $\beta + P_{50}(\alpha)$ (2) $P_{50}(\alpha) - \beta$ (3) $\beta - P_{50}(\alpha)$ (4) $-(\beta + P_{50}(\alpha))$

Sol. **4**

$\alpha \in (0,1), \beta = \log_e(1-\alpha)$

$P_n(x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \ldots\ldots + \frac{x^n}{n}, x \in (0, 1)$

$\int_0^\alpha \frac{t^{50}-1+1}{1-t}dt$

$-\int_0^\alpha \frac{1-t^{50}}{1-t}dt + \int_0^\alpha \frac{1}{1-t}dt$

$-\int_0^\alpha (1+t+t^2+\ldots\ldots t^{49})dt - [\ln(1-t)]_0^\alpha$

$-\left[t + \frac{t^2}{2} + \frac{t^3}{3} + \ldots\ldots \frac{t^{50}}{50}\right]_0^\alpha - \ln(1-\alpha)$

$-\left[\alpha + \frac{\alpha^2}{2} + \frac{\alpha^3}{3} + \ldots\ldots + \frac{\alpha^{50}}{50}\right] - \ln(1-\alpha)$

$-P_{50}(\alpha) - \ln(1-\alpha)$

$-(\beta + P_{50}(\alpha))$

Section : Mathematics Section B

81. Let $\alpha > 0$, be the smallest number such that the expansion of $\left(x^{\frac{2}{3}} + \frac{2}{x^3}\right)^{30}$ has a term

$\beta x^{-\alpha}, \beta \in \mathbb{N}$. Then α is equal to

Sol. **2**

$T_{r+1} = {}^{30}C_r \left(x^{\frac{2}{3}}\right)^{30-r} \left(\frac{2}{x^3}\right)^4$

$= {}^{30}C_r 2^r x^{\frac{60-11r}{3}}$

$\frac{60-11r}{3} < 0$

$11r > 60$

$r > \frac{60}{11}$

$r = 6$

$T_7 = {}^{30}C_6\ 2^6 x^{-2}$ then

$\beta = {}^{30}C_6 \times 2^6 \in N$

$\alpha = 2$

82. Let for $x \in \mathbb{R}$,

$$f(x) = \frac{x+|x|}{2} \text{ and } g(x) = \begin{cases} x, & x < 0 \\ x^2, & x \geq 0 \end{cases}.$$

Then area bounded by the curve $y = (f \circ g)(x)$ and the lines $y = 0, 2y - x = 15$ is equal to

Sol. **72**

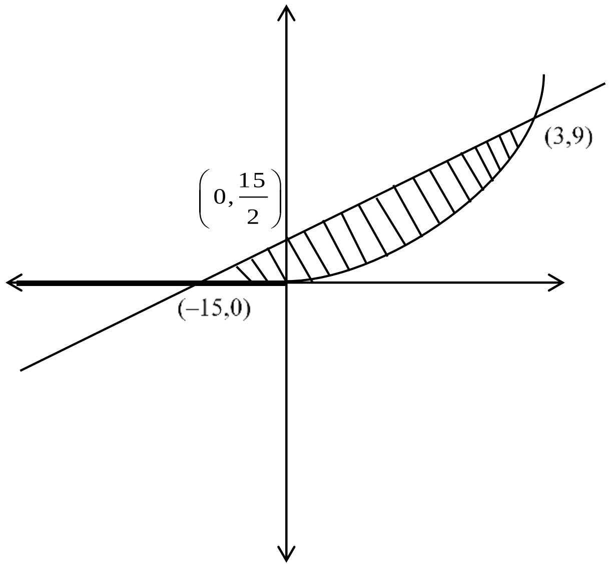
$f(x) = \frac{x+|x|}{2} = \begin{cases} x & x \geq 0 \\ 0 & x < 0 \end{cases}$

$g(x) = \begin{cases} x^2 & x \geq 0 \\ x & x < 0 \end{cases}$

$Fog(x) = f\{g(x)\} = \begin{cases} g(x) & g(x) \geq 0 \\ 0 & g(x) < 0 \end{cases}$

$fog(x) = \begin{cases} x^2 & x \geq 0 \\ 0 & x < 0 \end{cases}$

given lines are $2y - x = 15$ and $y = 0$



Area = $\int_0^3 \left(\frac{x+15}{2} - x^2\right)dx + \frac{1}{2}\times\frac{15}{2}\times 15$

$= \left[\frac{x^2}{4} + \frac{15x}{2} - \frac{x^3}{3}\right]_0^3 + \frac{225}{4}$

$= \frac{9}{4} + \frac{45}{2} - 9 + \frac{225}{4}$

Area = 72

83. Number of 4-digit numbers that are less than or equal to 2800 and either divisible by 3 or by 11 , is equal to

Sol. 710

4 digit number which are less then 2800 are 1000 – 2799

Number which are divisible by 3

$2799 = 1002 + (n-1)\,3$

$n = 600$

Number which are divisible by 11 in 1000 – 2799

= (Number which are divisible by 11 in 1 – 2799)

– (Number which are divisible by 11 in 1 – 999)

$= \left[\frac{2799}{11}\right] - \left[\frac{999}{11}\right]$

= 254 – 90

= 164

Number which are divisible by 33 in 1000 – 2799

= (Number which are divisible by 33 in 1 – 2799) – (Number which are divisible by 33 in 1 – 999)

$= \left[\frac{2799}{33}\right] - \left[\frac{999}{33}\right]$

= 84 – 30 = 54

total number = n(3) + n(11) – n(33)

= 600 + 164 – 54 = 710

84. If the variance of the frequency distribution

x_i	2	3	4	5	6	7	8
Frequency f_i	3	6	16	α	9	5	6

is 3 , then α is equal to

Sol. **5**

x_i	f_i	$d_i = x_i - 5$	$(f_i\, d_i)^2$	$f_i d_i$
2	3	–3	27	–9
3	6	–2	24	–12
4	16	–1	16	–16
5	α	0	0	0
6	9	1	9	9
7	5	2	20	10
8	6	3	54	18

$$\sigma^2 = \frac{\sum f_i d_i^2}{\sum f_i} - \left(\frac{\sum f_i d_i}{\sum f_i}\right)^2$$

$$= \frac{150}{45+\alpha} - 0 = 3$$

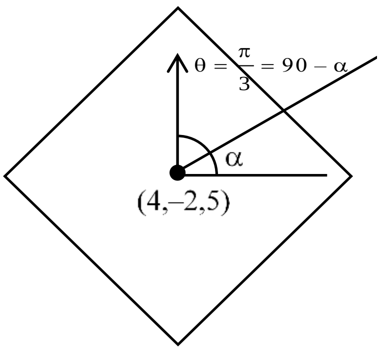
$\Rightarrow 150 = 135 + 3\alpha$

$\Rightarrow 3\alpha = 15$

$\Rightarrow \alpha = 5$

85. Let θ be the angle between the planes $P_1: \vec{r}\cdot(\hat{i}+\hat{j}+2\hat{k}) = 9$ and $P_2: \vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k}) = 15$. Let L be the line that meets P_2 at the point $(4,-2,5)$ and makes an angle θ with the normal of P_2. If α is the angle between L and P_2, then $(\tan^2\theta)(\cot^2\alpha)$ is equal to

Sol. **9**



$$\cos\theta = \frac{\langle 1,1,2\rangle . \langle 2,-1,1\rangle}{6}$$

$$= \frac{2-1+2}{6} = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

then $\frac{\pi}{2} - \alpha = \frac{\pi}{3}$

$$\alpha = \frac{\pi}{6}$$

$$(\tan^2\theta)(\cot^2\alpha) = (\sqrt{3})^2 \times (\sqrt{3})^2 = 9$$

86. Let 5 digit numbers be constructed using the digits 0,2,3,4,7,9 with repetition allowed, and are arranged in ascending order with serial numbers. Then the serial number of the number 42923 is

Sol. **2997**

2	$\frac{-}{6}$	$\frac{-}{6}$	$\frac{-}{6}$	$\frac{-}{6}$	= 1296
3	$\frac{-}{6}$	$\frac{-}{6}$	$\frac{-}{6}$	$\frac{-}{6}$	= 1296
4	0	$\frac{-}{6}$	$\frac{-}{6}$	$\frac{-}{6}$	=216
4	2	0	$\frac{-}{6}$	$\frac{-}{6}$	= 36
4	2	2	$\frac{-}{6}$	$\frac{-}{6}$	= 36
4	2	3	$\frac{-}{6}$	$\frac{-}{6}$	= 36
4	2	4	$\frac{-}{6}$	$\frac{-}{6}$	= 36
4	2	7	$\frac{-}{6}$	$\frac{-}{6}$	= 36
4	2	9	0	$\frac{-}{6}$	= 6
4	2	9	2	$\underline{0}$	= 1
4	2	9	2	$\underline{2}$	= 1
4	2	9	2	$\underline{3}$	= $\frac{1}{2997}$

87. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}| = \sqrt{14}, |\vec{b}| = \sqrt{6}$ and $|\vec{a} \times \vec{b}| = \sqrt{48}$.

Then $(\vec{a} \cdot \vec{b})^2$ is equal to

Sol. **36**

$$\left|\vec{a} \times \vec{b}\right|^2 = |\vec{a}|^2 \left|\vec{b}\right|^2 - \left(\vec{a} \cdot \vec{b}\right)^2$$

$$48 = 14 \times 6 - \left(\vec{a} \cdot \vec{b}\right)^2$$

$$\left(\vec{a} \cdot \vec{b}\right)^2 = 84 - 48$$

$$\left(\vec{a} \cdot \vec{b}\right)^2 = 36$$

88. Let the line $L: \frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{1}$ intersect the plane $2x + y + 3z = 16$ at the point P. Let the point Q be the foot of perpendicular from the point $R(1, -1, -3)$ on the line L. If α is the area of triangle PQR, then α^2 is equal to

Sol. **180**

Point on line L is $(2\lambda + 1, -\lambda - 1, \lambda + 3)$

If above point is intersection point of line L and plane then

$2(2\lambda + 1) + (-\lambda - 1) + 3(\lambda + 3) = 16$

$\lambda = 1$

Point P = (3, –2, 4)

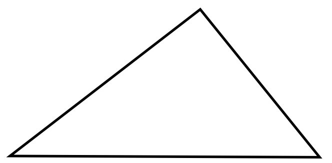
Dr of QR = $< 2\lambda, -\lambda, \lambda + 6 >$

Dr of L = $< 2, -1, 1 >$

$4\lambda + \lambda + \lambda + 6 = 0$

$\lambda = -1$

Q = (–1, 0, 2)



$\overrightarrow{QR} = 2\hat{i} - \hat{j} - 5\hat{k}$

$\overrightarrow{QP} = 4\hat{i} - 2\hat{j} + 2\hat{k}$

$$\overrightarrow{QR} \times \overrightarrow{QP} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & -5 \\ 4 & -2 & 2 \end{vmatrix} = -12\hat{i} - 24\hat{j}$$

$$\alpha = \frac{1}{2} \times \sqrt{144 + 576}$$

$$\alpha^2 = \frac{720}{4} = 180$$

$$\alpha^2 = 180$$

89. Let $a_1, a_2, \ldots, a_n$ be in A.P. If $a_5 = 2a_7$ and $a_{11} = 18$, then

$12\left(\frac{1}{\sqrt{a_{10}}+\sqrt{a_{11}}} + \frac{1}{\sqrt{a_{11}}+\sqrt{a_{12}}} + \cdots + \frac{1}{\sqrt{a_{17}}+\sqrt{a_{18}}}\right)$ is equal to

Sol. **8**

Given that

$a_5 = 2a_7$

$a_1 + 4d = 2(a_1 + 6d)$

$a_1 + 8d = 0$

$a_1 + 10d = 18$

$a_1 = -72$, $d = 9$

$a_{18} = a_1 + 17d = -72 + 153 = 81$

$a_{10} = a_1 + 9d = 9$

$$12\left(\frac{\sqrt{a_{11}}-\sqrt{a_{10}}}{d}+\frac{\sqrt{a_{12}}-\sqrt{a_{11}}}{d}+\ldots\ldots\frac{\sqrt{a_{18}}-\sqrt{a_{17}}}{d}\right)$$

$$= 12\left(\frac{\sqrt{a_{18}}-\sqrt{a_{10}}}{d}\right)$$

$$= \frac{12\times(9-3)}{9} = 8$$

90. The remainder on dividing 5^{99} by 11 is :

Sol. **9**

$5^{99} = 5^4\, 5^{95}$

$= 625\,(5^5)^{19}$

$= 625\,(3125)^{19}$

$= 625(3124 + 1)^{19}$

$= 625(11\lambda + 1)$

$= 11\,\lambda \times 625 + 625$

$= 11\,\lambda \times 625 + 616 + 9$

$= 11 \times k + 9$

Remainder = 9

मोशन है, तो भरोसा है।
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