JEE MAIN (Session 2) 2023
Paper Analysis

MATHS | 12th April 2023 _ Shift-1

Continuing to keep the pledge
of **imparting education** for the **last 16 Years**

49600+
SELECTIONS SINCE 2007

JEE (Advanced)	JEE (Main)	NEET/AIIMS	NTSE/OLYMPIADS
9395	26591	11383 (Under 50000 Rank)	2235 (6th to 10th class)

Most Promising RANKS
Produced by MOTION Faculties

NEET / AIIMS

AIR-1 to 10
25 Times

AIR-11 to 50
83 Times

AIR-51 to 100
81 Times

JEE MAIN+ADVANCED

AIR-1 to 10
8 Times

AIR-11 to 50
32 Times

AIR-51 to 100
36 Times

Nation's Best SELECTION
Percentage (%) Ratio

Student Qualified in NEET

(2022)
4837/5356 = **90.31%**
(2021)
3276/3411 = **93.12%**

Student Qualified in JEE ADVANCED

(2022)
1756/4818 = **36.45%**
(2021)
1256/2994 = **41.95%**

Student Qualified in JEE MAIN

(2022)
4818/6653 = **72.41%**
(2021)
2994/4087 = **73.25%**

NITIN VIIJAY (NV Sir)
Founder & CEO

SECTION - A

1. Let y = y (x), y > 0, be a solution curve of the differential equation $(1 + x^2)\, dy = y\,(x - y)dx$. If y(0) = 1 and $y\left(2\sqrt{2}\right) = \beta$, then

(1) $e^{3\beta^{-1}} = e(5+\sqrt{2})$ (2) $e^{3\beta^{-1}} = e(3+2\sqrt{2})$

(3) $e^{\beta^{-1}} = e^{-2}(3+2\sqrt{2})$ (4) $e^{\beta^{-1}} = e^{-2}(5+\sqrt{2})$

Sol. **2**

$(1+x^2)\, dy = y\,(x - y)dx$

$Y(0) = 1.\ \ y(2\sqrt{2}) = \beta$

$$\frac{dy}{dx} = \frac{yx - y^2}{1+x^2}$$

$$\frac{dy}{dx} + y\left(\frac{-x}{1+x^2}\right) = \left(\frac{-1}{1+x^2}\right)y^2$$

$$\frac{1}{y^2}\frac{dy}{dx} + \frac{1}{y}\left(\frac{-x}{1+x^2}\right) = \frac{-1}{1+x^2}$$

Put $\frac{1}{y} = t$ then $\frac{-1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$

$$\frac{dt}{dx} + t\frac{x}{1+x^2} = \frac{1}{1+x^2}$$

If $= e^{\int \frac{x}{1+x^2}dx} = e^{\frac{1}{2}\ln(1+x^2)}\sqrt{1+x^2}$

$$t\sqrt{1+x^2} = \int \frac{1}{\sqrt{1+x^2}}dx$$

$$\frac{\sqrt{1+x^2}}{y} = \ln\left(x+\sqrt{x^2+1}\right) + c$$

$y(0) = 1 \Rightarrow c = 1$

$$\Rightarrow \sqrt{1+x^2} = y\ln(e(x+\sqrt{x^2+1}))$$

$$\beta = \frac{3}{\ln(e(3+2\sqrt{2}))} \Rightarrow \frac{3}{\beta} = \ln(e(3+2\sqrt{2}))$$

$$e^{\frac{3}{\beta}} = e(3+2\sqrt{2})$$

2. Let D be the domain of the function $f(x) = \text{Sin}^{-1}\left(\log_{3x}\left(\frac{6+2\log_3 x}{-5x}\right)\right)$. If the range of the function $g : D \to R$ defined by g(x) = x – [x], ([x] is the greatest integer function), is (α, β), then $\alpha^2 + \frac{5}{\beta}$ is equal to

(1) 46 (2) 135 (3) 136 (4) 45

Sol. **Bonus**

$$\frac{6+2\log_3 x}{-5x} > 0 \;\&\; x > 0 \;\&\; x \neq \frac{1}{3}$$

this gives $x \in \left(0, \frac{1}{27}\right)$ ….(1)

$$-1 \le \log_{3x}\left(\frac{6+2\log_3 x}{-5x}\right) \le 1$$

$$3x \le \frac{6+2\log_3 x}{-5x} \le \frac{1}{3x}$$

$15x^2 + 6 + 2\log_3 x \ge 0 \qquad 6 + 2\log_3 x + \frac{5}{3} \ge 0$

$x \in \left(0, \frac{1}{27}\right)$ ….(2) $\qquad x \ge 3^{-\frac{23}{6}}$ ….(3)

form (1), (2) & (3)

$$x \in \left[3^{-\frac{23}{6}}, \frac{1}{27}\right)$$

$\therefore$ α is small positive quantity

$\&\ \beta = \frac{1}{27}$

$\therefore\ \alpha^2 + \frac{5}{\beta}$ is just greater than 135

3. Let C be the circle in the complex plane with centre $z_0 = \frac{1}{2}(1+3i)$ and radius r = 1. Let $z_1 = 1 + i$ and the complex number z_2 be outside the circle C such that $|z_1 - z_0|\,|z_2 - z_0| = 1$. If z_0. z_1 and z_2 are collinear, then the smaller value of $|z_2|^2$ is equal to

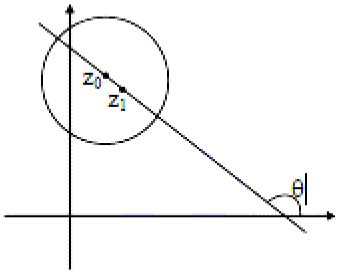
(1) $\frac{7}{2}$ (2) $\frac{13}{2}$ (3) $\frac{5}{2}$ (4) $\frac{3}{2}$

Sol. **3**

$$|z_1 - z_0| = \left|\frac{1-i}{2}\right| = \frac{1}{2}$$

$\Rightarrow |z_2 - z_0| = \sqrt{2}$: centre $\left(\frac{1}{2}, \frac{3}{2}\right)$

$z_\circ\left(\frac{1}{2}, \frac{3}{2}\right)$ and z_1 (1, 1)



$\tan\theta = -1 \Rightarrow \theta = 135°$

$z_2\left(\frac{1}{2}+\sqrt{2}\cos 135°, \frac{3}{2}+\sqrt{2}\sin 135°\right)$

or

$\left(\frac{1}{2}-\sqrt{2}\cos 135°, \frac{3}{2}-\sqrt{2}\sin 135°\right)$

$\Rightarrow z_2\left(-\frac{1}{2}, \frac{5}{2}\right) \text{or } z_2\left(\frac{3}{2}, \frac{1}{2}\right)$

$\Rightarrow |z_3|^2 = \frac{26}{4}, \frac{5}{2}$

$\Rightarrow |z_2|^2_{min} = \frac{5}{2}$

4. Let $< a_n >$ be a sequence such that $a_1 + a_2 + .. + a_n = \frac{n^2+3n}{(n+1)(n+2)}$. If $28\sum_{k=1}^{10}\frac{1}{a_k} = p_1p_2p_3\ldots p_m$, where $p_1, p_2\ldots\ldots p_m$ are the first m prime numbers, then m is equal to

(1) 8 (2) 5 (3) 6 (4) 7

Sol. **3**

$a_n = S_n - S_{n-1} = \frac{n^2+3n}{(n+1)(1+2)} - \frac{(n-1)(n+2)}{n(n+1)}$

$\Rightarrow a_n = \frac{4}{n(n+1)(1+2)}$

$\Rightarrow 28\sum_{k-1}^{10}\frac{1}{a_k} = 28\sum_{k-1}^{10}\frac{k(k+1)(k+2)}{4}$

$= \frac{7}{4}\sum_{k-1}^{10}(k(k+1)(k+2)(k+3) - (k-1)k(k+1)(k+2))$

$= \frac{7}{4}.10.11.12.13 = 2.3.5.7.11.13$

So m = 6

5. Let the plane P: $4x - y + z = 10$ be rotated by an angle $\frac{\pi}{2}$ about its line of intersection with the plane $x + y - z = 4$. If α is the distance of the point $(2, 3, -4)$ from the new position of the plane P, then 35α is equal to

(1) 90 (2) 105 (3) 85 (4) 126

Sol. **4**

Let equation in new position is

$(4x - y + z - 10) + \lambda (x + y - z - 4) = 0$

$4(4 + \lambda) - 1.(-1 + \lambda) + 1. (1 - \lambda) = 0$

$\Rightarrow \lambda = -9$

So equation in new position is

$-5x - 10y + 10z + 26 = 0$

$\Rightarrow \alpha = \frac{54}{15}$

6. Among the two statements

(S1) : $(p \Rightarrow q) \wedge (p \wedge (\sim q))$ is a contradiction and

(S2) : $(p \wedge q) \vee ((\sim p) \wedge q) \vee (p \wedge (\sim q)) \vee ((\sim p) \wedge (\sim q))$ is a tautology

(1) only (S2) is true (2) only (S1) is true (3) both are true (4) both are false

Sol. **3**

$S_1 : (p \rightarrow q) \wedge (p \wedge (\sim q))$

P	Q	$p \rightarrow q$	$p \wedge (\sim q)$	S1
T	T	T	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	F	F

$\Rightarrow S_1$ is Contradiction

S_2

p	q	$p \wedge q$	$(p \wedge \sim q)$	$(p \wedge \sim q)$	$(\sim p) \wedge (\sim p)$	S_2
T	T	T	F	F	F	T
T	F	F	F	T	F	T
F	T	F	T	F	F	T
F	F	F	F	F	T	T

S_2 is tautology

7. Two dice A and B are rolled. Let numbers obtained on A and B be α and β respectively. If the variance of $\alpha - \beta$ is $\frac{p}{q}$, where p and q are co-prime, then the sum of the positive division of p is equal to

(1) 36 (2) 31 (3) 48 (4) 72

Sol. **3**

$\alpha-\beta$	Case	P
5	(6, 1)	1/36
4	(6, 2) (5, 1)	2/36
3	(6, 3) (5, 2) (4, 1)	3/36
2	(6, 4) (5, 3) (4, 3) (3, 1)	4/36
1	(6, 5) (5, 4) (4, 3) (3, 2) (2, 1)	5/36
0	(6, 6) (5, 5) …… (1, 1)	6/36
– 1	- - - - -	5/36
– 2	- - - - -	4/36
– 3	- - - - -	3/36
– 4	(2, 6) (1, 5)	2/36
– 5	(1, 6)	1/36

$$\sum(x^2)=\sum x^2P(x)=2\left[\frac{25}{36}+\frac{32}{36}+\frac{27}{36}+\frac{16}{36}+\frac{5}{36}\right]$$

$$=\frac{105}{18}=\frac{35}{6}$$

$\mu=\sum(x)=0$ as data is symmetric

$$\sigma^2=\sum(x^2)=\sum x^2P(x)=\frac{35}{6}P=35=5\times 7$$

Sum of divisors = $(5^0+5^1)(7^0+7^1)=6\times 8=48$

8. The number of five digit numbers, greater then 40000 and divisible by 5, which can be formed using the digits 0, 1, 3, 5, 7 and 9 without repetition, is equal to

(1) 132 (2) 120 (3) 72 (4) 96

Sol. **(2)**

5	x	x	x	0
7	x	x	x	0
5	x	x	x	5
9	x	x	x	0
9	x	x	x	5

So Required numbers = $5\times {}^4P_3=120$

9. Let α, β be the roots of the quadratic equation $x^2+\sqrt{6}x+3=0$. Then $\dfrac{\alpha^{23}+\beta^{23}+\alpha^{14}+\beta^{14}}{\alpha^{15}+\beta^{15}+\alpha^{10}+\beta^{10}}$ is equal to

(1) 9 (2) 729 (3) 72 (4) 81

Sol. **(4)**

$$\alpha,\beta=\frac{-\sqrt{6}\pm\sqrt{6-12}}{2}=\frac{-\sqrt{6}\pm\sqrt{6}i}{2}$$

$$= \sqrt{3}e^{\pm\frac{3\pi i}{4}}$$

Required expression

$$\frac{(\sqrt{3})^{23}\left(2\cos\frac{69\pi}{4}\right)+(\sqrt{3})^{14}\left(2\cos\frac{42\pi}{4}\right)}{(\sqrt{3})^{15}\left(2\cos\frac{45\pi}{4}\right)+(\sqrt{3})^{10}\left(2\cos\frac{30\pi}{4}\right)}$$

$$(\sqrt{3})^8 = 81$$

10. If the local maximum value of the function $f(x)=\left(\frac{\sqrt{3}e}{2\sin x}\right)^{\sin^2 x}, x\in\left(0,\frac{\pi}{2}\right)$, is $\frac{k}{e}$, then $\left(\frac{k}{e}\right)^8+\frac{k^8}{e^5}+k^8$ is equal to

(1) $e^3 + e^6 + e^{11}$ (2) $e^5 + e^6 + e^{11}$ (3) $e^3 + e^5 + e^{11}$ (4) $e^3 + e^6 + e^{10}$

Sol. **(1)**

Let $y = \left(\frac{\sqrt{3}e}{2\sin x}\right)^{\sin^2 x}$

$\ln y = \sin^2 x.\ln\left(\frac{\sqrt{3}e}{2\sin x}\right)$

$$\frac{1}{y}y' = \ln\left(\frac{\sqrt{3}e}{2\sin x}\right)2\sin x\cos x+\sin^2 x\frac{2\sin x}{\sqrt{3}e}\frac{\sqrt{3}e}{2}\quad(-\text{cosec}x\cot x)$$

$$\Rightarrow \sin x\cos\left[2\ln\left(\frac{\sqrt{3}e}{2\sin x}\right)-1\right]=0$$

$$\Rightarrow \ln\left(\frac{3e}{4\sin^2 x}\right)=1 \Rightarrow \frac{3e}{4\sin^2 x}=e \Rightarrow \sin^2 x=\frac{3}{4}$$

$$\Rightarrow \sin x=\frac{\sqrt{3}}{2}\left(\text{as } x\in\left(0,\frac{\pi}{2}\right)\right)$$

$$\Rightarrow \text{ local max value } = \left(\frac{\sqrt{3}e}{\sqrt{3}}\right)^{3/4}=e^{3/8}=\frac{k}{e}$$

$$\Rightarrow k^8 = e^{11}$$

$$\Rightarrow \left(\frac{k}{e}\right)^8+\frac{k^8}{e^5}+k^8=e^3+e^6+e^{11}$$

11. Let $\lambda\in Z$, $\vec{a}=\lambda\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$. Let $\vec{c}$ be a vector such that $(\vec{a}+\vec{b}+\vec{c})\times\vec{c}=\vec{0}, \vec{a}.\vec{c} = -17$ and $\vec{b}.\vec{c} = -20$. Then $\left|\vec{c}\times\left(\lambda\hat{i}+\hat{j}+\hat{k}\right)\right|^2$ is equal to

(1) 62 (2) 53 (3) 49 (4) 46

Sol. **(4)**

$(\vec{a}+\vec{b}+\vec{c})\times\vec{c}=0$

$(\vec{a}+\vec{b})\times\vec{c}=0$

$\vec{c}=\alpha(\vec{a}+\vec{b})=\alpha(\lambda+3)\hat{i}+\alpha\hat{k}$

$\vec{b}.\vec{c}=-20\Rightarrow 3\alpha(\lambda+3)+2\alpha=-20$

$\vec{a}.\vec{c}=-17\Rightarrow \alpha\lambda(\lambda+3)-\alpha=-17$

$\Rightarrow\alpha(3\lambda+9+2)=-20$

$\alpha(\lambda^2+3\lambda-1)=-17$

$17(3\lambda+11)=20(\lambda^2+3\lambda-1)$

$20\lambda^2+9\lambda-207=0$

$\lambda=3 \qquad (\lambda\in Z)$

$\Rightarrow\alpha=-1 \ \Rightarrow\vec{c}=-(6\hat{i}+\hat{k})$

$\vec{v}=\vec{c}\times(3\hat{i}+\hat{j}+\hat{k})$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -6 & 0 & -1\\ 3 & 1 & 1\end{vmatrix}=\hat{i}+3\hat{j}-6\hat{k}$$

$|\vec{v}|^2=(-1)^2+3^2+6^2=46$

12. In a triangle ABC, if cos A + 2 cos B + cos C = 2 and the lengths of the sides opposite to the angles A and C are 3 and 7 respectively, then cos A – cos C is equal to

(1) $\frac{5}{7}$ (2) $\frac{9}{7}$ (3) $\frac{3}{7}$ (4) $\frac{10}{7}$

Sol. **(4)**

cosA+cosC = 2(1 – cosB)

$2\cos\frac{A+C}{2}\cos\frac{A-C}{2}=4\sin^2 B/2$

as $\cos\left(\frac{A+C}{2}\right)=\sin\frac{B}{2}$

so $\cos\frac{A-C}{2}=2\sin\frac{B}{2}$

$2\cos B/2\cos\frac{A-C}{2}=4\sin B/2\cos B/2$

$2\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right)=4\sin B/2\cos B/2$

$\sin A+\sin C=2\sin B$

$a+c=2b\Rightarrow a=3, c=7, b=5$

$$\cos A-\cos C=\frac{b^2+c^2-a^2}{2bc}-\frac{a^2+b^2-c^2}{2ab}$$

$$=\frac{25+49-9}{70}-\frac{9+25-49}{30}$$

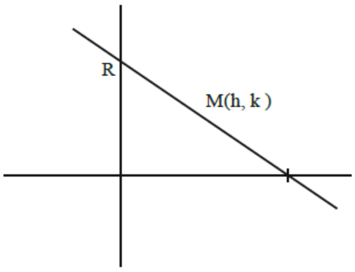
$$\frac{65}{70}+\frac{1}{2}=\frac{20}{14}=\frac{10}{7}$$

13. If the point $\left(\alpha,\frac{7\sqrt{3}}{3}\right)$ lies on the curve traced by the mid-points of the line segments of the lines x cos θ + y sin θ = 7, $\theta\in\left(0,\frac{\pi}{2}\right)$ between the co-ordinates axes, then α is equal to

(1) $7\sqrt{3}$ (2) –7 (3) 7 (4) $-7\sqrt{3}$

Sol. **(3)**

pt $\left(\alpha,\frac{7\sqrt{3}}{3}\right)$



$$x\cos\theta+y\sin\theta=7$$

x- intercept = $\frac{7}{\cos\theta}$

y – intercept = $\frac{7}{\sin\theta}$

$$A:\left(\frac{7}{\cos\theta},0\right)B:\left(0,\frac{7}{\sin\theta}\right)$$

Locus of mid pt M : (h, k)

$$h=\frac{7}{2\cos\theta},k=\frac{7}{2\sin\theta}$$

$$\frac{7}{2\sin\theta}=\frac{7\sqrt{3}}{3}\Rightarrow\sin\theta=\frac{\sqrt{3}}{2}\Rightarrow\theta=\frac{\pi}{3}$$

$$\alpha=\frac{7}{2\cos\theta}=7$$

14. Let $P\left(\frac{2\sqrt{3}}{\sqrt{7}}, \frac{6}{\sqrt{7}}\right)$, Q, R, and S be four points on the ellipse $9x^2 + 4y^2 = 36$. Let PQ and RS be mutually perpendicular and pass through the origin. If $\frac{1}{(PQ)^2} + \frac{1}{(RS)^2} = \frac{p}{q}$, where p and q are coprime, then p + q is equal to

(1) 137 (2) 143 (3) 157 (4) 147

Sol. **(3)**

Let R(2cosθ, 3sinθ)

As $OP \perp OR$

$$\text{so } \frac{3\sin\theta}{2\cos\theta} \times \frac{\frac{6}{\sqrt{7}}}{\frac{2\sqrt{3}}{\sqrt{7}}} = -1$$

$$\Rightarrow \tan\theta = \frac{-2}{3\sqrt{3}}$$

$$\Rightarrow R\left(\frac{-6\sqrt{3}}{\sqrt{31}}, \frac{6}{\sqrt{31}}\right) \text{ or } R\left(\frac{6\sqrt{3}}{\sqrt{31}}, \frac{-6}{\sqrt{31}}\right)$$

$$\text{Now} = \frac{1}{(PQ)^2} + \frac{1}{(RS)^2} = \frac{1}{4}\left(\frac{1}{(OP)^2} + \frac{1}{(OR)^2}\right)$$

$$= \frac{1}{4}\left(\frac{1}{\frac{48}{7}} + \frac{1}{\frac{144}{31}}\right) = \frac{1}{4}\left(\frac{7}{48} + \frac{31}{144}\right)$$

$$= \frac{13}{144}$$

$$\Rightarrow p + q = 157$$

15. Let the lines $l_1 : \frac{x+5}{3} = \frac{y+4}{1} = \frac{z-\alpha}{-2}$ and $l_2 : 3x + 2y + z - 2 = 0 = x - 3y + 2z - 13$ be coplanar. If the point P(a, b, c) on l_1 is nearest to the point Q(–4, –3, 2), then |a| + |b| + |c| is equal to

(1) 10 (2) 8 (3) 12 (4) 14

Sol. **(1)**

$(3x + 2y + z - 2) + \mu (x - 3y + 2z - 13) = 0$

$3(3 + \mu) + 1.(2 - 3\mu) - 2 (1+2\mu) = 0$

$9 - 4 \mu = 0$

$$\mu = \frac{9}{4}$$

$4(-15 - 8 + \alpha - 2) + 9 (-5 + 12 + 2\alpha - 13) = 0$

$-100 + 4\alpha - 54 + 18\alpha = 0$

$\Rightarrow \alpha = 7$

Let $P \equiv (3\lambda - 5, \lambda - 4, -2\lambda + 7)$

Direction ratio of PQ $(3\lambda - 1, \lambda - 1, -2\lambda + 5)$

But $PQ \perp \ell_1$

$\Rightarrow 3(3\lambda - 1) + 1.(\lambda - 1) - 2(-2\lambda + 5) = 0$

$\Rightarrow \lambda = 1$

P $(-2, -3, 5) \Rightarrow |a| + |b| + |c| = 10$

16. If $\frac{1}{n+1}\ {}^{n}C_{n} + \frac{1}{n}\ {}^{n}C_{n-1} + \ldots + \frac{1}{2}\ {}^{n}C_{1} + {}^{n}C_{0} = \frac{1023}{10}$ then n is equal to

(1) 7 (2) 9 (3) 6 (4) 8

Sol. **(2)**

$$\sum_{r=0}^{n} \frac{{}^{n}C_{r}}{r+1} = \frac{1}{n+1} \sum_{r=0}^{n} {}^{n+1}C_{r+1}$$

$$= \frac{1}{n+1}\left(2^{n+1} - 1\right) = \frac{1023}{10}$$

$n + 1 = 10 \Rightarrow n = 9$

17. The sum, of the coefficients of the first 50 terms in the binomial expansion of $(1 - x)^{100}$, is equal to

(1) $-{}^{101}C_{50}$ (2) ${}^{99}C_{49}$ (3) ${}^{101}C_{50}$ (4) $-{}^{99}C_{49}$

Sol. **(4)**

$(1-x)^{100} = C_0 - C_1 x + C_2 x^2 -$

$C_3 x^3 + \ldots\ldots C_{99} x^{99} + C_{100} x^{100}$

$\Rightarrow C_0 - C_1 + C_2 - C_3 + \ldots - C_{99} + C_{100} = 0$

$$C_0 - C_1 + C_2 + \ldots C_{99} = -\frac{1}{2}\ {}^{100}C_{50}$$

$$-\frac{1}{2}\frac{100!}{50!50!} = -\frac{1}{2} \times \frac{100 \times 99!}{50!50!} = -{}^{99}C_{49}$$

18. Let a, b, c be three distinct real numbers, none equal to one. If the vectors $a\hat{i} + \hat{j} + \hat{k}$, $\hat{i} + b\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + c\hat{k}$ are coplanar, then $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$ is equal to

(1) 1 (2) 2 (3) –2 (4) –1

Sol. **(1)**

$$\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$$

$C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$

$$\begin{vmatrix} a & 1-a & 1-a \\ 1 & b-1 & 0 \\ 1 & 0 & c-1 \end{vmatrix} = 0$$

a(b – 1)(c – 1) – (1 – a) (c – 1)+(1– a)(1 – b) = 0

a(1 – b)(1 – c)+(1 – a) (1 – c)+(1 – a)(1 – b) = 0

$$\frac{a}{1-a}+\frac{a}{1-b}+\frac{a}{1-c}=0$$

$$\Rightarrow -1+\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=0$$

$$\Rightarrow \frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=1$$

19. Let $A=\begin{bmatrix} 1 & \frac{1}{51} \\ 0 & 1 \end{bmatrix}$. If $B=\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} A \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$, then the sum of all the elements of the matrix $\sum_{n=1}^{50} B^n$ is equal to

(1) 50 (2) 75 (3) 125 (4) 100

Sol. **(4)**

Let $C=\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}, D=\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$

$$DC=\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=I$$

B = CAD

$$B^n = \underbrace{(CAD)(CAD)(CAD)\ldots\ldots(CAD)}_{n-\text{times}}$$

$$\Rightarrow B^n = CA^nD \quad \ldots (1)$$

$$A^2=\begin{bmatrix} 1 & \frac{1}{51} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & \frac{1}{51} \\ 0 & 1 \end{bmatrix}=\begin{bmatrix} 1 & \frac{2}{51} \\ 0 & 1 \end{bmatrix}$$

$$A^3=\begin{bmatrix} 1 & \frac{3}{51} \\ 0 & 1 \end{bmatrix}$$

Similarly $A^n=\begin{bmatrix} 1 & \frac{n}{51} \\ 0 & 1 \end{bmatrix}$

$$Bn=\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & \frac{n}{51} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$$

$$=\begin{bmatrix}1 & \frac{n}{51}+2\\ -1 & -\frac{n}{51}-1\end{bmatrix}\begin{bmatrix}-1 & -2\\ 1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}\frac{n}{51}+1 & \frac{n}{51}\\ -\frac{n}{51} & 1-\frac{n}{51}\end{bmatrix}$$

$$\sum_{n=1}^{50}B^n=\begin{bmatrix}25-50 & 25\\ -25 & -25-50\end{bmatrix}=\begin{bmatrix}75 & 25\\ -25 & 25\end{bmatrix}$$

Sum of the elements = 100

20. The area of the region enclosed by the curve $y = x^3$ and its tangent at the point $(-1, -1)$ is

(1) $\frac{23}{4}$ (2) $\frac{19}{4}$ (3) $\frac{31}{4}$ (4) $\frac{27}{4}$

Sol. **(4)**

equation of tangent : $y + 1 = 3(x + 1)$
i.e. $y = 3x + 2$

Point of intersection with curve (2, 8)

So Area $= \int_{-1}^{2}\left((3x+2)-x^3\right)dx=\frac{27}{4}$

SECTION - B

21. Let the positive numbers a_1, a_2, a_3, a_4 and a_5 be in a G.P. Let their mean and variance be $\frac{31}{10}$ and $\frac{m}{n}$ respectively, where m and n are co-prime. If the mean of their reciprocal is $\frac{31}{40}$ and $a_3 + a_4 + a_5 = 14$, then m + n is equal to ______.

Sol. (211)

Let $\frac{a}{r^2},\frac{a}{r},a,ar,ar^2$

Given $\frac{a}{r^2}+\frac{a}{r}+a+ar+ar^2=5\times\frac{31}{10}$ … (1)

And $\frac{r^2}{a}+\frac{r}{a}+\frac{1}{a}+\frac{1}{ar}+\frac{1}{ar^2}=5\times\frac{31}{40}$ … (2)

$(1)\div(2)a^2=4\Rightarrow a=2$

$\therefore r+\frac{1}{r}=5/2\ \ (a\neq-2)$

$\Rightarrow r = 2$

$\therefore$ Now $\frac{1}{2},1,2.4,8$

$$\therefore \sigma^2 = \frac{\sum x^2}{N} - \left(\frac{\sum x}{N}\right)^2$$

$$= \frac{186}{25} = \frac{M}{N} \Rightarrow 211 = m + n$$

22. Let $D_k = \begin{vmatrix} 1 & 2k & 2k-1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$. If $\sum_{k=1}^{n} D_k = 96$, then n is equal to

Sol. **(6.00)**

$$D_k = \begin{vmatrix} 1 & 2k & 2k-1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$$

$$\sum_{k=1}^{n} D_k = 96 \Rightarrow$$

$$\begin{vmatrix} \sum_{k=1}^{n} 1 & \sum 2k & \sum (2k-1) \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix} = 96$$

$$\Rightarrow \begin{vmatrix} n & n^2+n & n^2 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix} = 96$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\begin{vmatrix} n & n^2+n & n^2 \\ 0 & 2 & 0 \\ 0 & 0 & n+2 \end{vmatrix} = 96$$

$$\Rightarrow n(2n+4) = 96 \Rightarrow n(n+2) = 48 \Rightarrow n = 6$$

23. If $\int_{-0.15}^{0.15} |100x^2 - 1|\, dx = \frac{k}{3000}$, then k is equal to ________.

Sol. **(575)**

$$\int_{-0.15}^{0.15} |100x^2 - 1| dx = 2\int_{0}^{0.15} |100x^2 - 1| dx$$

Now $100x^2 - 1 = 0 \Rightarrow x^2 = \frac{1}{100} \Rightarrow x = 0.1$

$$I = 2\left[\int_0^{0.1} (1 - 100x^2) dx + \int_{0.1}^{0.15} (100x^2 - 1) dx\right]$$

$$I = 2\left[x - \frac{100}{3}x^3\right]_0^{0.1} + 2\left[\frac{100x^3}{3} - x\right]_{0.1}^{0.15}$$

$$= 2\left[0.1 - \frac{0.1}{3}\right] + 2\left[\frac{0.3375}{3} - 0.15 - \frac{0.1}{3} + 0.1\right]$$

$$= 2\left[0.2 - \frac{0.2}{3} + 0.1125 - 0.15\right]$$

$$= 2\left[\frac{5}{100} - \frac{2}{30} + \frac{1125}{10000}\right] = 2\left(\frac{1500 - 2000 + 3375}{30000}\right)$$

$$= \frac{575}{3000} \Rightarrow k = 575$$

24. Two circles in the first quadrant of radii r_1 and r_2 touch the coordinate axes. Each of them cuts off an intercept of 2 units with the line x + y = 2. Then $r_1^2 + r_2^2 - r_1r_2$ is equal to ______.

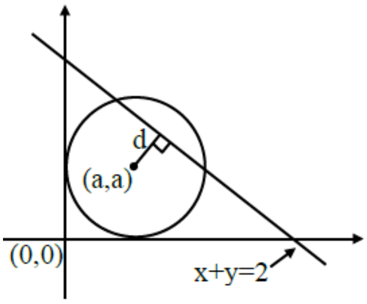
Sol. **(7)**

Circle $(x-a)^2 + (y-a)^2 = a^2$

$x^2 + y^2 - 2ax - 2ay + a^2 = 0$

Intercept = 2

$\Rightarrow 2\sqrt{a^2 - d^2} = 2$



Where d = perpendicular distance of centre from line x + y = 2

$$\Rightarrow 2\sqrt{a^2 - \left(\frac{a+a-2}{\sqrt{2}}\right)^2} = 2$$

$$\Rightarrow a^2 - \frac{(2a-2)^2}{2} = 1 \Rightarrow 2a^2 - 4a^2 + 8a - 4 = 2$$

$$\Rightarrow 2a^2 - 8a + 6 = 0 \Rightarrow a^2 - 4a + 3 = 0$$

$\therefore r_1 + r_2 = 4$ and $r_1r_2 = 3$

$\therefore r_1^2 + r_2^2 - r_1r_2 = (r_1 + r_2)^2 - 3r_1r_2$

= 16 – 9 = 7

25. A fair n(n > 1) faces die is rolled repeatedly until a number less than n appears. If the mean of the number of tosses required is $\frac{n}{9}$, then n is equal to ______.

Sol. **(10)**

$$\text{Mean} = 1.\frac{n-1}{n} + 2\frac{1}{n}\left(\frac{n-1}{n}\right) + 3\left(\frac{1}{n}\right)^2\left(\frac{n-1}{n}\right)\ldots$$

$$\frac{n}{9} = \left(\frac{n-1}{n}\right)\left(1 + 2\left(\frac{1}{n}\right) + 3\left(\frac{1}{n}\right)^2 \ldots\ldots\right)$$

$$\frac{n}{9} = \left(\frac{n-1}{n}\right)\left(1 - \frac{1}{n}\right)^{-2} = \left(\frac{n-1}{n}\right)\cdot\frac{n^2}{(n-1)^2}$$

$$\frac{n}{9} = \frac{n}{n-1} \Rightarrow n = 10$$

26. Let the digits a, b, c be in A.P. Nine-digit numbers are to be formed using each of these three digits thrice such that three consecutive digits are in A.P. at least once. How many such numbers can be formed?

Sol. **(1260)**

abc or cba

abc

– – – – –

cba

$$\frac{{}^7C_1 \times 2 \times 6!}{2!2!2!} = 1260$$

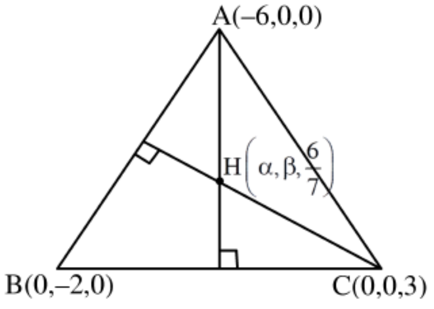
27. Let the plane x + 3y – 2z + 6 = 0 meet the co-ordinate axes at the points A, B, C. If the orthocenter of the triangle ABC is $\left(\alpha, \beta, \frac{6}{7}\right)$, then 98 $(\alpha + \beta)^2$ is equal to _____.

Sol. **(288.00)**

A(–6, 0, 0) B(0, –2, 0) C = (0, 0, 3)

$\overrightarrow{AB} = 6\hat{i} - 2\hat{j}, \overrightarrow{BC} = 2\hat{j} + 3\hat{k},$

$\overrightarrow{AC} = 6\hat{i} + 3\hat{k}$



$\overrightarrow{AH} \cdot \overrightarrow{BC} = 0$

$\left(\alpha+6,\beta,\frac{6}{7}\right)\cdot(0,2,3)=0$

$\beta=\frac{-9}{7}$

$\overrightarrow{CH}\cdot\overrightarrow{AB}=0$

$\left(\alpha,\beta,\frac{-15}{7}\right)\cdot(6,-2,0)=0$

$6\alpha-2\beta=0$

$\alpha=\frac{-3}{7}$

$98(\alpha+\beta)^2=(98)\frac{(144)}{49}=288$

28. The number of the relations, on the set {1, 2, 3} containing (1, 2) and (2, 3), which are reflexive and transitive but not symmetric, is _______.

Sol. **(3)**

A = {1, 2, 3}

For Reflexive (1, 1) (2, 2), (3, 3) $\in$ R
For transitive : (1, 2) and (2, 3) $\in$R $\Rightarrow$ (1, 3) $\in$ R
Not symmetric : (2, 1) and (3, 2) $\notin$ R
R_1 = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)}
R_2 = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3) (2, 1)}
R_3 = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3) (2, 1)}

29. Let [x] be the greatest integer $\le$ x. Then the number of points in the interval (–2, 1), where the function $f(x)=|[x]|+\sqrt{x-[x]}$ is discontinuous, is _______.

Sol. **(2)**

Need to check at doubtful points

discont at x I only
at x = – 1 $\Rightarrow$ f (-1^+) = 1 + 0 = 1
$\Rightarrow$ f (-1^-) = 2 + 1 = 3
at x = 0 $\Rightarrow$ f (0^+) = 0 + 0 = 0
$\Rightarrow$ f (0^-) = 1 + 1 = 2
at x = 1 $\Rightarrow$ f (1^+) = 1 + 0 = 1
$\Rightarrow$ f (1^-) = 0 + 1 = 1
discont. at two points

30. Let $I(x)=\int\sqrt{\frac{x+7}{x}}\,dx$ and I(9) = 12 + 7 $\log_e$ 7. If I(1) = α + 7 $\log_e$ $\left(1+2\sqrt{2}\right)$, then α^4 is equal to ______.

Sol. **(64)**

$\int\sqrt{\frac{x+7}{x}}dx$

Put $x = t^2$

$dx = 2tdt$

$$\int 2\sqrt{t^2+7}dt = 2\int\sqrt{t^2+\sqrt{7}^2}dt$$

$$I(t) = 2\left[\frac{t}{2}\sqrt{t^2+7}+\frac{7}{2}\ln\left|\sqrt{t^2+7}\right|\right]+C$$

$$I(x) = \sqrt{x}\sqrt{x+7}+7\ln\left|\sqrt{x}+\sqrt{x+7}\right|+C$$

$$I(9) = 12+7\ln 7 = 12+7(\ln(3+4))+C$$

$$\Rightarrow C = 0$$

$$I(x) = \sqrt{x}\sqrt{x+7}+7\ln\left(\sqrt{x}+\sqrt{x+7}\right)$$

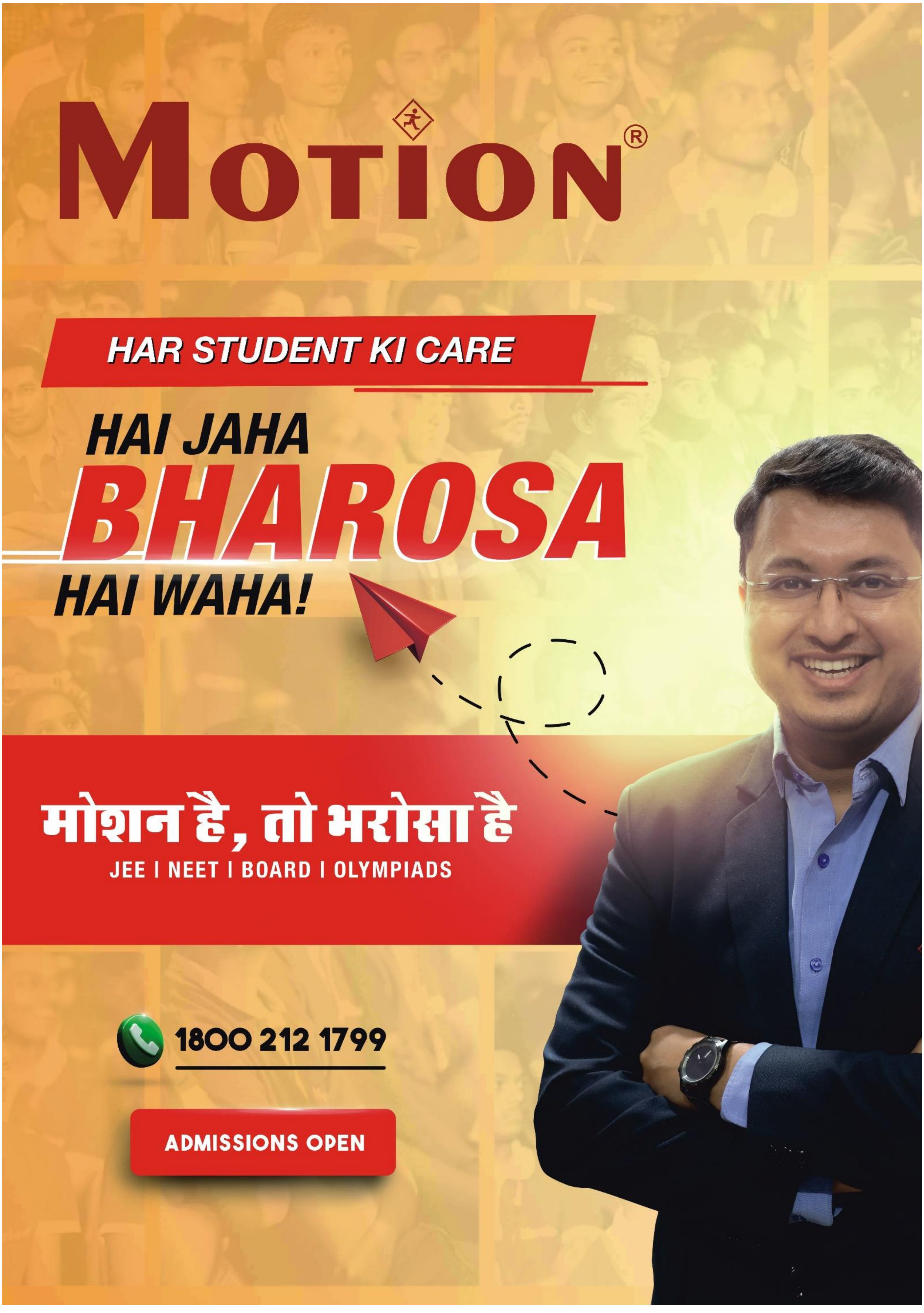
$$I(1) = 1\sqrt{8}+7\ln\left(1+\sqrt{8}\right)$$

$$I(1) = \sqrt{8}+7\ln\left(1+2\sqrt{2}\right)$$

$$\alpha = \sqrt{8}$$

$$\alpha^4 = \left(8^{1/2}\right)^4$$

$$\alpha^4 = 8^2 = 64$$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