

# JEE MAIN (Session 2) 2023 Paper Analysis

MATHS | 08<sup>th</sup> April 2023 \_ Shift-2



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in NEET

(2022)

$4837/5356 = 90.31\%$

(2021)

$3276/3411 = 93.12\%$

Student Qualified  
in JEE ADVANCED

(2022)

$1756/4818 = 36.45\%$

(2021)

$1256/2994 = 41.95\%$

Student Qualified  
in JEE MAIN

(2022)

$4818/6653 = 72.41\%$

(2021)

$2994/4087 = 73.25\%$



**NITIN VIJAY (NV Sir)**  
Founder & CEO

## SECTION - A

1. Let  $A = \left\{ \theta \in (0, 2\pi) : \frac{1+2i\sin\theta}{1-i\sin\theta} \text{ is purely imaginary} \right\}$ . Then the sum of the elements in A is.

(1)  $\pi$  (2)  $3\pi$  (3)  $4\pi$  (4)  $2\pi$

Sol. (3)

$$z = \frac{1+2i\sin\theta}{1-i\sin\theta} \times \frac{1+i\sin\theta}{1+i\sin\theta}$$

$$z = \frac{1-2\sin^2\theta + i(3\sin\theta)}{1+\sin^2\theta}$$

$$\operatorname{Re}(z) = 0$$

$$\frac{1-2\sin^2\theta}{1+\sin^2\theta} = 0$$

$$\sin\theta = \frac{\pm 1}{\sqrt{2}}$$

$$A = \left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \right\}$$

$$\text{sum} = 4\pi \text{ (Option 3)}$$

2. Let P be the plane passing through the line  $\frac{x-1}{1} = \frac{y-2}{-3} = \frac{z+5}{7}$  and the point  $(2, 4, -3)$ . If the image of the point  $(-1, 3, 4)$  in the plane P is  $(\alpha, \beta, \gamma)$  then  $\alpha + \beta + \gamma$  is equal to

(1) 12 (2) 9 (3) 10 (4) 11

Sol. (3)

Equation of plane is given by

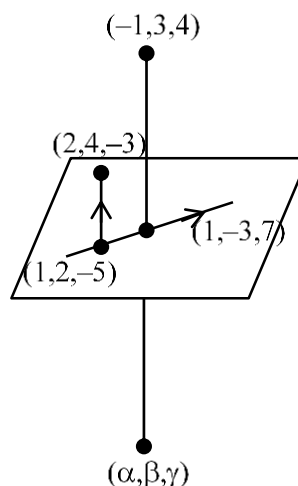
$$\begin{vmatrix} x-1 & y-2 & z+5 \\ 1 & 2 & 2 \\ 1 & -3 & 7 \end{vmatrix} = 0$$

$$4x - y - z = 7$$

$$\frac{\alpha+1}{4} = \frac{\beta-3}{-1} = \frac{\gamma-4}{-1} = \frac{-2(-4-3-4-7)}{16+1+1} = 2$$

$$\alpha = 7, \beta = 1, \gamma = 2$$

$$\alpha + \beta + \gamma = 10 \text{ (Option 3)}$$



3. If  $A = \begin{bmatrix} 1 & 5 \\ \lambda & 10 \end{bmatrix}$ ,  $A^{-1} = \alpha A + \beta I$  and  $\alpha + \beta = -2$ , then  $4\alpha^2 + \beta^2 + \lambda^2$  is equal to :

(1) 14 (2) 12 (3) 19 (4) 10

Sol. (1)

$$|A - xI| = 0 \Rightarrow \begin{vmatrix} 1-x & 5 \\ \lambda & 10-x \end{vmatrix} = 0 \Rightarrow x^2 - 11x + 10 - 5\lambda = 0$$

$$\Rightarrow (10 - 5\lambda)A^{-1} = -A + 11I$$

$$\therefore \alpha = \frac{-1}{10-5\lambda} \text{ and } \beta = \frac{+11}{10-5\lambda}$$



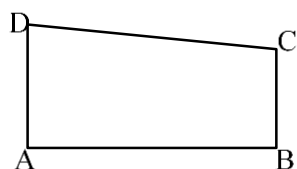
$$\alpha + \beta = -2 \Rightarrow \frac{10}{10-5\lambda} = -2 \Rightarrow 10-5\lambda = -5 \Rightarrow \lambda = 3$$

$$\therefore \alpha = \frac{1}{5} \quad \& \quad \beta = \frac{-11}{5}$$

$$\therefore 4a^2 + \beta^2 + \lambda^2 = \frac{4}{25} + \frac{121}{25} + 3^2 = 14 \text{ Ans.}$$

4. The area of the quadrilateral ABCD with vertices A(2,1,1), B (1,2, 5), C(-2,-3, 5) and D (1, -6, -7) is equal to  
 (1) 54 (2)  $9\sqrt{38}$  (3) 48 (4)  $8\sqrt{38}$

Sol. (4)



Vector Area =  $\vec{v}$

$$= \frac{1}{2} \overrightarrow{AB} \times \overrightarrow{AC} + \frac{1}{2} \overrightarrow{AC} \times \overrightarrow{AD}$$

$$= \frac{1}{2} (\overrightarrow{AB} - \overrightarrow{AD}) \times \overrightarrow{AC}$$

$$\left( \begin{array}{l} \overrightarrow{AB} = -\hat{i} + \hat{j} + 4\hat{k} \\ \overrightarrow{AD} = -\hat{i} - 7\hat{j} - 8\hat{k} \\ \overrightarrow{AC} = -4\hat{i} - 4\hat{j} + 4\hat{k} \end{array} \right)$$

$$= \frac{1}{2} (8\hat{j} + 12\hat{k}) \times (-4)(\hat{i} + \hat{j} - \hat{k})$$

$$= \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 8 & 12 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= (-2) (-20\hat{i} + 12\hat{j} - 8\hat{k})$$

$$= 8(5\hat{i} - 3\hat{j} + 2\hat{k})$$

$$\therefore \text{Area} = |\vec{v}| = 8\sqrt{25 + 9 + 4} = 8\sqrt{38} \text{ Ans.}$$

5.  $25^{190} - 19^{190} - 8^{190} + 2^{190}$  is divisible by  
 (1) 34 but not by 14 (2) 14 but not by 34 (3) Both 14 and 34 (4) Neither 14 nor 34

Sol. (1)

$$25^{190} - 8^{190} \text{ is divisible by } 25 - 8 = 17$$

$$19^{190} - 2^{190} \text{ is divisible by } 19 - 2 = 17$$

$$25^{190} - 19^{190} \text{ is divisible by } 25 - 19 = 6$$

$$8^{190} - 2^{190} \text{ is divisible by } 8 - 2 = 6$$

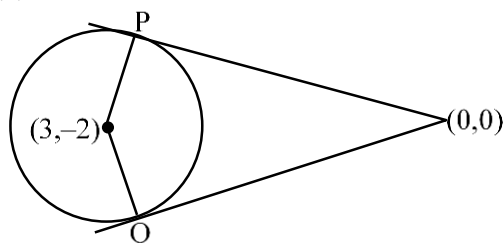
$$\text{L.C.M. of } 1746 = 34$$

$$\therefore \text{divisible by 34 but not by 14}$$

6. Let O be the origin and OP and OQ be the tangents to the circle  $x^2 + y^2 - 6x + 4y + 8 = 0$  at the points P and Q on it. If the circumcircle of the triangle OPQ passes through the point  $\left(\alpha, \frac{1}{2}\right)$ , then a value of  $\alpha$  is.

- (1)  $-\frac{1}{2}$  (2)  $\frac{5}{2}$  (3) 1 (4)  $\frac{3}{2}$

Sol. (2)



Circumcircle of  $\triangle OPQ$

$$(x-0)(x-3) + (y-0)(y+2) = 0$$

$$x^2 + y^2 - 3x + 2y = 0$$

passes through  $\left(\alpha, \frac{1}{2}\right)$

$$\therefore \alpha^2 + \frac{1}{4} - 3\alpha + 1 = 0$$

$$\Rightarrow \alpha^2 - 3\alpha + \frac{5}{4} = 0 \Rightarrow 4\alpha^2 - 12\alpha + 5 = 0$$

$$\Rightarrow 4\alpha^2 - 10\alpha - 2\alpha + 5 = 0$$

$$(2\alpha - 1)(2\alpha - 5) = 0 \therefore \alpha = \frac{1}{2}, \frac{5}{2} \text{ Ans.}$$

7. Let  $a_n$  be the  $n^{\text{th}}$  term of the series  $5 + 8 + 14 + 23 + 35 + 50 + \dots$  and  $S_n = \sum_{k=1}^n a_k$ . Then  $S_{30} - a_{40}$  is equal to

- (1) 11260 (2) 11280 (3) 11290 (4) 11310

Sol. (3)

$$S_n = 5 + 8 + 14 + 23 + 35 + 50 + \dots + a_n$$

$$S_n = 5 + 8 + 14 + 23 + 35 + \dots + a_n$$

$$O = 5 + 3 + 6 + 9 + 12 + 15 + \dots - a_n$$

$$a_n = 5 + (3 + 6 + 9 + \dots (n-1) \text{ terms})$$

$$a_n = \frac{3n^2 - 3n + 10}{2}$$

$$a_{40} = \frac{3(40)^2 - 3(40) + 10}{2} = 2345$$

$$S_{30} = \frac{3 \sum_{n=1}^{30} n^2 - 3 \sum_{n=1}^{30} n + 10 \sum_{n=1}^{30} 1}{2}$$

$$= \frac{\frac{3 \times 30 \times 31 \times 61}{6} - \frac{3 \times 30 \times 31}{2} + 10 \times 30}{2}$$

$$S_{30} = 13635$$

$$S_{30} - a_{40} = 13635 - 2345$$

$$= 11290 \text{ (Option (3))}$$

8. If  $\alpha > \beta > 0$  are the roots of the equation  $ax^2 + bx + 1 = 0$ , and  $\lim_{x \rightarrow \frac{1}{\alpha}} \left( \frac{1 - \cos(x^2 + bx + a)}{2(1 - ax)^2} \right)^{\frac{1}{2}} = \frac{1}{k} \left( \frac{1}{\beta} - \frac{1}{\alpha} \right)$ , then k

is equal to

- (1)  $\beta$  (2)  $2\alpha$  (3)  $2\beta$  (4)  $\alpha$

Sol. (2)

$$\therefore ax^2 + bx + 1 = a(x - \alpha)(x - \beta) \therefore \alpha\beta = \frac{1}{a}$$

$$\therefore x^2 + bx + a = a(1 - \alpha x)(1 - \beta x)$$

$$\therefore \lim_{x \rightarrow \frac{1}{\alpha}} \left\{ \frac{1 - \cos(x^2 + bx + a)}{2(1 - \alpha x)^2} \right\}^{\frac{1}{2}} = \lim_{x \rightarrow \frac{1}{\alpha}} \left\{ \frac{1 - \cos a(1 - \alpha x)(1 - \beta x)}{2\{a(1 - \alpha x)(1 - \beta x)\}^2} \cdot a^2(1 - \beta x)^2 \right\}^{\frac{1}{2}}$$

$$= \left[ \frac{1}{2} \cdot \frac{1}{2} a^2 \left( 1 - \frac{\beta}{\alpha} \right)^2 \right]^{\frac{1}{2}}$$

$$= \frac{1}{2} \frac{1}{\alpha\beta} \left( 1 - \frac{\beta}{\alpha} \right) = \frac{1}{2} \left( \frac{1}{\alpha\beta} - \frac{1}{\alpha^2} \right)$$

$$= \frac{1}{2\alpha} \left( \frac{1}{\beta} - \frac{1}{\alpha} \right) = \frac{1}{k} \left( \frac{1}{\beta} - \frac{1}{\alpha} \right)$$

$$\therefore k = 2\alpha \text{ Ans.}$$

9. If the number of words, with or without meaning, which can be made using all the letters of the word MATHEMATICS in which C and S do not come together, is  $(6!)k$ , is equal to

- (1) 1890 (2) 945 (3) 2835 (4) 5670

Sol. (4)

$M_2A_2T_2HEICS$

= total words – when C & S are together

$$\frac{11!}{2!2!2!} - \frac{10!}{2!2!2!} \times 2$$

$$\frac{10!}{2!2!2!} \times 9$$

$$= \frac{9 \times 10 \times 9 \times 8 \times 7}{8} \times 6$$

$$= 5670 \times 6$$

$$k = 5670 \text{ (Option 4)}$$

10. Let S be the set of all values of  $\theta \in [-\pi, \pi]$  for which the system of linear equations

$$x + y + \sqrt{3}z = 0$$

$$-x + (\tan \theta)y + \sqrt{7}z = 0$$

$$x + y + (\tan \theta)z = 0$$

has non-trivial solution. Then  $\frac{120}{\pi} \sum_{\theta \in S} \theta$  is equal to

- (1) 20 (2) 40 (3) 30 (4) 10

**Sol. (1)**

For non trivial solutions

$$D = 0$$

$$\begin{vmatrix} 1 & 1 & \sqrt{3} \\ -1 & \tan \theta & \sqrt{7} \\ 1 & 1 & \tan \theta \end{vmatrix} = 0$$

$$\tan^2 \theta - (\sqrt{3} - 1) - \sqrt{3} = 0$$

$$\tan \theta = \sqrt{3}, -1$$

$$\theta = \left\{ \frac{\pi}{3}, \frac{-2\pi}{3}, \frac{-\pi}{4}, \frac{3\pi}{4} \right\}$$

$$\frac{120}{\pi}(\Sigma \theta) = \frac{120}{\pi} \times \frac{\pi}{6} = 20 \text{ (Option 1)}$$

- 11.** For  $a, b \in \mathbb{Z}$  and  $|a - b| \leq 10$ , let the angle between the plane  $P : ax + y - z = b$  and the line  $l : x - 1 = a - y = z + 1$  be  $\cos^{-1}\left(\frac{1}{3}\right)$ . If the distance of the point  $(6, -6, 4)$  from the plane  $P$  is  $3\sqrt{6}$ , then  $a^4 + b^2$  is equal to

(1) 85

(2) 48

(3) 25

(4) 32

**Sol. (4)**

$$\theta = \cos^{-1} \frac{1}{3} \therefore \sin \theta = \sqrt{1 - \frac{1}{9}} = \frac{2\sqrt{2}}{3}$$

$$\sin \theta = \frac{a \cdot 1 + 1(-1) + (-1) \cdot 1}{\sqrt{a^2 + 1 + 1} \cdot \sqrt{3}} = \frac{2\sqrt{2}}{3}$$

$$\Rightarrow \{3(a - 2)\}^2 = 24(a^2 + 2)$$

$$\Rightarrow 3(a^2 - 4a + 4) = 8a^2 + 16$$

$$\Rightarrow 5a^2 + 12a + 4 = 0$$

$$\Rightarrow 5a^2 + 10a + 2a + 4 = 0$$

$$\therefore a = -2, \frac{-2}{5} \therefore a \in \mathbb{Z}$$

$$\therefore a = -2$$

Distance of  $(6, -6, 4)$  from

$$-2x + y - z - b = 0 \text{ is } 3\sqrt{6}$$

$$\therefore \left| \frac{-12 - 6 - 4 - b}{\sqrt{4 + 1 + 1}} \right| = 3\sqrt{6}$$

$$\Rightarrow |b + 22| = 18 \therefore b = -40, -4$$

$$\therefore |a - b| \leq 10$$

$$\therefore b = -4$$

$$\therefore a^4 + b^2$$

$$= 32 \text{ Ans.}$$

12. Let the vectors  $\vec{u}_1 = \hat{i} + \hat{j} + a\hat{k}$ ,  $\vec{u}_2 = \hat{i} + b\hat{j} + \hat{k}$  and  $\vec{u}_3 = c\hat{i} + \hat{j} + \hat{k}$  be coplanar. If the vectors  $\vec{v}_1 = (a+b)\hat{i} + c\hat{j} + c\hat{k}$ ,  $\vec{v}_2 = a\hat{i} + (b+c)\hat{j} + a\hat{k}$  and  $\vec{v}_3 = b\hat{i} + b\hat{j} + (c+a)\hat{k}$  are also coplanar, then  $6(a+b+c)$  is equal to

(1) 4 (2) 12 (3) 6 (4) 0

Sol. (2)

$$[\vec{u}_1 \vec{u}_2 \vec{u}_3] = 0 \quad \therefore \begin{vmatrix} 1 & 1 & a \\ 1 & b & 1 \\ c & 1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow b - 1 + c - 1 + a(1 - bc) = 0$$

$$\therefore abc = a + b + c - 2$$

$$[\vec{v}_1 \vec{v}_2 \vec{v}_3] = 0 \quad \therefore \begin{vmatrix} a+b & c & c \\ a & b+c & a \\ b & b & c+a \end{vmatrix} = 0$$

$$R_3 \rightarrow R_3 - (R_1 + R_2) \Rightarrow \begin{vmatrix} a+b & c & c \\ a & b+c & a \\ -2a & -2c & 0 \end{vmatrix} = 0$$

$$\Rightarrow -2a(ac - bc - c^2) + 2c(a^2 + ab - ac) = 0$$

$$\Rightarrow -2a^2c + 2abc + 2ac^2 + 2a^2c + 2abc - 2ac^2 = 0$$

$$\Rightarrow 4abc = 0 \quad \therefore abc = 0$$

$$\therefore a + b + c = 2 \quad \therefore 6(a+b+c) = 12 \text{ Ans.}$$

13. The absolute difference of the coefficients of  $x^{10}$  and  $x^7$  in the expansion of  $\left(2x^2 + \frac{1}{2x}\right)^{11}$  is equal to

(1)  $10^3 - 10$  (2)  $11^3 - 11$  (3)  $12^3 - 12$  (4)  $13^3 - 13$

Sol. (3)

$$T_{r+1} = {}^{11}C_r (2x^2)^{11-r} \left(\frac{1}{2x}\right)^r$$

$$= {}^{11}C_r 2^{11-2r} x^{22-3r}$$

$$22 - 3r = 10 \quad \text{and} \quad 22 - 3r = 7$$

$$r = 4 \quad \text{and} \quad r = 5$$

$$\text{Coefficient of } x^{10} = {}^{11}C_4 \cdot 2^3$$

$$\text{Coefficient of } x^7 = {}^{11}C_5 \cdot 2^1$$

$$\text{difference} = {}^{11}C_4 \cdot 2^3 - {}^{11}C_5 \cdot 2$$

$$= \frac{11 \times 10 \times 9 \times 8}{24} \times 8 - \frac{11 \times 10 \times 9 \times 8 \times 7}{120} \times 2$$

$$= 11 \times 10 \times 3 \times 8 - 11 \times 3 \times 4 \times 7$$

$$= 11 \times 3 \times 4 \times (20 - 7)$$

$$= 11 \times 12 \times 13$$

$$= 12(12 - 1)(12 + 1)$$

$$= 12(12^2 - 1)$$

$$= 12^3 - 12 \text{ (Option 3)}$$



14. Let  $A = \{1, 2, 3, 4, 5, 6, 7\}$ . Then the relation  $R = \{(x, y) \in A \times A : x + y = 7\}$  is

- (1) Symmetric but neither reflexive nor transitive
- (2) Transitive but neither symmetric nor reflexive
- (3) An equivalence relation
- (4) Reflexive but neither symmetric nor transitive

**Sol.** (1)

$$R = \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$$

15. If the probability that the random variable  $X$  takes values  $x$  is given by  $P(X=x) = k(x+1)3^{-x}$ ,  $x = 0, 1, 2, 3, \dots$ , where  $k$  is a constant, then  $P(X \geq 2)$  is equal to

- (1)  $\frac{7}{27}$
- (2)  $\frac{11}{18}$
- (3)  $\frac{7}{18}$
- (4)  $\frac{20}{27}$

**Sol.** (1)

$$\sum_{x=0}^{\infty} P(X=x) = 1$$

$$k(1 + 2 \cdot 3^{-1} + 3 \cdot 3^{-2} + 4 \cdot 3^{-3} + \dots \infty) = 1$$

$$\text{Let } s = 1 + \frac{2}{3} + \frac{3}{3^2} + \frac{4}{3^3} + \dots \infty$$

$$\frac{s}{3} = \frac{1}{3} + \frac{2}{3^2} + \frac{3}{3^3} + \dots \infty$$

$$\frac{2s}{3} = 1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots \infty$$

$$\frac{2s}{3} - \frac{1}{1 - \frac{1}{3}} = \frac{3}{2}$$

$$s = \frac{9}{4}$$

$$\text{so } \boxed{k = \frac{4}{9}}$$

$$P(X \geq 2) = 1 - P(x=0) - P(x=1)$$

$$= 1 - \frac{4}{9} \left( 1 + \frac{2}{3} \right)$$

$$= \frac{7}{27} \text{ (Option 1)}$$

16. The integral  $\int \left( \left( \frac{x}{2} \right)^x + \left( \frac{2}{x} \right)^x \right) \log_2 x \, dx$  is equal to

- (1)  $\left( \frac{x}{2} \right)^x \log_2 \left( \frac{2}{x} \right) + C$
- (2)  $\left( \frac{x}{2} \right)^x - \left( \frac{2}{x} \right)^x + C$
- (3)  $\left( \frac{x}{2} \right)^x \log_2 \left( \frac{x}{2} \right) + C$
- (4)  $\left( \frac{x}{2} \right)^x + \left( \frac{2}{x} \right)^x + C$

**Sol.** (2) **Bonus**

$$\int (x^x 2^{-x} + 2^x x^{-x}) \log_2 x \, dx$$

$$\int (e^{x \ln x} \cdot e^{-x \ln 2} + e^{x \ln 2} \cdot e^{-x \ln x}) dx$$

$$\int (e^{x \ln x - x \ln 2} + e^{x \ln 2 - x \ln x}) \frac{\ln x}{\ln 2} dx$$

let  $x \ln x - x \ln 2 = t$   
 $(\ln x + 1 - \ln 2) dx = dt$

17. The value of  $36 (4 \cos^2 9^\circ - 1)(4 \cos^2 27^\circ - 1)(4 \cos^2 81^\circ - 1)(4 \cos^2 243^\circ - 1)$  is  
 (1) 27 (2) 54 (3) 18 (4) 36

Sol. (4)

$$4 \cos^2 \theta - 1 = 4(1 - \sin^2 \theta) - 1 = 3 - 4 \sin^2 \theta = \frac{\sin 3\theta}{\sin \theta}$$

so given expression can be written as

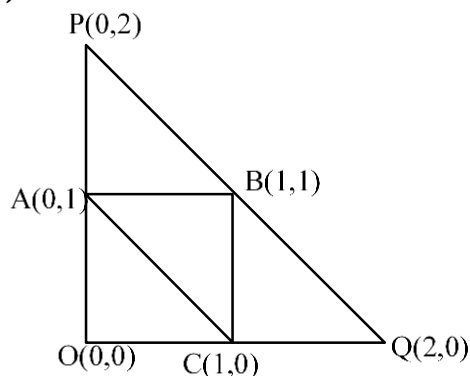
$$36 \times \frac{\sin 27^\circ}{\sin 9^\circ} \times \frac{\sin 81^\circ}{\sin 27^\circ} \times \frac{\sin 243^\circ}{\sin 81^\circ} \times \frac{\sin 729^\circ}{\sin 243^\circ}$$

$$36 \times \frac{\sin 729^\circ}{\sin 9^\circ} = 36$$

18. Let A (0, 1), B(1,1) and C (1,0) be the mid-points of the sides of a triangle with incentre at the point D. If the focus of the parabola  $y^2 = 4ax$  passing through D is  $(\alpha + \beta\sqrt{3}, 0)$ , where  $\alpha$  and  $\beta$  are rational numbers, then  $\frac{\alpha}{\beta^2}$  is equal to

- (1) 6 (2) 8 (3)  $\frac{9}{2}$  (4) 12

Ans. (2)



$$a = OP = 2 \quad b = OQ = 2 \quad c = PQ = 2\sqrt{2}$$

$$D\left(\frac{4}{2+2+2\sqrt{2}}, \frac{4}{2+2+2\sqrt{2}}\right) \equiv D\left(\frac{2}{2+\sqrt{2}}, \frac{2}{2+\sqrt{2}}\right)$$

$$y^2 = 4ax \Rightarrow \left(\frac{2}{2+\sqrt{2}}\right)^2 = 4a \cdot \left(\frac{2}{2+\sqrt{2}}\right)$$

$$\therefore 4a = \frac{2}{2+\sqrt{2}} \therefore a = \frac{1}{2} \cdot \frac{2-\sqrt{2}}{4-2} = \frac{1}{4}(2-\sqrt{2})$$

$$\therefore \alpha = \frac{2}{4} = \frac{1}{2} \quad \beta = \frac{-1}{4}$$

$$\therefore \frac{\alpha}{\beta^2} = 8 \text{ Ans.}$$

19. The negation of  $(p \wedge (\sim q)) \vee (\sim p)$  is equivalent to

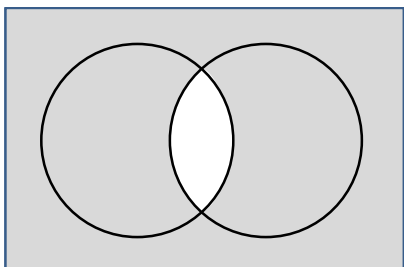
(1)  $p \wedge (\sim q)$

(2)  $p \wedge (q \wedge (\sim p))$

(3)  $p \vee (q \vee (\sim p))$

(4)  $p \wedge q$

Sol. 4



20. Let the mean and variance of 12 observations be  $\frac{9}{2}$  and 4 respectively. Later on, it was observed that two observations were considered as 9 and 10 instead of 7 and 14 respectively. If the correct variance is  $\frac{m}{n}$ , where m and n are coprime, then m + n is equal to

(1) 316

(2) 317

(3) 315

(4) 314

Sol. 2

$$\frac{\sum x}{12} = \frac{9}{2}$$

$$\sum x = 54$$

$$\frac{\sum x^2}{12} - \left(\frac{9}{2}\right)^2 = 4$$

$$\sum x^2 = 291$$

$$\sum x_{\text{new}} = 54 - (9 + 10) + 7 + 14 = 56$$

$$\sum x_{\text{new}}^2 = 291 - (81 + 100) + 49 + 196 = 355$$

$$\sigma_{\text{new}}^2 = \frac{355}{12} - \left(\frac{56}{12}\right)^2$$

$$\sigma_{\text{new}}^2 = \frac{281}{36} = \frac{m}{n}$$

$$\boxed{m + n = 317} \text{ Option (2)}$$

### SECTION - B

21. Let  $R = \{a, b, c, d, e\}$  and  $S = \{1, 2, 3, 4\}$ . Total number of onto functions  $f : R \rightarrow S$  such that  $f(a) \neq 1$ , is equal to \_\_\_\_\_.

Sol. 180

Total onto function

$$\frac{|5|}{|3|} \times |4| = 240$$

Now when  $f(a) = 1$

$$|4| + \frac{|4|}{|2|} \times |3| = 24 + 36 = 60$$

$$\text{so required } f^n = 240 - 60 = 180$$

22. Let  $m$  and  $n$  be the numbers of real roots of the quadratic equations  $x^2 - 12x + [x] + 31 = 0$  and  $x^2 - 5|x + 2| - 4 = 0$  respectively, where  $[x]$  denotes the greatest integer  $\leq x$ . Then  $m^2 + mn + n^2$  is equal to \_\_\_\_\_.

**Sol.** 9

$$x^2 - 12x + [x] + 31 = 0$$

$$\{x\} = x^2 - 11x + 31$$

$$0 \leq x^2 - 11x + 31 < 1$$

$$x^2 - 11x + 30 < 0$$

$$x \in (5, 6)$$

$$\text{so } [x] = 5$$

$$x^2 - 12x + 5 + 31 = 0$$

$$x^2 - 12x + 36 = 0$$

$$\boxed{x = 6} \text{ but } x \in (5, 6)$$

$$\text{so } x \in \phi$$

$$\boxed{m = 0}$$

Now

$$x^2 - 5|x + 2| - 4 = 0$$

$$x \geq -2$$

$$x^2 - 5x - 14 = 0$$

$$(x - 7)(x + 2) = 0$$

$$x = 7, -2$$

$$x < -2$$

$$x^2 + 5x + 6 = 0$$

$$(x + 3)(x + 2) = 0$$

$$x = -3, -2$$

$$x = \{7, -2, -3\}$$

$$n = 3$$

$$m^2 + mn + n^2 = 0 + 0 + 9 = 9$$

23. Let  $P_1$  be the plane  $3x - y - 7z = 11$  and  $P_2$  be the plane passing through the points  $(2, -1, 0)$ ,  $(2, 0, -1)$ , and  $(5, 1, 1)$ . If the foot of the perpendicular drawn from the point  $(7, 4, -1)$  on the line of intersection of the planes  $P_1$  and  $P_2$  is  $(\alpha, \beta, \gamma)$ , then  $\alpha + \beta + \gamma$  is equal to \_\_\_\_\_.

**Sol.** 11

$P_2$  is given by

$$\begin{vmatrix} x-5 & y-1 & z-1 \\ 3 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix} = 0$$

$$\boxed{x - y - z = 3}$$

DR of line intersection of  $P_1$  &  $P_2$

$$\begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 3 & -1 & -7 \end{vmatrix}$$

$$+6\hat{i} + 4\hat{j} + 2\hat{k}$$

$$\text{Let } z = 0, \quad x - y = 3$$

$$3x - y = 11$$

$$2x = 8$$

$$x = 4$$

$$y = 1$$

So Line is

$$\frac{x-4}{6} = \frac{y-1}{4} = \frac{z-0}{2} = r$$

$$(\alpha, \beta, \gamma) = (6r+4, 4r+1, 2r)$$

$$6(\alpha-7) + 4(\beta-4) + 2(\gamma+1) = 0$$

$$6\alpha - 42 + 4\beta - 16 + 2\gamma + 2 = 0$$

$$36r + 24 + 16r + 4 + 4r - 56 = 0$$

$$56r = 28$$

$$r = \frac{1}{2}$$

$$\alpha + \beta + \gamma = 12r + 5 \\ = 6 + 5 = 11$$

24. If domain of the function  $\log_e \left( \frac{6x^2 + 5x + 1}{2x - 1} \right) + \cos^{-1} \left( \frac{2x^2 - 3x + 4}{3x - 5} \right)$  is  $(\alpha, \beta) \cup (\gamma, \delta]$ , then,  $18(\alpha^2 + \beta^2 + \gamma^2 + \delta^2)$  is equal to

Sol. 20

$$\frac{6x^2 + 5x + 1}{2x - 1} > 0$$

$$\frac{(3x+1)(2x+1)}{2x-1} > 0$$

$$\begin{array}{c} - \quad + \quad - \quad + \\ \hline -\frac{1}{2} \quad -\frac{1}{3} \quad \frac{1}{2} \end{array}$$

$$x \in \left( -\frac{1}{2}, -\frac{1}{3} \right) \cup \left( \frac{1}{2}, \infty \right) \quad \dots(A)$$

$$-1 \leq \frac{2x^2 - 3x + 4}{3x - 5} \leq 1$$

and

$$\frac{2x^2 - 1}{3x - 5} \geq 0 \quad \text{and} \quad \frac{2x^2 - 6x + 9}{3x - 5} \leq 0$$

$$\begin{array}{c} - \quad + \quad - \quad + \\ \hline -\frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}} \quad \frac{5}{3} \end{array}$$

$$\text{and} \quad 3x - 5 < 0$$

$$x \in \left[ \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right] \cup \left( \frac{5}{3}, \infty \right) \quad \dots(B)$$

$$x < \frac{5}{3} \quad \dots(C)$$

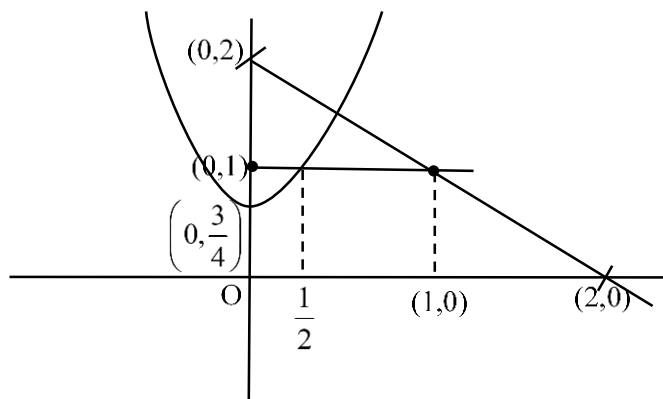
$$A \cap B \cap C \equiv \left( -\frac{1}{2}, -\frac{1}{3} \right) \cup \left( \frac{1}{2}, \frac{1}{\sqrt{2}} \right]$$

$$\text{So } 18(\alpha^2 + \beta^2 + \gamma^2 + \delta^2) = 18 \left( \frac{1}{4} + \frac{1}{9} + \frac{1}{4} + \frac{1}{2} \right) \\ = 18 + 2 = 20$$



25. Let the area enclosed by the lines  $x + y = 2$ ,  $y = 0$ ,  $x = 0$  and the curve  $f(x) = \min\left\{x^2 + \frac{3}{4}, 1 + [x]\right\}$  where  $[x]$  denotes the greatest integer  $\leq x$ , be  $A$ . Then the value of  $12A$  is\_\_\_\_\_.

**Sol.** 17



$$\int_0^{\frac{1}{2}} \left(x^2 + \frac{3}{4}\right) dx + \frac{1}{2} \times \left(\frac{3}{2} + \frac{1}{2}\right) \times 1$$

$$= \left[ \frac{x^3}{3} + \frac{3x}{4} \right]_0^{\frac{1}{2}} + 1$$

$$A = \frac{1}{24} + \frac{3}{8} + 1$$

$$12A = \frac{1}{2} + \frac{36}{8} + 12$$

$$= \frac{1}{2} + \frac{9}{2} + 12$$

$$= 5 + 12$$

$$= 17$$

26. Let  $0 < z < y < x$  be three real numbers such that  $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$  are in an arithmetic progression and  $x, \sqrt{2}y, z$  are in a geometric progression. If  $xy + yz + zx = \frac{3}{\sqrt{2}}xyz$ , then  $3(x + y + z)^2$  is equal to\_\_\_\_\_.

**Sol.** 150

$$\frac{2}{y} = \frac{1}{x} + \frac{1}{z}$$

$$2y^2 = xz$$

$$\frac{2}{y} = \frac{x+z}{xz} = \frac{x+z}{2y^2}$$

$$x + z = 4y$$

$$xy + yz + zx = \frac{3}{\sqrt{2}}xyz$$

$$y(x+z) + zx = \frac{3}{\sqrt{2}}xz \cdot y$$

$$4y^2 + 2y^2 = \frac{3}{\sqrt{2}} y \cdot 2y^2$$

$$6y^2 = 3\sqrt{2}y^3$$

$$y = \sqrt{2}$$

$$x + y + z = 5y = 5\sqrt{2}$$

$$3(x + y + z)^2 = 3 \times 50 = 150$$

27. Let the solution curve  $x = x(y)$ ,  $0 < y < \frac{\pi}{2}$ , of the differential equation  $(\log_e (\cos y))^2 \cos y \, dx - (1 + 3x \log_e (\cos y)) \sin y \, dy = 0$  satisfy  $x\left(\frac{\pi}{3}\right) = \frac{1}{2\log_e 2}$ . If  $x\left(\frac{\pi}{6}\right) = \frac{1}{\log_e m - \log_e n}$ , where  $m$  and  $n$  are co-prime, then  $mn$  is equal to

**Sol.**

**12**

$$\cos y \ln^2 \cos y \, dx = (1 + 3x \ln \cos y) \sin y \, dy$$

$$\frac{dx}{dy} = \tan y \left( \frac{3x}{\ln \cos y} + \frac{1}{\ln^2 \cos y} \right)$$

$$\frac{dx}{dy} - \left( \frac{3 \tan y}{\ln \cos y} \right) x = \frac{\tan y}{\ln^2 \cos y}$$

$$\text{If } = e^{-\int \frac{3 \tan y}{\ln \cos y \cos y} dy}$$

$$\ln \cos y = t$$

$$\frac{1}{\cos y} - \sin y \, dy = dt$$

$$\text{If } = e^{\frac{3}{t} dt} = e^{3 \ln t} = t^3 = \ln^3 \cos y$$

$$\text{solution is } x \cdot \ln^3 \cos y = \left\{ \frac{\sin y}{\cos y} \cdot \ln \cos y \, dy + C \right.$$

$$x \ln^3 \cos y = \frac{-\ln^2 \cos y}{2} + C$$

$$x\left(\frac{\pi}{3}\right) = \frac{1}{2 \ln 2} \text{ so } \frac{1}{2 \ln 2} \times \ln^3\left(\frac{1}{2}\right) = -\frac{\ln^3\left(\frac{1}{2}\right)}{2} + C$$

$$C = 0$$

$$y = \frac{\pi}{6} \quad x \ln^3 \frac{\sqrt{3}}{2} = -\frac{1}{2} \ln^2 \frac{\sqrt{3}}{2} + 0$$

$$x = -\frac{1}{2 \ln\left(\frac{\sqrt{3}}{2}\right)}$$

$$x = \frac{1}{\ln \frac{4}{3}} = \frac{1}{\ln 4 - \ln 3}$$

$$mn = 12$$

28. Let  $[t]$  denote the greatest integer function. If  $\int_0^{2.4} [x^2] dx = \alpha + \beta\sqrt{2} + \gamma\sqrt{3} + \delta\sqrt{5}$ , then  $\alpha + \beta + \gamma + \delta$  is equal to \_\_\_\_\_.

**Sol.** 6

$$\begin{aligned} & \int_0^1 0 dx + \int_1^{\sqrt{2}} 1 dx + \int_{\sqrt{2}}^{\sqrt{3}} 2 dx + \int_{\sqrt{3}}^2 3 dx + \int_2^{\sqrt{5}} 4 dx + \int_{\sqrt{5}}^{2.4} 5 dx \\ &= \sqrt{2} - 1 + 2(\sqrt{3} - \sqrt{2}) + 3(2 - \sqrt{3}) + 4(\sqrt{5} - 2) + 5((2.4) - \sqrt{5}) \\ &= 9 - \sqrt{2} - \sqrt{3} - \sqrt{5} \\ &\alpha + \beta + \gamma + \delta = 9 - 1 - 1 - 1 = 6 \end{aligned}$$

29. The ordinates of the points P and Q on the parabola with focus (3,0) and directrix  $x = -3$  are in the ratio 3 : 1  $\therefore$  If R ( $\alpha, \beta$ ) is the point of intersection of the tangents to the parabola at P and Q, then  $\frac{\beta^2}{\alpha}$  is equal to \_\_\_\_\_.

**Sol.** 16

Parabola is  $y^2 = 12x$

Let Q( $3t^2, 6t$ )

so P( $27t^2, 18t$ )

$$\begin{aligned} R(\alpha, \beta) &= (at_1t_2, a(t_1 + t_2)) \\ &= (3t \cdot 3t, 3(t + 3t)) \end{aligned}$$

$$R(\alpha, \beta) = (9t^2, 12t)$$

$$\frac{\beta^2}{\alpha} = \frac{(12t)^2}{9t^2} = \frac{144}{9} = 16$$

30. Let k and m be positive real numbers such that the function  $f(x) = \begin{cases} 3x^2 + k\sqrt{x+1}, & 0 < x < 1 \\ mx^2 + k^2, & x \geq 1 \end{cases}$  is differentiable for all  $x > 0$ . Then  $\frac{8f'(8)}{f'(\frac{1}{8})}$  is equal to \_\_\_\_\_.

**Sol.** 309

function is differentiable  $\forall x < 0$

so  $f(1^-) = f(1)$

$$3 + \sqrt{2}k = m + k^2 \quad \dots(1)$$

and  $f_+^1(1^-) = f_-^1(1^+)$

$$2mx \Big|_{x=1} = 6x + \frac{k}{2\sqrt{x+1}} \Big|_{x=1}$$

$$2m = 6 + \frac{k}{2\sqrt{2}}$$

$$m = 3 + \frac{k}{4\sqrt{2}} \quad \dots(2)$$

$$k^2 + 3 + \frac{k}{4\sqrt{2}} = 3 + \sqrt{2}k$$

$$\boxed{k = \frac{7}{4\sqrt{2}}}, 0$$

$$m = 3 + \frac{7}{32}$$

$$\boxed{m = \frac{103}{32}}$$

So  $\frac{8f'(8)}{f'\left(\frac{1}{8}\right)} = 8 \times \frac{2mx|_{x=8}}{6x + \frac{k}{2\sqrt{x+1}}|_{x=\frac{1}{8}}}$

$$= \frac{8 \times 2 \times 8 \times \frac{103}{32}}{\frac{16}{12}}$$

$$= 103 \times 3 = 309$$

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