JEE MAIN 2024 SESSION-2
Paper with Solution

MATHS | 06th April 2024 _ Shift-1

SECTION – A

1. If A(3, 1, –1), B$\left(\frac{5}{3},\frac{7}{3},\frac{1}{3}\right)$, C (2, 2, 1) and D $\left(\frac{10}{3},\frac{2}{3},\frac{-1}{3}\right)$ are the vertices of a quadrilateral ABCD, then its area is

(1) $2\sqrt{2}$ (2) $\frac{4\sqrt{2}}{3}$ (3) $\frac{2\sqrt{2}}{3}$ (4) $\frac{5\sqrt{2}}{3}$

Sol. **2**

$\overrightarrow{AC} = -\hat{i}+\hat{j}+2\hat{k}$

$\overrightarrow{BD} = \frac{5}{3}\hat{i}-\frac{5}{3}\hat{j}-\frac{2}{3}\hat{k}$

area of quadrilateral = $\frac{1}{2}\left|\overrightarrow{AC}\times\overrightarrow{BD}\right|$

$= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 2 \\ \frac{5}{3} & \frac{-5}{3} & \frac{-2}{3} \end{vmatrix}$

$= \frac{8}{6}(\hat{i}+\hat{j}) = \frac{4}{3}\sqrt{2}$

2. For α, β, ∈ R and a natural number n, let $A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2}+\alpha \\ 2r & 2 & n^2-\beta \\ 3r-2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$. Then $2A_{10} - A_8$ is

(1) 2α + 4β (2) 0 (3) 2n (4) 4α + 2β

Sol. **(4)**

$A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2}+\alpha \\ 2r & 2 & n^2-\beta \\ 3r-2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$

$R_2 \rightarrow R_2 - 2R_1$

$Ar = \begin{vmatrix} r & 1 & \frac{n^2}{2}+\alpha \\ 0 & 0 & -\beta-2\alpha \\ 3r-2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$

Ar = (β + 2α) (3r – 3r + 2)

Ar = 4α + 2β

Hence $2A_{10} - A_8$

$= A_{10} = 4\alpha + 2\beta$

3. The shortest distance between the lines $\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}$ and $\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$ is

(1) $4\sqrt{3}$ (2) $8\sqrt{3}$ (3) $6\sqrt{3}$ (4) $5\sqrt{3}$

Sol. **(1)**

L_1 ; $\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5} \Rightarrow \begin{matrix} P(x_1, y_1, z_1)=(3,-15,9) \\ a_1, b_1, c_1=2,-7,5 \end{matrix}$

L_2 ; $\frac{x+1}{2}=\frac{y+1}{1}=\frac{z-9}{-3} \Rightarrow \begin{matrix} Q(x_2, y_2, z_2)=(-1,1,9) \\ a_2, b_2, c_2,=2,1,-3 \end{matrix}$

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} -4 & 16 & 0 \\ 2 & -7 & 5 \\ 2 & 1 & -3 \end{vmatrix} = 192$$

and $|\vec{b}_1 \times \vec{b}_2| = \sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}$

$= \sqrt{(16)^2+(16)^2+(16)^2} = 16\sqrt{3}$

shortest distance $= \frac{192}{16\sqrt{3}} = 4\sqrt{3}$ Ans.

4. $\int_0^{\pi/4} \frac{\cos^2 x \sin^2 x}{(\cos^3 x+\sin^3 x)^2} dx$ is equal to

(1) 1/9 (2) 1/3 (3) 1/12 (4) 1/6

Sol. **(4)**

$$\int_0^{\pi/4} \frac{\cos^2 x \sin^2 x}{(\cos^3 x+\sin^3 x)^2} dx$$

$$\Rightarrow \int_0^{\pi/4} \frac{\tan^2 x.\sec^2 x}{(1+\tan^3 x)^2}.dx$$

Let $1 + \tan^3 x = t$

$3.\tan^2 x.\sec^2 x.\ dx = dt$

$$\Rightarrow \frac{1}{3}\int_1^2 \frac{dt}{(t^2)^2}.$$

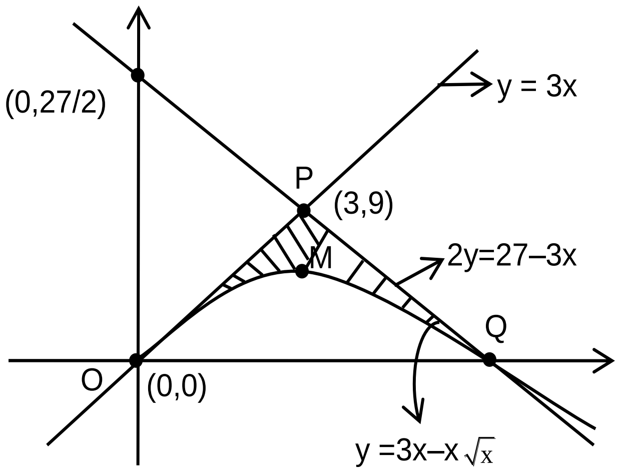
$$\Rightarrow \frac{1}{3}\left[-\frac{1}{t}\right]_1^2$$

$$\Rightarrow -\frac{1}{3}\left[+\frac{1}{2}-1\right] \Rightarrow \frac{1}{6}$$

5. Let the area of the region enclosed by the curves $y = 3x$, $2y = 27 - 3x$ and $y = 3x - x\sqrt{x}$ be A. Then 10A is equal to:

(1) 162 (2) 154 (3) 184 (4) 172

Sol. **(1)**



$y = 3x \quad 2y = 27 - 3x$

$3y = 27$

$y = 9 \Rightarrow x = 3 \rightarrow P(3, 9)$

$y = 3x \quad y = 3x - x\sqrt{x}$

$x = 0, \quad y = 0 \quad 0(0, 0)$

$2y = 27 - 3x \quad y = 3x - x\sqrt{x}$

$x = 9; y = 0 \quad Q(9, 0)$

Required Area

= Area of ΔOPQ – curve OMQ

$\Rightarrow \frac{1}{2}\times 9\times 9 - \int_0^9 (3x - x^{3/2}).dx$

$\Rightarrow \frac{81}{2} - \left[\frac{3\times x^2}{2} - \frac{x^{5/2}}{5/2}\right]_0^9$

$A = \frac{162}{10} \Rightarrow 10A = 162$

6. The interval in which the function $f(x) = x^x$, $x > 0$, is strictly increasing is

(1) $\left[\frac{1}{e}, \infty\right)$ (2) $(0, \infty)$ (3) $\left(0, \frac{1}{e}\right]$ (4) $\left[\frac{1}{e^2}, 1\right)$

Sol. **(1)**

$f(x) = x^x$

$f'(x) = x^x (1 + \ln x) \geq 0$

$x > 0 \Rightarrow 1 + \ln x \geq 0$

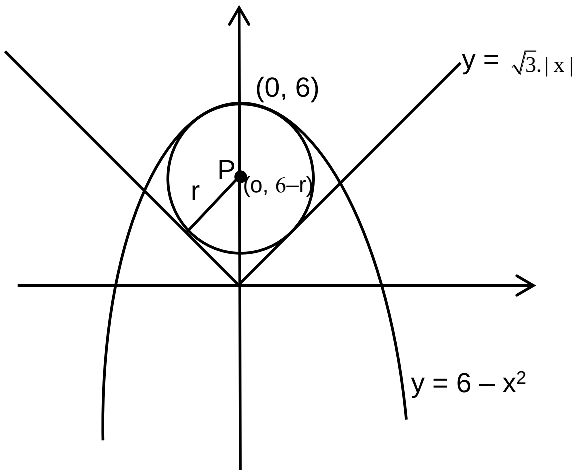
$x \geq \frac{1}{e}$

$f'(x) \geq 0$ for $x \in \left[\frac{1}{e}, \infty\right)$

7. Let C be the circle of minimum area touching the parabola $y = 6 - x^2$ and the lines $y = \sqrt{3}\,|x|$. Then, which one of the following points lies on the circle C ?

(1) (2, 4) (2) (2, 2) (3) (1, 1) (4) (1, 2)

Sol. **(1)**



Equation of circle $x^2 + (y - 6 + r)^2 = r^2$

distance b/w P and line = r

$\left|\frac{6-r}{2}\right| = r$

$r = 2$

Equation is $x^2 + (y - 4)^2 = 4$ Point (2, 4) is lie on circle

8. If $f(x)\begin{cases} x^3 \sin\left(\frac{1}{x}\right), x \neq 0 \\ 0 \quad\quad ,x = 0 \end{cases}$, then

(1) $f''(0) = 0$ (2) $f''(0) = 1$

(3) $f''\left(\frac{2}{\pi}\right) = \frac{24-\pi^2}{2\pi}$ (4) $f''\left(\frac{2}{\pi}\right) = \frac{12-\pi^2}{2\pi}$

Sol. **(3)**

$f(x)\begin{cases} x^3 \sin\left(\frac{1}{x}\right), x \neq 0 \\ 0 \quad\quad ,x = 0 \end{cases}$

$\lim_{h\to 0^+} f(0+h) = \lim_{h\to 0}(0+h)^3.\sin\left(\frac{1}{0+h}\right)$

$\Rightarrow \lim_{h\to 0^+} h^3.\sin(1/h)$

$\Rightarrow 0 \times (-1, 1] = 0$

$\lim_{h\to 0^-} f(0-h) = \lim_{h\to 0}(0-h)^3 \sin\left(-\frac{1}{h}\right)$

$\Rightarrow 0 \times [-1, 1] = 0$

f(x) is continuous.

$$f'(x) = 3.x^2.\sin\left(\frac{1}{x}\right) - x.\cos\frac{1}{x}$$

$$f''(x) = 6x\ .\sin\frac{1}{x} - 3\cos\frac{1}{x} - \cos\left(\frac{1}{x}\right) - \frac{\sin\frac{1}{x}}{x}$$

$$f''\left(\frac{2}{\pi}\right) = \frac{24-\pi^2}{2\pi}$$

9. Let A = {n ∈ [100, 700] ∩ N : n is neither a multiple of 3 nor a multiple of 4}. Then the number of elements in A is

(1) 300 (2) 290 (3) 280 (4) 310

Sol. (1)

A = {100, 101, 102, 103,......, 700}

n (multiple of 3) = 200

n (multiple of 4) = 151

n (multiple of 3 ∩ multiple of 4)

n (multiple of 12) = 50

n(3 ∪ 4) = n (3 divisible) + n(4 divisible) – n(3 div ∩ 4div)

= 200 + 151 – 50 = 301

Total number = 601

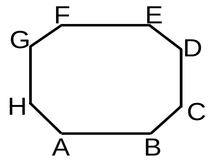
hence number of elements in A

= 601 – 301 = 300

10. The number of triangles whose vertices are at the vertices of a regular octagon but none of whose sides is a side of the octagon is

(1) 16 (2) 24 (3) 56 (4) 48

Sol. (1)



Total triangle = $^8C_3 = 56$

None of sides are common

= total – (1 sides common + 2 side common)

= $56 - (^8C_1 \times 4 + 8) = 16$

11. Let the relations R_1 and R_2 on the set X = {1, 2, 3, ….., 20} be given by R_1 = {(x, y): 2x – 3y = 2} and R_2 = {(x, y): –5x + 4y = 0}. If M and N be the minimum number of element required to be added in R_1 and R_2, respectively, in order to make the relations symmetric, then M + N equals

(1) 8 (2) 12 (3) 16 (4) 10

Sol. **(4)**

$R_1 = \{(x,y); 2x-3y=2\}$

$R_1 = \{(4,2)\ (7,4)\ (10,6)\ (13,8)\ (16,10)\ (19,12)\}$

M = 6

$R_2 = \{(x,y); -5x+4y=0\}$

= {(4, 5) (8, 10) (12, 15) (16, 20)} N = 4

Number of elements to added to make symmetric = 10

12. Let y = y(x) be the solution of the differential equation $(1 + x^2)\dfrac{dy}{dx} + y = e^{\tan^{-1}x}, y(1) = 0$. Then y(0) is

(1) $\frac{1}{4}\left(e^{\pi/2}-1\right)$ (2) $\frac{1}{2}\left(e^{\pi/2}-1\right)$ (3) $\frac{1}{2}\left(1-e^{\pi/2}\right)$ (4) $\frac{1}{4}\left(1-e^{\pi/2}\right)$

Sol. **(3)**

$(1 + x^2) \cdot \dfrac{dy}{dx} + y = e^{\tan^{-1}x}$

let $\tan^{-1} x = t$

$\dfrac{1}{1+x^2}.dx = dt$

$\dfrac{dy}{dt} + y = e^t$

If = $e^{\int 1.dt} = e^t$

$y.\ e^t = \dfrac{e^{2t}}{2} + C$

$y.e^{\tan^{-1}x} = \dfrac{e^{2\tan^{-1}x}}{2} + C$

$x = 1 \quad y = 0 \quad C = -\dfrac{e^{\pi/2}}{2}$

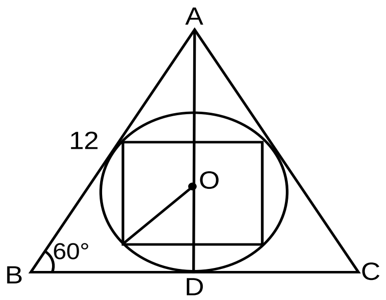
$y.\ \tan^{-1}x = \dfrac{e^{2\tan^{-1}x}}{2} - \dfrac{e^{\pi/2}}{2}$

$x = 0 \Rightarrow y = \dfrac{1}{2}(1-e^{\pi/2})$

13. A circle is inscribed in an equilateral triangle of side of length 12. If the area and perimeter of any square inscribed in this circle are m and n. respectively. then m + n^2 is equal to

(1) 396 (2) 408 (2) 312 (4) 414

Sol. **(2)**



radius of circle $\Rightarrow \dfrac{1}{3} \times AD$

AD = AB sin 60 = $6\sqrt{3}$

r = $2\sqrt{3}$

Perimeter = 4a

= 4 × $\sqrt{2}r$

= $8\sqrt{6}$ = n

area = $4 \times \text{area.}\Delta OPQ$

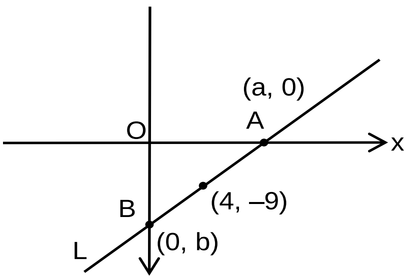
= 4 × $\frac{1}{2} \times r^2 = 24 = M$

M + n^2 = 24 + 384 = 408

14. Let a variable line of slope m > 0 passing through the point (4, – 9) intersect the coordinate axes at the points A and B. The minimum value of the sum of the distance of A and B from the origin is

(1) 30 (2) 15 (3) 10 (4) 25

Sol. **(4)**



L ; y + 9 = m (x – 4)

A (a, 0) = $\left(4 + \frac{9}{m}, 0\right)$

B (0, b) = (0, –9 – 4 m)

OA + OB = 4 + $\frac{9}{m} + 9 + 4m$

T = $\frac{9}{m} + 4m + 13$

for minimum $\frac{dT}{dm} = 0$ $\quad \frac{-9}{m^2} + 4 = 0$

⇒ m = 3/2

T = $\frac{9}{3} \times 2 + 4 \times \frac{3}{2} + 13 = 25$

15. Let α, β be the distinct roots of the equation $x^2 - (t^2 - 5t + 6)x + 1 = 0$, $t \in R$ and $a_n = \alpha^n + \beta^n$. Then the minimum value of $\frac{a_{2023} + a_{2025}}{a_{2024}}$ is

(A) 1/4 (2) –1/4 (3) –1/2 (D) 1/2

Sol. **(2)**

From Newton's formula

$\alpha_{n+2} - (t^2 - 5t + 6)\alpha_{n+1} + \alpha_n = 0$

n = 2023

$\alpha_{2025} - (t^2\ 5t + 6)\ \alpha_{2024} + \alpha_{2023} = 0$

$\frac{\alpha_{2025} + \alpha_{2023}}{\alpha_{2024}} = t^2 - 5t + 6$

$= \frac{-D}{4a} = -\frac{1}{4}$

16. The function $f(x) = \frac{x^2+2x-15}{x^2-4x+9}, x \in R$ is

(1) both one-one and onto
(2) one-one but not onto
(3) onto but not one-one.
(4) neither one-one nor onto

Sol. **Bonus**

$f(x) = \frac{x^2+2x-15}{x^2-4x+9}$

$f'(x) = \frac{(x^2-4x+9)(2x+2)-(x^2+2x-15)(2x-4)}{(x^2-4x+9)^2}$

$f'(x) = \frac{-6x^2+48x-42}{(x^2-4x+9)^2}$

So f′(x) is +ve & –ve so many one and into

17. The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On rechecking, it was found that an observation by mistake was taken 8 instead of 12. The correct standard deviation is

(1) 1.8
(2) $\sqrt{3.96}$
(3) $\sqrt{3.86}$
(4) 1.94

Sol. **(2)**

mean $(\bar{x}) = 10$ standard deviation $\sigma = 2$

n = 20

$(\bar{x}) = 10$

$\frac{x_1 + x_2 + ... + x_{20}}{20} = 10$

$x_1 + x_2 + ... + x_{20} = 200$

Let mistake observation is x_{20}

$\bar{x}_{new} = \frac{200-8+12}{20} = 10.2$

$\sigma^2 = \frac{\sum x_i^2}{n} - \bar{x}^2 = 4$

$\Rightarrow x_1^2 + x_2^2 + ... + x_{19}^2 + 64 = 2080$

$\Rightarrow x_1^2 + ... + x_{19}^2 = 2016$

$\sigma_{new}^2 = \frac{2016+144}{20} - (10.2)^2$

$\sigma_{new} = \sqrt{3.96}$

18. Let f: (–∞, ∞) – {0} →R be a differentiable function such that $f'(1) = \lim_{a\to\infty} a^2 f\left(\frac{1}{a}\right)$.

Then $\lim_{a\to\infty} \frac{a(a+1)}{2}\tan^{-1}\left(\frac{1}{a}\right) + a^2 - 2\log_e a$ is equal to

(1) $\frac{5}{2}+\frac{\pi}{8}$ (2) $\frac{3}{2}+\frac{\pi}{4}$ (3) $\frac{3}{8}+\frac{\pi}{4}$ (4) $\frac{3}{4}+\frac{\pi}{8}$

Sol. **(1)**

$f : (-\infty, \infty) - \{0\} \to R$

$f'(1) = \lim_{a\to\infty} a^2 f\left(\frac{1}{a}\right)$

$\lim_{a\to\infty} \frac{a(a+1)}{2}\tan^{-1}\left(\frac{1}{a}\right) + a^2 - 2\ln(a)$

$\lim_{a\to\infty} a^2\left(\frac{\left(1+\frac{1}{a}\right)}{2}\tan^{-1}\left(\frac{1}{a}\right) + 1 - \frac{2}{a^2}\ln(a)\right)$

$f(x) = \frac{1}{2}(1+x)\tan^{-1}(x) + 1 - 2x^2\ln(x)$

$f'(x) = \frac{1}{2}\left(\frac{1+x}{1+x^2} + \tan^{-1}(x) + 4x\ell n(x)\right) + 2x$

$f'(1) = \frac{1}{2}\left(1+\frac{\pi}{4}\right) + 2$

$f'(1) = \frac{5}{2}+\frac{\pi}{8}$

19. Let y = y(x) be the solution of the differential equation

$(2x\log_e x)\frac{dy}{dx} + 2y = \frac{3}{x}\log_e x, x > 0$ and $y(e^{-1}) = 0.$ Then, y(e) is equal to

(1) $-\frac{3}{e}$ (2) $-\frac{2}{3e}$ (3) $-\frac{3}{2e}$ (4) $-\frac{2}{e}$

Sol. **(1)**

$2x\log x\frac{dy}{dx} + 2y = \frac{3}{x}\log x$

$\frac{dy}{dx} + \frac{y}{x\log x} = \frac{3}{2x^2}$

If = $e^{\int \frac{1}{x\log x}dx} = e^{dx\log x} = \log x$

y. logx = $\int \frac{3}{2x^2}.\log x dx + C$

= $\frac{3}{2}\log x.\left(\frac{-1}{x}\right) - \frac{3}{2}\int .\frac{1}{x}\left(\frac{-1}{x}\right)dx$

y log x = $\frac{-3}{2a}\log x - \frac{3}{2}.\frac{1}{x} + C$

$\Rightarrow x = \frac{1}{e}$ $\quad y = 0$

$O = \frac{-3}{2e^{-1}}.\log e^{-1} - \frac{3}{2e^{-1}} + C$

$\frac{3e}{2} - \frac{3e}{2} + C$

$C = 0$

$y \log x = \frac{-3\log x}{2x} - \frac{3}{2x}$

$y(e).1 = \frac{-3\times 1}{2e} - \frac{3}{2e}$

$= \frac{-6}{2e} = \frac{-3}{e}$

20. A company has two plants A and B to manufacture motorcycles. 60% motorcycles are manufactured at plant A and the remaining are manufactured at plant B. 80% of the motorcycles manufactured at plant A are rated of the standard quality, while 90% of the motorcycles manufactured at plant B are rated of the standard quality. A motorcycles picked up randomly from the total production is found to be of the standard quality. If p is the probability that it was manufactured at plant B, then 126p is.

(1) 66 (2) 56 (3) 54 (4) 64

Sol. **(3)**

	Plant A	Plant B
Manufactory	60%	40%
standard quality	$\frac{8}{10}$	$\frac{9}{10}$

$$P\left(\frac{B}{S}\right) = \frac{P\left(\frac{S}{B}\right).P(B)}{P\left(\frac{S}{A}\right).P(A) + P\left(\frac{S}{B}\right)P(B)}$$

$$= \frac{\frac{4}{10}\times\frac{9}{10}}{\frac{6}{10}\times\frac{8}{10} + \frac{4}{10}\times\frac{9}{10}}$$

$P = \frac{36}{48+36} = \frac{36}{84} = \frac{9}{21}$

$126P = 126 \times \frac{9}{21} = 54$

SECTION – B

21. For $n \in N$, if $\cot^{-1} 3 + \cot^{-1} 4 + \cot^{-1} 5 + \cot^{-1} n = \frac{\pi}{4}$, then n is equal to ____.

Sol. **47**

$\cot^{-1} 3 + \cot^{-1} 4 + \cot^{-1} 5 + \cot^{-1} n = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{n}\right) = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{1/3+1/4}{1-1/3.1/4}\right)+\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{n}\right)=\frac{\pi}{4}$

$\Rightarrow \tan^{-1}\frac{7}{11}+\tan^{-1}\frac{1}{5}+\tan^{-1}\left(\frac{1}{n}\right)=\frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{\frac{7}{11}+\frac{1}{5}}{1-\frac{7}{11}.\frac{1}{5}}\right)+\tan^{-1}\left(\frac{1}{n}\right)=\frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{35+11}{55-7}\right)+\tan^{-1}\left(\frac{1}{n}\right)=\frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{\frac{23}{24}+\frac{1}{n}}{1-\frac{23}{24}\frac{1}{n}}\right)=\frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{23n+24}{24n-23}\right)=\frac{\pi}{4}$

$\Rightarrow 23n + 24 = 24n - 23$

$\Rightarrow n = 47$

22. Let L_1, L_2 be the lines passing through the point P(0, 1) and touching the parabola $9x^2 + 12x + 18y - 14 = 0$. Let Q and R be the point on the lines L_1 and L_2 such that the ΔPQR is an isosceles triangle with base QR. If the slopes of the line QR are m_1 and m_2, then $16\left(m_1^2+m_2^2\right)$ is equal to ____.

Sol. **68**

Parabola

$9x^2 + 12x + 18y - 14 = 0$

Let the equation of line be

$y - 1 = m (x - 0)$

$y = mx + 1$

$9x^2 + 12x + 18(mx + 1) - 14 = 0$

$\Rightarrow 9x^2 + x (12 + 18m) + 4 = 0$

$\boxed{D=0}$

$(12 + 18m)^2 - 4(9)(4) = 0$

$\Rightarrow 36\{(2 + 3m)^2 - 4\} = 0$

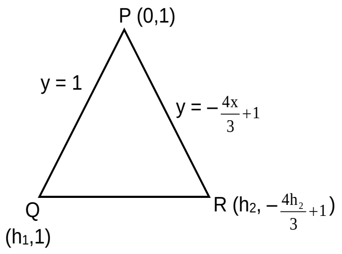
$\Rightarrow (2 + 3m + 2) (2 + 3m - 2) = 0$

$(3m + 4) (3m) = 0$

$m = \frac{-4}{3}, 0$

lines are L_1 $\boxed{y=\frac{-4x}{3}+1}$

$\boxed{y=1}$



PQ = PR

$$\Rightarrow h_1^2 + o = h_2^2 + \left(\frac{-4h_2}{3}\right)^2$$

$$= h_2^2 + \frac{16h_2^2}{9}$$

$$h_1^2 = \frac{25h_2^2}{9}$$

$$h_1 + \frac{5h_2}{3} \text{ or } h_1 = \frac{-5h_2}{3}$$

Slope of QR

$$m_1 = \frac{\frac{-4h_2}{3}}{h_2 - h_1}$$

$$= \frac{-4h_2}{3} \times \frac{1}{h_2 - \frac{5h_2}{3}}$$

$$= \frac{-4h_2}{3h_2 - 5h_2}$$

$$= \frac{-4h_2}{-2h_2} = 2$$

$$m_2 = \frac{\frac{-4h_2}{3}}{h_2 + \frac{5h_2}{3}}$$

$$= \frac{-4h_2}{3} \times \frac{1}{\frac{3h_2 + 5h_2}{3}}$$

$$= \frac{-4h_2}{8h_2} = \frac{-1}{2}$$

Now $16(m_1^2 + m_2^2)$

$$= 16\left(4 + \frac{1}{4}\right) = 16 \times \frac{17}{4} = 68 \text{ Ans.}$$

23. Let the first term of a series be $T_1 = 6$ and its r^{th} term $T_r = 3\,T_{r-1} + 6^r$, r = 2, 3, ….., n. If the sum of the first n terms of this series is $\frac{1}{5}(n^2 - 12n + 39)(4.6^n - 5.3^n + 1)$, then n is equal to ___.

Sol. **6**

$T_r = 3T_{r-1} + 6^r$, r = 2, 3, 4, ... n

$T_2 = 3.T_1 + 6^2$

$T_2 = 3.6 + 6^2$ ...(1)

$T_3 = 3T_2 + 6^3$

$T_3 = 3(3.6 + 6^2) + 6^3$

$T_3 = 3^2.6 + 3.6^2 + 6^3$ ...(2)

$T_r = 3^{r-1} \cdot 6 + 3^{r-2} \cdot 6^2 + \ldots + 6^r$

$$T_r = 3^{r-1}\cdot 6\left[1+\frac{6}{3}+\left(\frac{6}{3}\right)^2+\ldots+\left(\frac{6}{3}\right)^{r-1}\right]$$

$$T_r = 3^{r-1}\cdot 6\left(1+2+2^2+\ldots+2^{r-1}\right)$$

$$T_r = 6\cdot 3^{r-1}1\cdot\frac{\left(1-2^r\right)}{(-1)}$$

$$T_r = 6\cdot 3^{r-1}\cdot\left(2^r-1\right)$$

$$T_r = \frac{6\cdot 3^r}{3}\cdot\left(2^r-1\right)$$

$$T_r = 2.\left(6^r-3^r\right)$$

$$S_n = 2\Sigma\left(6^r-3^r\right)$$

$$S_n = 2\cdot\left[\frac{6\cdot\left(6^n-1\right)}{5}-\frac{3\cdot\left(3^n-1\right)}{2}\right]$$

$$S_n = 2\left[\frac{12\left(6^n-1\right)-15\left(3^n-1\right)}{10}\right]$$

$$S_n = \frac{3}{5}\left[4.6^4-5.3^n+1\right]$$

$$\therefore n^2-12n+39=3$$

$n^2 - 12n + 36 = 0 \Rightarrow n = 6$

24. Let $\vec{a}=2\hat{i}-3\hat{j}+4\hat{k}, \vec{b}=3\hat{i}+4\hat{j}-5\hat{k}$ and a vector $\vec{c}$ be such that $\vec{a}\times(\vec{b}+\vec{c})+\vec{b}\times\vec{c}=\hat{i}+8\hat{j}+13\hat{k}$. If $\vec{a}.\vec{c}=13$, then $(24-\vec{b}.\vec{c})$ is equal to ____.

Sol. **46**

$$\vec{a}\times\vec{b}+\vec{a}\times\vec{c}+\vec{b}\times\vec{c}=(1,8,13)$$

$$\vec{a}\times(\vec{a}\times\vec{b})+\vec{a}\times(\vec{a}\times\vec{c})+\vec{a}\times(\vec{b}\times\vec{c})$$

$$=\vec{a}\times(\hat{i}+8\hat{j}+13k)$$

$$(\vec{a}\cdot\vec{b})\vec{a}-a^2\vec{b}+(\vec{a}\cdot\vec{c})\vec{a}-a^2\vec{c}+(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}=\vec{a}\times(\hat{i}+8\hat{j}+13\hat{k})$$

$$\Rightarrow -26\vec{a}-29\vec{b}+13\vec{a}-29\vec{c}+13\vec{b}+26\vec{c}=\vec{a}\times(\hat{i}+8\hat{j}+13\hat{k})$$

$$\Rightarrow\quad -13\vec{a}-16\vec{b}-3\vec{c}=\vec{a}\times(\hat{i}+8\hat{j}+13k)$$

$$\Rightarrow\quad -13\vec{a}.\vec{b}-16b^2-3\vec{b}.\vec{c}=\vec{a}\times(\hat{i}+8\hat{j}+13k).\vec{b}$$

$$\Rightarrow\quad (-13)(-26)-16(50)-3\vec{b}\cdot\vec{c}=\begin{vmatrix}2 & -3 & 4\\ 1 & 8 & 13\\ 3 & 4 & -5\end{vmatrix}$$

$$\Rightarrow\quad -462-3\vec{b}\cdot\vec{c}=-396$$

$$\Rightarrow\quad \vec{b}\cdot\vec{c}=-22$$

Hence $24-\vec{b}.\vec{c}=46$

25. Let a conic C pass through the point (4, –2) and P(x, y), x $\geq$3, be any point on C. Let the slope of the line touching the conic C only at a single point P be half the slope of the line joining the points P and (3, – 5). If the focal distance of the point (7, 1) on C is d, then 12d equals ____.

Sol. **75**

$y = f(x)$

$\frac{dy}{dx} = f'(x) =$ slope of line tanget at p(x, y)

Now the slope of line joining p(x, y)

& (3, –5) is $\frac{y+5}{x-3}$

Now $\frac{dy}{dx} = \frac{1}{2}\left(\frac{y+5}{x-3}\right)$

$\int \frac{dy}{y+5} = \frac{1}{2}\int \frac{dx}{x-3}$

$\Rightarrow 2\ell n(y + 5) = \ell n(x - 3) + \ell nk$

It pass through (4, –2)

$2\ \ell n\ (-2 + 5) = \ell n(4 - 3) + \ell nk$

$2\ell n3 = \ell n1 + \ell nk$

$\ell nk = 2\ell n3$

$k = 9$

$2\ell n(y + 5) = \ell n(x - 3) + \ell n9$

$\ell n(y + 5)^2 = \ell n9(x - 3)$

$(y + 5)^2 = 9\ (x - 3)$

Direction $x - 3 = -\frac{9}{4}$

$x = -\frac{9}{4} + 3$

$x = \frac{3}{4}$

Focal distance of point (7, 1) = Distance of (7, 1) from the directrix

$\Rightarrow d = 7 - \frac{3}{4}$

$d = \frac{25}{4}$

$12d = 12 \times \frac{25}{4}$

12d = 75 Ans.

26. Let $r_k = \frac{\int_0^1 \left(1-x^7\right)^k dx}{\int_0^1 \left(1-x^7\right)^{k+1} dx}, k \in N$. Then the value of $\sum_{k=1}^{10} \frac{1}{7(r_k - 1)}$ is equal to ____.

Sol. **65**

$$r_k = \frac{\int_0^1 (1-x^7)^k dx}{\int_0^1 (1-x^7)^{k+1} dx}$$

$$\int_0^1 (1-x^7)^{k+1} dx = \int_0^1 (1-x^7)^k.(1-x^7)dx$$

$$= \int_0^1 (1-x^7)^k dx - \int_0^1 x^7(1-x^7)^k dx$$

$$= \int_0^1 (1-x^7)^k - \int_0^1 x.x^6(1-x^7)^k dx$$

$$= \int_0^1 (1-x^7)^k - x\int_0^1 x^6(1-x^7)^k dx + \int_0^1 \left[1.\int x^6.(1-x^7)^k dx\right]dx$$

$$= \int_0^1 (1-x^7)^k - x\left.\frac{(1-x^7)^{k+1}}{-7(k+1)}\right|_0^1 + \int_0^1 \frac{(1-x^7)^{k+1}}{-7(k+1)}dx$$

$$\Rightarrow \int_0^1 (1-x^7)^{k+1}dx\left\{1+\frac{1}{7(k+1)}\right\} = \int_0^1 (1-x^7)^k dx - 0$$

$$\frac{7k+8}{7(k+1)} = \frac{\int_0^1 (1-x^7)^k dx}{\int_0^1 (1-x^7)^{k+1} dx}$$

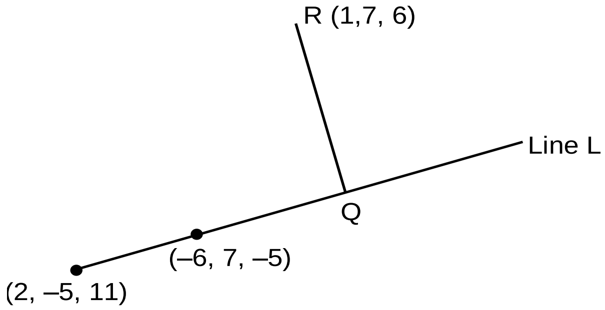
$$\sum_{k=1}^{10} \frac{1}{7(r_k - 1)}$$

$$\Rightarrow \sum_{k=1}^{10} \frac{1}{7(1+\frac{1}{7k+7}-1)}$$

$$\Rightarrow \sum_{k=1}^{10} (K+1) \Rightarrow \frac{10\times 11}{2}+10 \Rightarrow 55 + 10 = 65 \text{ Ans.}$$

27. Let P be the point (10, –2, –1) and Q be the foot of the perpendicular drawn from the point R(1, 7, 6) on the line passing through the point (2, –5, 11) and (–6, 7, – 5). Then the length of the line segment PQ is equal to ________.

Sol. **13**



equation of L :

$$\frac{x-2}{-6-2} = \frac{y+5}{7+5} = \frac{z-11}{-5-11}$$

$\frac{x-2}{-8}=\frac{y+5}{12}=\frac{z-11}{-16}$

$\frac{x-2}{-2}=\frac{y+5}{3}=\frac{z-11}{-4}=\lambda(\text{Let})$

Let Q (–2λ + 2, 3λ – 5, –4λ + 11)

DR's of QR

$(-2\lambda+2-1,\ 3\lambda-5-7, -4\lambda+11-6)$

$(-2\lambda+1,\ 3\lambda-12, -4\lambda+5)$

$\because$ RQ ⊥ Line L

$\therefore$ –2(–2λ+1) + 3(3λ – 12) – 4 (–4λ + 5) = 0

⇒ 4λ – 2 + 9λ – 36 + 16λ – 20 = 0

⇒ 29λ = 58 ⇒ $\boxed{\lambda=2}$

$\therefore$ Q (–2 × 2 + 2, 3× 2 – 5, –4 × 2 + 11)

Q (–2, 1, 3)

$\therefore$ PQ = $\sqrt{(10+2)^2+(-2-1)^2+(-1-3)^2}$

$= \sqrt{144+9+16}$

$= \sqrt{169}$

$\boxed{PQ=13}$ Ans.

28. Let αβγ = 45; α, β, γ ∈ R. If x(α, 1, 2) + y(1, β, 2) + z(2, 3, γ) = (0, 0, 0) for some x, y, z ∈ R, xyz ≠ 0, Then 6α + 4β + γ is equal to___.

Sol. **55**

αβγ = 45 x(α, 1, 2) + y(1, β, 2) + z(2, 3, γ) = (0, 0, 0)

αx + y + 2z = 0

x + βy + 3z = 0

2x + 2y + γz = 0

$$\begin{vmatrix}\alpha & 1 & 2\\ 1 & \beta & 3\\ 2 & 2 & \gamma\end{vmatrix}=0$$

$\alpha(\beta\gamma-6)-1(\gamma-6)+2(2-2\beta)=0$

⇒ αβγ – 6α – γ + 6 + 4 – 4β = 0

⇒ 10 + 45 = 6α + 4β + γ

⇒ 6α + 4β + γ = 55 Ans.

29. If the second, third and fourth terms in the expansion of $(x + y)^n$ are 135, 30 and $\frac{10}{3}$, respectively, then $6(n^3 + x^2 + y)$ is equal to ________

Sol. **806**

$T_{r+1}={}^nC_r\ x^{n-r}\ y^r$

$T_2={}^nC_1\ x^{n-1}\ y=135$ ....(1)

$T_3 = {}^nC_2\ x^{n-2}\ y^2 = 30$ ....(2)

$T_4 = {}^nC_3\ x^{x-3}\ y^3 = \frac{10}{3}$ ....(3)

$\frac{{}^nC_2}{{}^nC_1} . \frac{y}{x} = \frac{30}{135}$

$\frac{n-2+1}{2} . \frac{y}{x} = \frac{2}{9}$

$9(n-1)y = 4x$ ....(4)

$\frac{{}^nC_3}{{}^nC_2} . \frac{y}{x} = \frac{1}{9}$

$\frac{n-3+1}{3} . \frac{y}{x} = \frac{1}{9}$

$3(n-2).\ y = x$ ....(5)

from (4) & (5)

$3n - 3 = 4n - 8$

$\Rightarrow n = 5$

$\boxed{9y = x}$

${}^5C_1 . x^4 \left(\frac{x}{9}\right) = 135 \Rightarrow \boxed{x = 3}$ $\boxed{y = \frac{1}{3}}$

Hence

$6(n^3 + x^2 + y) = 6\ (125 + 9 + \frac{1}{3})$

$= 806$ Ans.

30. Let x_1, x_2, x_3, x_4, be the solution of the equation $4x^4 + 8x^3 - 17x^2 - 12x + 9 = 0$ and $\left(4+x_1^2\right)\left(4+x_2^2\right)\left(4+x_3^2\right)\left(4+x_4^2\right) = \frac{125}{16}m$. Then the value of m is ____.

Sol. **221**

$4x^4 + 8x^3 - 17x^2 - 12x + 9 = 0$

x_1, x_2, x_3, x_4 are the solution of equation

$4x^4 + 8x^3 - 17x^2 - 12x + 9 = 4(x - x_1)(x - x_2)(x - x_3)(x - x_4)$

put $x = 2i$

$4(2i)^4 + 8(2i)^3 - 17(2i)^2 - 12(2i) + 9$

$= 4[(2i - x_1)(2i - x_2)(2i - x_3)(2i - x_4)]$

$\Rightarrow$ $64 - 64i + 68 - 24i + 9$

$\Rightarrow$ $|141 - 88i| = 4|(2i - x_1)(2i - x_2)(2i - x_3)(2i - x_4)|$

$\Rightarrow$ $(141)^2 + (88)^2 = 16[(4 + x_1^2)(4 + x_2^2)(4 + x_3^2)(4 + x_4^2)]$

$\Rightarrow$ $\frac{19881 + 7744}{16} = (4+x_1^2)(4+x_2^2)(4+x_2^2)(4+x_4^2)$

$\Rightarrow$ $\frac{27625}{16} = \frac{125}{16}m$

$\boxed{m = 221}$ Ans.

REVISED