JEE MAIN 2024 SESSION-2

Paper with Solution

MATHS | 04th April 2024 _ Shift-2

SECTION – A

1. Let PQ be a chord of the parabola $y^2 = 12x$ and the midpoint of PQ be at (4, 1). Then, which of the following point lies on the line passing through the points P and Q?

(1) $\left(\frac{3}{2},-16\right)$ (2) $(3,-3)$ (3) $(2,-9)$ (4) $\left(\frac{1}{2},-20\right)$

Sol. **(4)**

$T = S_1$

$y \times 1 - 6(x + 4) = 1 - 48$

$y - 6x = -47 + 24$

$6x - y - 23 = 0$

option 4 satisfying the equation.

2. Let $f(x)=\int_0^x \left(i+\sin\left(1-e^t\right)\right)dt, x \in R$. Then, $\lim\limits_{x\to0}\frac{f(x)}{x^3}$ is equal to

(1) $-\frac{1}{6}$ (2) $\frac{2}{3}$ (3) $\frac{1}{6}$ (4) $-\frac{2}{3}$

Sol. **(1)**

$$\lim_{x\to0}\frac{x+\sin(1-e^x)}{3x^2} \Rightarrow \lim_{x\to0}\frac{1-e^x\cos(1-e^x)}{6x}$$

$$\Rightarrow \lim_{x\to0}-\frac{(e^x)^2\sin(1-e^x)-e^x\cos(1-e^x)}{6}$$

$$=\frac{-1}{6}$$

3. If the function

$$f(x)=\begin{cases}\dfrac{72^x-9^x-8^x+1}{\sqrt{2}-\sqrt{1+\cos x}} & , \quad x\neq0 \\ a\log_e 2\log_e 3 & , \quad x=0\end{cases}$$

is continuous at x = 0, then the value of a^2 is equal to

(1) 746 (2) 1250 (3) 1152 (4) 968

Sol. **(3)**

$$f(0^+)=\lim_{x\to0}\frac{72^h-9^h-8^h+1}{1-\cos h}\times2\sqrt{2}$$

$$=\lim_{x\to0}\frac{(9^h-1)(8^h-1)}{h^2\times\frac{1}{2}}\times2\sqrt{2}$$

$$\ell n9\times\ell n8\times4\sqrt{2}$$

$$=24\sqrt{2}\ell n2\times\ell n3$$

4. If the coefficients of x^4, x^5 and x^6 in the expansion of $(1 + x)^n$ are in the arithmetic progression, then the maximum value of n is:

(1) 28 (2) 21 (3) 7 (4) 14

Sol. **(4)**

$$2\,{}^nC_5 = {}^nC_4 + {}^nC_6$$

$$\frac{2.n!}{5!(n-5)!} = \frac{n!}{4!(n-4)!} + \frac{n!}{6!(n-6)!}$$

$$\frac{2!}{5\times 4!(n-5)(n-6)!} = \frac{1}{4!(n-4)(n-5)(n-6)!} + \frac{1}{6\times 5\times 4!(n-6)!}$$

$$\frac{2}{5(n-5)} = \frac{1}{(n-4)(n-5)} + \frac{1}{30}$$

$$\frac{2(n-4)}{5} = 1 + \frac{(n-4)(n-5)}{30}$$

$$12n - 40 = 30 + n^2 - 9n + 20$$

$$n^2 - 21n + 98 = 0$$

$$n = 14, 7$$

5. If the mean of the following probability distribution of a random variable X :

X	0	2	4	6	8
P(X)	A	2a	a + b	2b	3b

is $\frac{46}{9}$, then the variance of the distribution is

(1) $\frac{151}{27}$ (2) $\frac{566}{81}$ (3) $\frac{581}{81}$ (4) $\frac{173}{27}$

Sol. **(2)**

$$\sum P_i = 1 \Rightarrow 4a + 6b = 1$$

$$\text{Mean} = \sum p_i x_i$$

$$\frac{46}{9} = 0 + 4a + 4a + 4b + 12b + 24b$$

$$\frac{46}{9} = 8a + 40b$$

$$\frac{46}{3} = 24a + 120b \Rightarrow 4a + 20b = \frac{23}{9}$$

$$\Rightarrow a = \frac{1}{12}, b = \frac{1}{9}$$

$$\text{var} = \sum = p_i x_i^2 - (\sum p_i x_i)^2$$

$$= 8a + 16a + 16b + 72b + 192b - \left(\frac{46}{9}\right)^2 = \frac{566}{81}$$

6. Given that the inverse trigonometric function assumes principal values only. Let x, y be any two real numbers in [–1, 1] such that $\cos^{-1} x - \sin^{-1} y = \alpha$, $\frac{-\pi}{2} \le \alpha \le \pi$. Then, the minimum value of $x^2 + y^2 + 2xy \sin\alpha$ is

(1) –1 (2) 0 (3) $\frac{-1}{2}$ (4) $\frac{1}{2}$

Sol. **(2)**

$$\cos^{-1}x + \cos^{-1}y = \frac{\pi}{2} + \alpha$$

$$xy - \sqrt{1-x^2}\sqrt{1-y^2} = -\sin\alpha$$

$$(xy + \sin\alpha)^2 = 1 - y^2 - x^2 + x^2y^2$$

$$x^2 + y^2 + 2xy\sin\alpha = 1 - \sin^2\alpha = \cos^2\alpha$$

7. Let y = y(x) be the solution of the differential equation $(x^2 + 4)^2 dy + (2x^3y + 8xy - 2)dx = 0$. If y(0) = 0, then y(2) is equal to

(1) $\frac{\pi}{8}$ (2) $\frac{\pi}{16}$ (3) $\frac{\pi}{32}$ (4) 2π

Sol. **(3)**

$$\frac{dy}{dx} + y\left\{\frac{2x^3 + 8x}{(x^2+4)^2}\right\} = \frac{2}{(x^2+4)^2}$$

$$I.F = e^{\int \frac{2x(x^2+4)}{(x^2+4)^2}dx} = (x^2+4)$$

$$y(x^2+4) = \int \frac{2}{x^2+4}dx = \tan^{-1}\left(\frac{x}{2}\right) + C$$

$$C = 0 \Rightarrow y(2) = \frac{\pi}{4(8)}$$

8. Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = I + \text{adj}(A) + (\text{adj } A)^2 + \ldots + (\text{adj } A)^{10}$. Then, the sum of all the elements of the matrix B is :

(1) –110 (2) 22 (3) –88 (4) –124

Sol. **(3)**

$$\text{adj} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$$

$$(\text{adj}A)^2 = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix} + \ldots\ldots + \begin{bmatrix} 1 & -20 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 11 & -110 \\ 0 & 11 \end{bmatrix}$$

9. Let a relation R on N × N be defined as :

(x_1, y_1) R (x_2, y_2) if and only if $x_1 \le x_2$ or $y_1 \le y_2$.

Consider the two statements:

(I) R is reflexive but not symmetric.

(II) R is transive

Then which one of the following is true?

(1) Neither (I) nor (II) is correct.
(2) Both (I) and (II) are correct.
(3) Only (I) is correct.
(4) Only (II) is correct.

Sol. **(3)**

If (3, 7) R(4, 3) & (4, 3) R(1, 3)

$\Rightarrow$ (3, 7) R(1, 3)

(5, 4) R(5, 4) $\Rightarrow 5 \le 5$ (Reflexive)

(1, 2) R(3, 4) $\Rightarrow 1 \le 3$ or $2 \le 4$

$\Rightarrow$ (3, 4) R(1, 2)

$\Rightarrow$ (Not Symmetric)

10. Let $f(x) = 3\sqrt{x-2} + \sqrt{4-x}$ be a real valued function. If α and β are respectively the minimum and the maximum values of *f*, then $\alpha^2 + 2\beta^2$ is equal to

(1) 38
(2) 24
(3) 44
(4) 42

Sol. **(4)**

$$f'(x) = \frac{3}{2\sqrt{x-2}} - \frac{1}{2\sqrt{4-x}} = 0$$

$$3\sqrt{4-x} = \sqrt{x-2}$$

36 – 9x = x – 2

10x = 38

$$x = \frac{38}{10} = \frac{19}{5}$$

Also, $x_{\in}[2,4]$

$$f(2) = \sqrt{2}$$

$$f\left(\frac{19}{5}\right) = \frac{9}{\sqrt{5}} + \frac{1}{\sqrt{5}} = 2\sqrt{5}$$

$$f(4) = 3\sqrt{2}$$

11. If the value of the integral $\int_{-1}^{1} \frac{\cos \alpha x}{1+3^x} dx$ is $\frac{2}{\pi}$. Then, a value of α is

(1) $\frac{\pi}{2}$
(2) $\frac{\pi}{3}$
(3) $\frac{\pi}{6}$
(4) $\frac{\pi}{4}$

Sol. **(1)**

$$I=\int_{-1}^{1}\frac{3^x\cos(\alpha x)}{1+3^x}$$

$$2I=\int_{-1}^{1}(\cos\alpha x)dx$$

$$2I=2\int_{0}^{1}(\cos\alpha x)dx$$

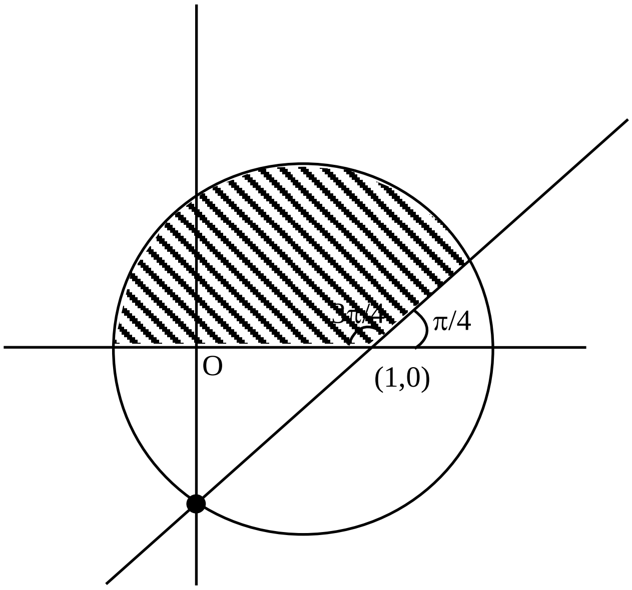
$$I=\left(\frac{\sin\alpha x}{\alpha}\right)_0^1\Rightarrow\frac{\sin\alpha}{\alpha}=\frac{2}{\pi}$$

12. The area (in sq. units) of the region

$S=\{z\in\mathbb{C}:|z-1|\le 2;(z+\bar{z})+i(z-\bar{z})\le 2,\text{Im}(z)\ge 0\}$

(1) $\frac{7\pi}{3}$ (2) $\frac{17\pi}{8}$ (3) $\frac{7\pi}{4}$ (4) $\frac{3\pi}{2}$

Sol. **(4)**



$(x-1)^2+y^2\le 2$ & $x-y\le 1$

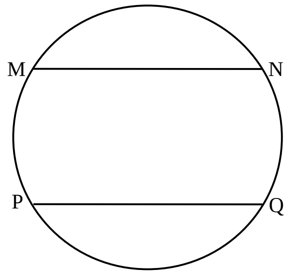
Area $\frac{1}{2}\times r^2\times\theta\ \Rightarrow\frac{3\pi}{2}$

13. Let C be a circle with radius $\sqrt{10}$ units and centre at the origin. Let the line x + y = 2 intersects the circle C at the points P and Q. Let MN be chord of C of length 2 unit and slope –1. Then, a distance (in units) between the chord PQ and the chord MN is

(1) $2-\sqrt{3}$ (2) $\sqrt{2}-1$ (3) $\sqrt{2}+1$ (4) $3-\sqrt{2}$

Sol. **(4)**

C : $x^2 + y^2 = 10$



MN : $y = -x + k$

$x + y = k$

$\ell_{MN} = 2$

$$2\sqrt{10 - \frac{k^2}{2}} = 2$$

$$\frac{k^2}{2} = 9$$

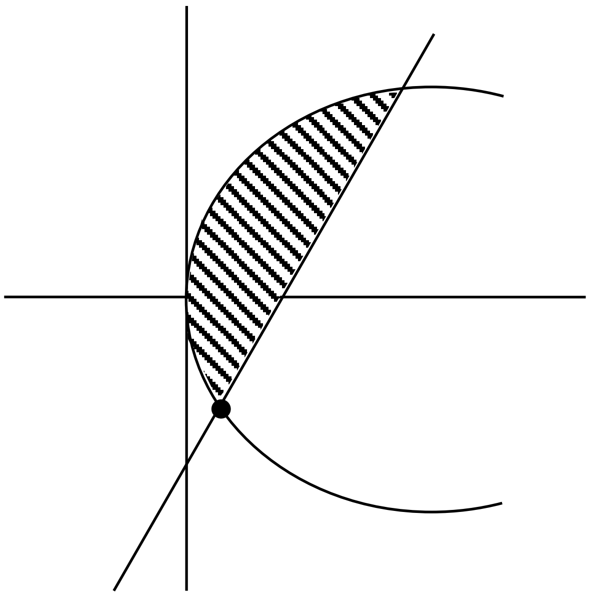
$$k = 3\sqrt{2}$$

$$d = \frac{|k-2|}{\sqrt{2}} = 3 - \sqrt{2}$$

14. The area (in sq. units) of the region described by $\{(x, y) : y^2 \le 2x, \text{ and } y \ge 4x - 1\}$ is

(1) $\frac{8}{9}$ (2) $\frac{11}{32}$ (3) $\frac{9}{32}$ (4) $\frac{11}{12}$

Sol. **(3)**



$$(4x-1)^2 2x \Rightarrow x = \frac{1}{2}, \frac{1}{8}$$

$y = 1, -\frac{1}{2}$

$$\int_{-\frac{1}{2}}^{1}\left(\frac{y+1}{4}-\frac{y^2}{2}\right)dy$$

$$\left(\frac{y^2}{8}+\frac{y}{4}-\frac{y^3}{6}\right)_{-1/2}^{1}$$

$$\left(\frac{1}{8}+\frac{1}{4}-\frac{1}{6}\right)-\left(\frac{1}{32}-\frac{1}{8}+\frac{1}{48}\right)=\frac{9}{32}$$

15. Let P be the point of intersection of the lines $\frac{x-2}{1}=\frac{y-4}{5}=\frac{z-2}{1}$ and $\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-3}{2}$. Then the shortest distance of P from the line 4x = 2y = z is

(1) $\frac{\sqrt{14}}{7}$ (2) $\frac{6\sqrt{14}}{7}$ (3) $\frac{3\sqrt{14}}{7}$ (4) $\frac{5\sqrt{14}}{7}$

Sol. **(3)**

$2+\lambda = 3+2\mu$

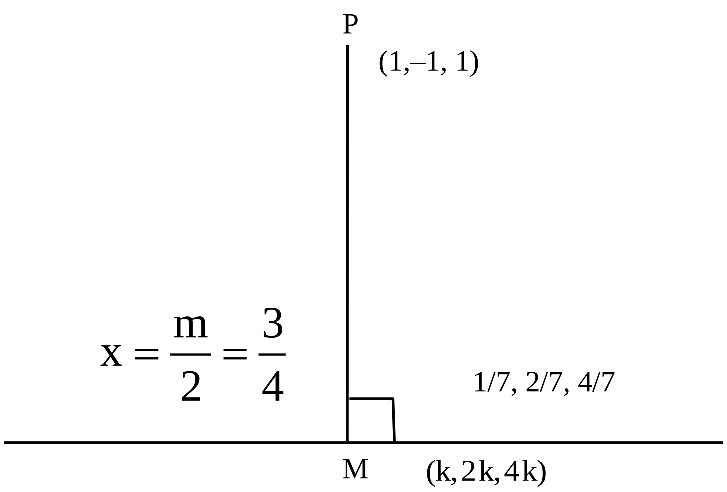
$4 + 5\lambda = 3\mu + 2$

$\lambda - 2\mu = 1$

$5\lambda - 3\mu = -2$

$\lambda = -1$

$(\lambda+2, 5\lambda+4, \lambda+2)$



$(k-1), (2k+1), (4k-1), (1, 2, 4) = 0$

$k - 1 + 4k + 2 + 16k - 4 = 0$

$21k = 3$

$k = \frac{1}{7}$

$$PM = \sqrt{(1/7-1)^2 + (2/7+1)^2 + (4/7-1)^2}$$

$$= \sqrt{\frac{36}{49} + \frac{81}{49} + \frac{9}{49}} = \sqrt{\frac{126}{7}}$$

16. Let three real numbers a, b, c be in arithmetic progression and a + 1, b, c +3 be in geometric progression. If a > 10 and the arithmetic mean of a, b and c is 8 , then the cube of the geometric mean of a, b and c is

(1) 312 (2) 128 (3) 316 (4) 120

Sol. **(4)**

$2b = a + c$, $b^2 = (a+1)(c+3)$; $a + b + c = 24$

$3b = 24$

$B = 8$

Now,

$64 = ac + 3a + c + 3$

$3a + c + ac = 61$

$3a + c\{a + 1\} = 61$

$3a + (16 - a)(1 + a) = 61$

$a = 3, 15$

$c = 13, 1$

3, 8, 13

Or

15, 8, 1

$G.M = (120)^{1/3}$

17. The value of $\dfrac{1\times 2^2 + 2\times 3^2 + ... + 100\times(101)^2}{1^2\times 2 + 2^2\times 3 + ... + 100^2\times 101}$ is

(1) $\dfrac{32}{31}$ (2) $\dfrac{305}{301}$ (3) $\dfrac{31}{30}$ (4) $\dfrac{306}{305}$

Sol. **(2)**

$$S_1 = \sum_{n=1}^{100} n(n+1)^2 \Rightarrow \sum (n^3 + 2n^2 + n)$$

$$\Rightarrow \frac{100^2\times 101^2}{4} + \frac{100\times 101\times 201}{3} + \frac{100\times 101}{2}$$

$$S_2 = \sum_{n=1}^{100} n^2(n+1) \Rightarrow \sum_{n=1}^{100} (n^3 + n^2) = \frac{100^2\times 101^2}{4} + \frac{100\times 101\times 201}{6}$$

$$\frac{S_1}{S_2} = \frac{305}{301}$$

18. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$ and $\vec{c}=x\hat{i}+2\hat{j}+3\hat{k},\ x\in R$. If $\vec{d}$ is the unit vector in the direction of $\vec{a}+\vec{b}$ such that $\vec{a}\cdot\vec{d}=1$, then $(\vec{a}\times\vec{b}).\vec{c}$ is equal to

(1) 3 (2) 11 (3) 6 (4) 9

Sol. **(2)**

$$\vec{d}=\frac{\pm(\vec{b}+\vec{c})}{|\vec{b}+\vec{c}|}=$$

$$\vec{d}=\frac{\pm(x+2)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(x+2)^2+40}}$$

$$\vec{a}.\vec{d}=1$$

$$\pm\{x+2+6-2\}=\sqrt{(x+2)^2+40}$$

$(x + 6)^2 = (x + 2)^2 + 40 \Rightarrow (2x + 8) \times 40 \Rightarrow x = 1$

$$\Rightarrow(\vec{a}\times\vec{b}).\vec{c}=\begin{vmatrix}1 & 1 & 1\\ 2 & 4 & -5\\ 1 & 2 & 3\end{vmatrix}=22-11=11$$

19. For $\lambda > 0$, let θ be the angle between the vectors $\vec{a}=\hat{i}+\lambda\hat{j}-3\hat{k}$ and $\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$. If the vectors $\vec{a}+\vec{b}$ b and $\vec{a}-\vec{b}$ are mutually perpendicular, then the value of $(14 \cos\theta)^2$ is equal to

(1) 20 (2) 50 (3) 25 (4) 40

Sol. **(3)**

$(4, \lambda - 1, -1)\ (-2, \lambda + 1, -5) = 0$

$-8 + \lambda^2 - 1 + 5 = 0$

$\lambda = 2, -2$

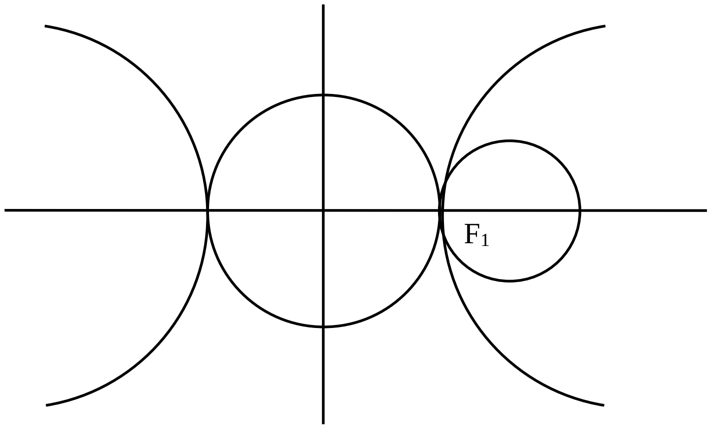
$$\cos\theta=\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}=\frac{-5}{14}$$

$(14 \cos\theta)^2 = 25$

20. Consider a hyperbola H having centre at the origin and foci on the x-axis. Let C_1 be the circle touching the hyperbola H and having the centre at the origin. Let C_2 be the circle touching the hyperbola H at its vertex and having the centre at one of its foci. If areas (in sq units) of C_1 and C_2 are 36π and 4π, respectively, then the length (in units) of latus rectum of H is

(1) $\frac{11}{3}$ (2) $\frac{28}{3}$ (3) $\frac{14}{3}$ (4) $\frac{10}{3}$

Sol. **(2)**



$H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$C_1 : x^2 + y^2 = a^2$

$C_2 : (x - ae)^2 + y^2 = (ae - a)^2$

(Area) $C_1 = \pi a^2 = 36\pi \Rightarrow a = 6$

(Area) C_2 $\pi a^2(e-1)^2 = 4\pi$

$a(e-1)^2 = 2$

$e-1 = 1/3$

$e = 4/3$

$LR = \frac{2b^2}{a}$

$= 2\{a\}(e^2 - 1) = 12\left\{\frac{7}{9}\right\} \Rightarrow \frac{28}{3}$

21. Let $S = \left\{\sin^2 2\theta : \left(\sin^4\theta + \cos^4\theta\right)x^2 + (\sin 2\theta)x + \left(\sin^6\theta + \cos^6\theta\right) = 0 \text{ has real root}\right\}$. If α and β be the smallest and largest elements of the S, respectively, then $3\left((\alpha - 2)^2 + (\beta - 1)^2\right)$ equals ______

Sol. **(4)**

$(-2\sin^2 2\theta)\, x^2 + (\sin 2\theta)x + 1 - 3\sin^2\theta\cos^2\theta = 0$

$D \geq 0$

$\sin^2 2\theta - 4(1 - 2\text{Sin}^2 2\theta)\left(1 - \frac{3}{4}\sin^2 2\theta\right) \geq 0$

$\sin^2 2\theta \in \left(2 - \frac{2}{\sqrt{3}}, 2 + \frac{2}{\sqrt{3}}\right) \Rightarrow$ But $\sin^2\theta \in [0, 1] \Rightarrow \alpha = 2 - \frac{2}{\sqrt{3}}$, $\beta = 1$

22. In a tournament, a team plays 10 matches with probabilities of winning and losing each match as $\frac{1}{3}$ and $\frac{2}{3}$ respectively. Let x be the number of matches that the team wins, and y be the number of matches that team loses. If the probability $P(|x - y| \leq 2)$ is p, then 3^9 p equals. equals ______.

Sol. **8288**

$P(W)=\frac{1}{3}, P(L)=\frac{2}{3}$

Now, $|x - y| \le 2$ and $x + y = 10$

(No. of winning) x	No. of losing y
10	0
9	1
8	2
7	3
6	4
5	5
4	6
3	7
2	8
1	9
0	10

$\Rightarrow \Rightarrow {}^{10}C_6\left(\frac{1}{3}\right)^6\left(\frac{2}{3}\right)^4 + {}^{10}C_5\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^5 + {}^{10}C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^6 = 1$

$3^9 p = 8288$

23. If $\int \text{cosec}^5 x dx = \alpha \cot x \,\text{cosec}\, x\left(\text{cosec}^2 x + \frac{3}{2}\right) + \beta \log_e\left|\tan\frac{x}{2}\right| + C$ where α, $\beta \in R$ and C is the constant of integration, then the value of $8(\alpha + \beta)$ equals _____

Sol. **1**

$I = \int \text{cosec}^3 x\, \text{cosec}^2 x dx \Rightarrow -\text{cosec}^3 x \cot x + \int \cot x(-3\text{cosec}^3 x \cot x)dx$

$I = -\text{cosec}^3 x \cot x - 3\int \text{cosec}^3 x \cot^2 x dx$

$I = -\text{cosec}^3 x \cot x - 3\int \text{cosec}^3 x(\text{cosec}^2 x - 1)$

$4I = -\text{cosec}^3 x \cot x + 3\int \text{cosec} x . \text{cosec}^2 x dx \quad \int \cot^2 x(\text{cosec} x)dx$

$I = -\frac{1}{4}\text{cosec} x \cot x\left[\text{cosec}^2 x + \frac{3}{2}\right] + \frac{3}{8}\ell n\left|\tan\frac{x}{2}\right| + C$

$\therefore \alpha = -\frac{1}{4}, \beta = \frac{3}{8}$

$\Rightarrow 8(\alpha + \beta) = 1$

24. There are 4 men and 5 women in Group A, and 5 men and 4 women in Group B. If 4 persons are selected from each group, then the number of ways of selecting 4 men and 4 women is ______

Sol. **5626**

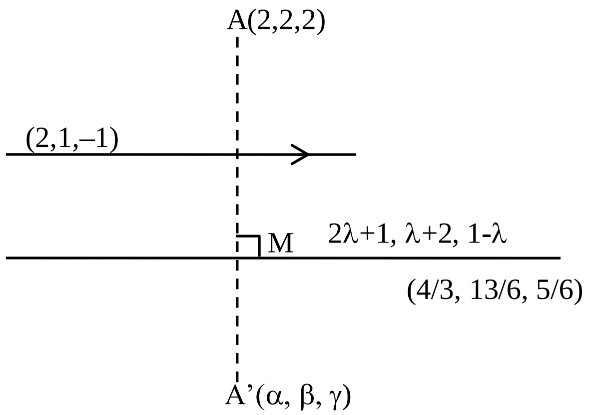
$G-A\begin{pmatrix}5w\\4m\end{pmatrix}\ \ G-B\begin{pmatrix}4w\\5m\end{pmatrix}$

${}^4C_4 \times {}^5C_0 \times {}^5C_0 \times {}^4C_4 + {}^4C_3 \times {}^5C_1 \times {}^5C_1 \times {}^4C_3 + {}^4C_2 \times {}^5C_2 \times {}^4C_2 \times {}^5C_2$

$+ {}^4C_1 \times {}^5C_3 \times {}^5C_3 \times {}^4C_1 + {}^4C_0 \times {}^5C_4 \times {}^5C_4 \times {}^4C_0 = 5626$

25. Consider a line L passing through the points P(1,2,1) and Q(2,1,–1). If the mirror image of the point A(2,2,2) in the line L is (α,β,γ), then $\alpha + \beta + 6\gamma$ is equal to equal to ______.

Sol. **(6)**



$A' = \left(\frac{8}{3}-2, \frac{13}{3}-2, \frac{5}{3}-2\right)$

$= \left(\frac{2}{3}, \frac{7}{3} - \frac{1}{3}\right)$

$\vec{r} = (1,2,1) + \lambda(2,1,-1)$

$\vec{AM}.(2,1,-1) = 0$

$(2\lambda-1, \lambda, -\lambda-1)\ \ (2, 1, -1) = 0$

$4\lambda - 2 + \lambda + \lambda + 1 = 0$

$6\lambda = 1 \Rightarrow \lambda = \frac{1}{6}$

$\Rightarrow \alpha + \beta + 6\gamma = 6$

26. Let y = y(x) be the solution of the differential equation $(x + y + 2)^2\,dx = dy$, $y(0) = -2$. Let the maximum and minimum values of the function y = y (x) in $\left[0, \frac{\pi}{3}\right]$ be α and β, respectively. If $(3\alpha+\pi)^2 + \beta^2 = \gamma + \delta\sqrt{3}$ $\gamma, \delta \in Z$, then $\gamma + \delta$ equals _______.

Sol. **31**

$$\frac{dy}{dx}=(x+y+z)^2$$

Put x + y + z = t

$$1+\frac{dy}{dx}=\frac{dt}{dx}$$

$$\frac{dt}{dx}-1=t^2$$

$$\tan^{-1}t=x+c$$

$$\tan^{-1}(x+y+z)=x+c$$

$$\Downarrow (x=0, y=-2)$$

C = 0

$$y=\tan x-x-2$$

$$\frac{dy}{dx}=\tan^2 x\ge 0$$

$$\forall x\in\left[0,\frac{\pi}{3}\right]$$

$\Rightarrow y_{min} = y(0) = -2 = \beta$

And $y_{max}=y\left(\frac{\pi}{3}\right)=\sqrt{3}-2-\frac{\pi}{3}=\alpha$

Now, $(3\alpha+\pi)^2+\beta^2$

$$(3\sqrt{3}-6-\pi+\pi)^2+4$$

$27 + 36 - 36\sqrt{3}+4$

$$67-36\sqrt{3}$$

So, $\gamma = 67$

$\delta = -36$

$\Rightarrow \gamma + \delta = 31$

27. Let A be 2 × 2 symmetric matrix such that $A\begin{bmatrix}1\\1\end{bmatrix}=\begin{bmatrix}3\\7\end{bmatrix}$ and the determinatnt of A be 1. If $A^{-1} = \alpha A + \beta I$, where I is an identity matrix of order 2 × 2 then $\alpha + \beta$ equal ______.

Sol. **(5)**

Let $A=\begin{bmatrix}a & b\\ b & d\end{bmatrix}$

Now $A=\begin{bmatrix}a & b\\ b & d\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}=\begin{bmatrix}3\\7\end{bmatrix}$

a + b = 3, b + d = 7

$ad - b^2 = 1$

$(3 - b)(7 - b) - b^2 = 1$

$21 - 10b = 1$

$\Rightarrow b = 2, a = 1, d = 5$

Now, $A^{-1} = \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}$

$= \begin{bmatrix} \alpha & 2\alpha \\ 2\alpha & 5\alpha \end{bmatrix} + \begin{bmatrix} \beta & 0 \\ 0 & \beta \end{bmatrix}$

$= \alpha + \beta = 5$

28. Consider a triangle ABC having the vertices A(1, 2), B(α, β) and C(γ, δ) and angle $\angle ABC = \frac{\pi}{6}$ and $\angle BAC = \frac{2\pi}{3}$. If the points B and C lie on the line y = x + 4 then $\alpha^2 + \delta^2$ is equal to ____.

Sol. **14**

$y = x + y$ ...(1)

$\frac{m-1}{1+m} = \pm \tan\frac{\pi}{6} = \pm\frac{1}{\sqrt{3}}$

(+) $\sqrt{3}m - \sqrt{3} = 1 + m$

$m = \frac{\sqrt{3}+1}{\sqrt{x}-1} = 2 + \sqrt{3}$

(–) $\sqrt{3}m - \sqrt{3} = -m - 1$

$m = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2 - \sqrt{3}$

equation of AB :

$y - 2 = (2 + \sqrt{3})(x - 1)$ ...(2)

equation of AC:

$y - 2 = (2 - \sqrt{3})(x - 1)$ ...(3)

from (1) & (2)

$x + 2 = 2x - 2 + \sqrt{3}(x - 1)$

$x = 4 - \sqrt{3}x + \sqrt{3}$

$x = \frac{4+\sqrt{3}}{\sqrt{3}+1} = \frac{(4+\sqrt{3})(\sqrt{3}-1)}{2}$

from (1) & (3)

$x + 2 = 2x - 2 - \sqrt{3}\ (x - 1)$

$x = \frac{\sqrt{3}-4}{\sqrt{3}-1} = \frac{(\sqrt{3}-4)(\sqrt{3}+1)}{2}$

$$\alpha^2+\gamma^2=\frac{(4\sqrt{3}-4+3-\sqrt{3})^2+(3+\sqrt{3}-4\sqrt{3}-4)^2}{4}$$

$$=\frac{(3\sqrt{3}-1)^2+(3\sqrt{3}+1)^2}{4}=\frac{56}{4}=14$$

29. Consider the function $f: R \to R$ defined by $f(x)=\frac{2x}{\sqrt{1+9x^2}}$.

If the composition of $f, \underbrace{(f\circ f\circ f\circ\cdots\circ f)}_{10\text{ times}}(x)=\frac{2^{10}x}{\sqrt{1+9\alpha x^2}}$, then the value of $\sqrt{3\alpha+1}$ is equal to ______.

Sol. **1024**

$$f(f(x))=\frac{2f(x)}{\sqrt{1+9f^2(x)}}==\frac{4x}{\sqrt{1+9x^2+9.2^2x^2}}$$

$$f(f(f)(x))=\frac{\frac{2^3x}{\sqrt{1+9x^2}}}{\sqrt{1+9(1+2^2)\frac{2^2x^2}{1+9x^2}}}=\frac{2^3x}{\sqrt{1+9x^2(1+2^2+2^4)}}$$

$\therefore$ By observation

$$\alpha=1+2^2+2^4+.....2^{18}=1\left(\frac{(2^2)^{10}-1}{2^2-1}\right)=\frac{2^{20}-1}{3}$$

$$3\alpha+1=2^{20}\to\sqrt{3\alpha+1}=2^{10}=1024$$

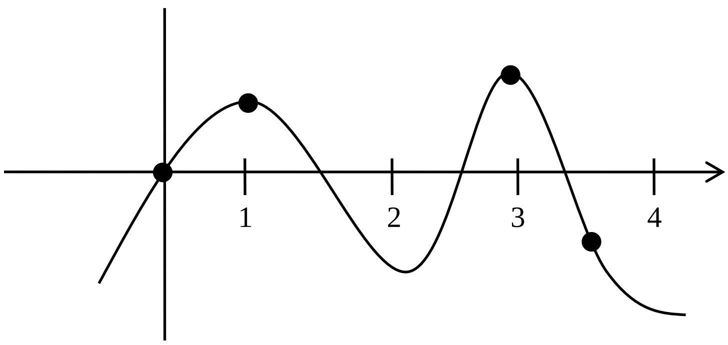
30. Let $f: R \to R$ be a thrice differentiable function such that f(0) = 0, f(1) = 1, f(2) = –1, f(3) = 2 and f(4) = –2. Then, the minimum number of zeros of (3f'f" + ff'"}(x) is _________

Sol. **5**

(3f' f" + ff"') (x) = (ff" + (f')^2(x))'

(ff" & (f')^2) (x) = (((ff')) (x))'

$\therefore$ (3f'f" & ff"') (x) = (f(x).f(x))"



Minimum roots of f(x) $\to$ 4

$\therefore$ minimum roots of f'(x) $\to$ 3

$\therefore$ minimum roots of (f(n). f'(x)) $\to$ 7

$\therefore$ minimum roots of (f(n)f'(x))'' $\to$ 5

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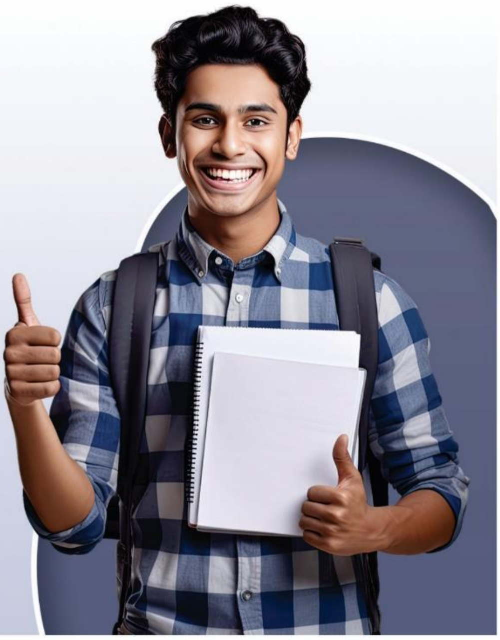

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