JEE MAIN 2024 SESSION-2

Paper with Solution

Maths | 08th April 2024 _ Shift-1

SECTION – A

1. Let $f(x)$ be a positive function such that the area bounded by $y = f(x)$, $y = 0$ from $x = 0$ to $x = a > 0$ is $e^{-a} + 4a^2 + a - 1$. Then the differential equation, whose general solution is $y = c_1 f(x) + c_2$, where c_1 and c_2, where c_1 and c_2 are arbitrary constants, is

(1) $(8e^x - 1)\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$ (2) $(8e^x - 1)\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

(3) $(8e^x + 1)\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ (4) $(8e^x + 1)\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

Sol. **(3)**

$$\int_0^a f(a)\,dx = e^{-a} + 4a^2 + a - 1$$

$f(a) = -e^{-a} + 8a + 1$

$f(x) = -e^{-x} + 8x + 1$

$y = c_1 f(x) + c_2$

$y = C_1(-e^{-x} + 8x + 1) + C_2$

$\frac{dy}{dx} = C_1(e^{-x} + 8) + O$

$y'' = C_1(-e^{-x})$

$C_1 = -y''e^{+x}$

$y' = -y''\, e^x(e^{-x} + 8)$

$y' = -y'' - 8y''\, e^x$

$y''\,(1 + 8e^x) + y' = 0$

2. Let z be a complex number such that $|z + 2| = 1$ and $1m\left(\frac{z+1}{z+2}\right) = \frac{1}{5}$. Then the value of $\left|\text{Re}\left(\overline{z+2}\right)\right|$ is

(1) $\frac{2\sqrt{6}}{5}$ (2) $\frac{24}{5}$ (3) $\frac{\sqrt{6}}{5}$ (4) $\frac{1+\sqrt{6}}{5}$

Sol. **(1)**

Let $z = x + iy$

$$= \text{im}\left(\frac{(x+1)+iy}{(x+2)+iy}\right) = \frac{1}{5}$$

$$= \text{im}\left(\frac{((x+1)+iy)((x+2)-iy)}{(x+2)^2+y^2}\right) = \frac{1}{5}$$

$$= \text{im}\left(\frac{(x+1)(x+2)-iy(x+1)+iy(x+2)+y^2}{(x+2)^2+y^2}\right) = \frac{1}{5}$$

$$= \frac{y(x+2)-y(x+1)}{(x+2)^2+y^2} = \frac{1}{5}$$

$$= \frac{y}{x^2+2x+4+y^2} = \frac{1}{5}$$

$$= x^2 + y^2 + 2x - 5y + 4 = 0$$

$= (x+2)^2 + y^2 - 5y = 0$ ...(1)

$|z + 2| = 1$

$|x + 2 + iy| = 1$

$(x + 2)^2 + y^2 = 1$

$(x+ 2)^2 = 1 - y^2$

Put value (1)

$1 - y^2 + y^2 - 5y = 0$

$y = \frac{1}{5}$

$(x + 2)^2 = \frac{24}{25}$

$x + 2 = \pm\frac{\sqrt{24}}{5}$

$x = -2 \pm \frac{\sqrt{24}}{5}$

$z + 2 = \left(-2 \pm \frac{\sqrt{24}}{5}\right) + \frac{1}{5}i + 2$

$z + 2 = \pm\frac{\sqrt{24}}{5} + \frac{1}{5}i$

$\text{Re}(z+2) = \pm\frac{\sqrt{24}}{5}$

$|\text{Re}(z+2)| = \frac{2\sqrt{6}}{5}$ Ans.

3. The number of critical points of the function $f(x) = (x - 2)^{2/3} (2x + 1)$ is

(1) 2 (2) 1 (3) 0 (4) 3

Sol. **(1)**

$f(x) = (x-2)^{2/3}(2x+1)$

$f'(x) = \frac{2(2x+1)}{3(x-2)^{\frac{1}{3}}} + 2(x-2)^{\frac{2}{3}}$

$f'(x) = \frac{2(2x+1) + 6(x-2)}{(x-2)^{\frac{1}{3}}} = 0$

$f'(x) = \frac{4x + 2 + 6x - 12}{(x-2)^{\frac{1}{3}}} = 0$

$10x - 10 = 0$

$\boxed{x = 1}$

$f'(x) = 0$at $\boxed{x = 1}$ and Non diff. at $\boxed{x = 2}$

2 critical point.

4. Let $f(x) = 4\cos^3 x + 3\sqrt{3}\cos^2 x - 10$. The number of points of local maxima of f in interval $(0, 2\pi)$ is

(1) 3 (2) 2 (3) 1 (4) 4

Sol. **(2)**

$f(x) = 4\cos^3x + 3\sqrt{3}\cos^2x - 10$

$f'(x) = 12\cos^2x(-\sin x) + 6\sqrt{3}\cos x(-\sin x)$

$f'(x) = -\sin 2x(6\cos + 3\sqrt{3})$

$\boxed{f'(x) = 0}$

$\sin 2x = 0$ or $\cos x = \frac{-\sqrt{3}}{2}$

$x = (0, 2\pi)$

$2x = (0, 4\pi)$ $x = \frac{5\pi}{6}, \frac{7\pi}{6}$

$2x = \pi, 2\pi, 3\pi$

$x = \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

$f''(x) = -2\cos 2x(6\cos x + 3\sqrt{3}) + 6\sin x \sin 2x$

$f''\left(\frac{\pi}{2}\right) = -2(-1)(3\sqrt{3}) + 0 \Rightarrow +ve$

$f''(\pi) \Rightarrow +ve$

$f''\left(\frac{3\pi}{2}\right) = -2(-1)(3\sqrt{3}) + 0 \Rightarrow +ve$

$f''\left(\frac{5\pi}{6}\right) = -2\left(\frac{1}{2}\right)\left(6\left(-\frac{\sqrt{3}}{2}\right) + 3\sqrt{3}\right) + 6\left(\frac{1}{2}\right)\left(\frac{-\sqrt{3}}{2}\right) = \frac{-3\sqrt{3}}{2} \Rightarrow -ve$ local max.

$f''\left(\frac{7\pi}{6}\right) = -2\left(\frac{1}{2}\right)\left(6 + \left(-\frac{\sqrt{3}}{2}\right) + 3\sqrt{3}\right) + 6\left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) \Rightarrow -ve$ local max.

5. Let $y = y(x)$ be the solution of the differential equation

$(1 + y^2)e^{\tan x}\,dx + \cos^2x\,(1 + e^{2\tan x})\,dy = 0$, $y(0) = 1$. Then $y\left(\frac{\pi}{4}\right)$ is equal to

(1) $\frac{1}{e^2}$ (2) $\frac{1}{e}$ (3) $\frac{2}{e}$ (4) $\frac{2}{e^2}$

Sol. **(2)**

$(1 + y^2)e^{\tan x}\,dx + \cos^2x\,(1 + e^{2\tan x})dy = 0$

$$\frac{dy}{1+y^2} = -\frac{e^{\tan x}dx}{\cos^2 x(1+e^{2\tan x})}$$

Let $e^{\tan x} = t$

then $e^{\tan x} \cdot \sec^2x\,dx = dt$

$\Rightarrow \tan^{-1} y = -\frac{dt}{(1+t^2)}$

$\Rightarrow \tan^{-1} y = -\tan^{-1}(t) + C$

$\Rightarrow \tan^{-1} y + \tan^{-1}(e^{\tan x}) = C$

$\boxed{y(0) = 1}$

$\Rightarrow \tan^{-1}(1) + \tan^{-1}(1) = C$

$C = \frac{\pi}{2}$

$\rightarrow \tan^{-1}y + \tan^{-1}(e^{\tan x}) = \frac{\pi}{2}$

$\rightarrow y\left(\frac{\pi}{4}\right) = ?$

$\Rightarrow \tan^{-1}(y) + \tan^{-1}(e) = \frac{\pi}{2}$

$\Rightarrow \tan^{-1}\left(\frac{y+e}{1-ye}\right) = \frac{\pi}{2}$

$\frac{y+e}{1-ye} = \tan\left(\frac{\pi}{2}\right)$

$\boxed{y = \frac{1}{e}}$

6. If the set $R = \{(a,\ b) : a + 5b = 42, a, b \in \mathbb{N}\}$ has m elements and $\sum_{n=1}^{m}\left(1 - i^{n!}\right) = x + iy$, where $i = \sqrt{-1}$, then the value of m + x + y is

(1) 4 (2) 8 (3) 12 (4) 5

Sol. **(3)**

R = {(37, 1), (32, 2), (27, 3), ………… $\boxed{m = 8}$

$\sum_{n=1}^{8}(1 - i^{n!}) = x + iy$

$\Rightarrow 8 - (i + i^2 + i^6 + i^{24} + i^{120} + \ldots + \ldots +)$

$\Rightarrow 8 - (i + i^2 + i^6 + 5)$

$\Rightarrow 5 - i = x + iy$

$x = 5,\ y = -1$ $\boxed{m = 8}$

m + x + y = 12 Ans.

7. The sum of all the solutions of the equation $(8)^{2x} - 16 \cdot (8)^x + 48 = 0$ is:

(1) $1 + \log_6(8)$ (2) $\log_8(6)$ (3) $1 + \log_8(6)$ (4) $\log_8(4)$

Sol. **(3)**

$(8)^{2x} - 16.8^x + 48 = 0$

$8^x = t$

$t^2 - 16t + 48 = 0$

$t^2 - 12t - 4t + 48 = 0$

$t(t - 12) - 4(t - 12) = 0$

$(t - 4)(t - 12) = 0$

$t = 4, t = 12$

$8^x = 4$

$x_1 = \log_8 4$

& $8^x = 12$

$x_2 = \log_8 12$

$x_1 + x_2 = \log_8 4 + \log_8 12$

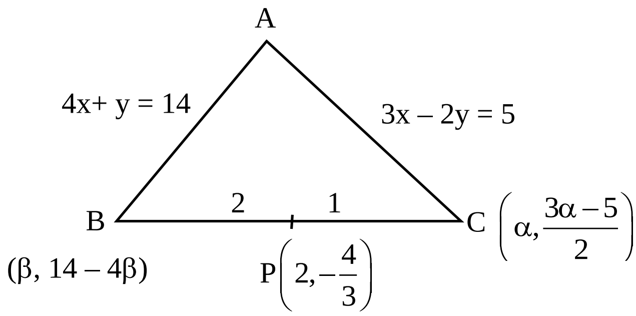
$= \log_8 48$

$= 1 + \log_8 6$

8. The equations of two sides AB and AC of a triangle ABC are $4x + y = 14$ and $3x - 2y = 5$, respectively. The point $\left(2, -\frac{4}{3}\right)$ divides the third side BC internally in the ratio 2:1. The equation of the side BC is

(1) $x + 6y + 6 = 0$ (2) $x - 3y - 6 = 0$ (3) $x - 6y - 10 = 0$ (4) $x + 3y + 2 = 0$

Sol. **(4)**



$\frac{2\alpha + \beta}{3} = 2$

$2\alpha + \beta = 6$ ……(1)

$\frac{3\alpha - 5 + 14 - 4\beta}{3} = \frac{-4}{3}$

$3\alpha - 4\beta = -13$ … (2)

$4(2\alpha + \beta = 6)$ Point C(1, –1)

$3\alpha - 4\beta = -13$ Point B(4, –2)

$11\alpha = 11$ eq. of BC

$\alpha = 1$ $y + 1 = \frac{-2+1}{4-1}(x - 1)$

$2 + \beta = 6$ $3y + 3 = -(x - 1)$

$\boxed{\beta = 4}$ $\boxed{3y + x + 2 = 0}$

9. Let $I(x)=\int \frac{6}{\sin^2 x(1-\cot x)^2}dx$. If I(0) = 3, then $I\left(\frac{\pi}{12}\right)$ is equal to

(1) $2\sqrt{3}$ (2) $\sqrt{3}$ (3) $6\sqrt{3}$ (4) $3\sqrt{3}$

Sol. **(4)**

$I(x) = \int \frac{6\,dx}{\sin^2(1-\cot x)^2}$ $I(0) = 3$

$I\left(\frac{\pi}{12}\right) = ?$

$I(x) = \int \frac{6\,dx}{\sin^2 x \frac{(\sin x-\cos x)^2}{\sin^2 x}} = \int \frac{6\,dx}{\sin^2 x+\cos^2 x-2\sin\cos x}$

$= \int \frac{6\,dx}{1-2\sin x\cos x} = \int \frac{6\sec^2 x\,dx}{1+\tan^2 x-2\tan x}$

$= \int \frac{6\sec^2 x\,dx}{(\tan x-1)^2}$

$= \boxed{\tan x = t}$

$= \sec^2 x\,dx = dt$

$\int \frac{6dt}{(t-1)^2} = 6\int (t-1)^{-2}dt$

$I(x) = \frac{-6}{(t-1)} + C$

$I(x) = -\frac{6}{\tan x-1} + C$

$\because$ I(0) = 3

3 = 6 + C

$\boxed{C = -3}$

$I(x) = -\frac{6}{\tan x-1} - 3$

$I\left(\frac{\pi}{12}\right) = -\frac{6}{2-\sqrt{3}-1} - 3$

$= -\frac{6}{1-\sqrt{3}} - 3 = -\left(\frac{6+3-3\sqrt{3}}{1-\sqrt{3}}\right)$

$= -\left(\frac{9-3\sqrt{3}}{1-\sqrt{3}}\times\frac{1+\sqrt{3}}{1+\sqrt{3}}\right)$

$= -\left(\frac{9+9\sqrt{3}-3\sqrt{3}-9}{1-3}\right) = \frac{6\sqrt{3}}{2} = 3\sqrt{3}$

10. The value of $k \in \mathbb{N}$ for which the integral $I_n = \int_0^1 \left(1-x^k\right)^n dx$, $n \in \mathbb{N}$, satisfies 147 I_{20} = 148 I_{21} is

(1) 7 (2) 14 (3) 10 (4) 8

Sol. **(1)**

$I_n = \int_0^1 (1-x^k)^n \, dx$

$I_{21} = \int_0^1 (1-x^k)^{21} \, dx$

$\Rightarrow \int_0^1 (1-x^k)(1-x^k)^{20} \, dx$

$\Rightarrow \int_0^1 (1-x^k)^{20} - \int_0^1 x^k (1-x^k)^{20} \, dx$

$I_{21} = I_{20} - \int_0^1 x^k (1-x^k)^{20} dx$

$\int_0^1 x \; x^{(k-1)} (1-x^k)^{20} dx$

$1 - x^k = t$

$-K\,(x^{K-1}\,dx) = dt$

Integration by part

$x\left[-\frac{(1-x^K)^{21}}{21K}\right]_0^1 - \frac{1}{21k}\int_0^1 (1-x^k)^{21} dx$

$I_{21} = I20 + 0 - \frac{I_{21}}{21K}$

$= \left(\frac{21K+1}{21K}\right) I_{21} = I_{20}$

$= (21K + 1)I_{21} = 21KI_{20}$

$\boxed{K = 7}$

11. Let the circles $C_1 : (x-\alpha)^2 + (y-\beta)^2 = r_1^2$ and $C_2 : (x-8)^2 + \left(y - \frac{15}{2}\right)^2 = r_2^2$ touch each other externally at the point (6, 6). If the point (6, 6) divides the line segment joining the centres of the circles C_1 and C_2 internally in the ratio 2:1, then $(\alpha+\beta) + 4\left(r_1^2 + r_2^2\right)$ equals

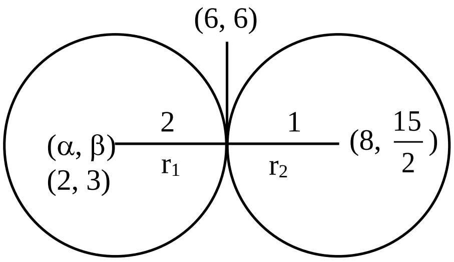
(1) 145 (2) 110 (3) 130 (4) 125

Sol. **(3)**

$\frac{16+\alpha}{3} = 6$

$\alpha = 2$

$\frac{15+\beta}{3} = 6$



$\frac{r_1}{r_2} = \frac{2}{1}$

$\beta = 3$

$r_1 = \sqrt{4^2 + 3^2}$

$r_1 = 5$

$r_2 = \frac{5}{2}$

$(\alpha + \beta) + 4\ (r_1^2 + r_2^2)$

$= 5 + 4\left(25 + \frac{25}{4}\right) = 5 + 125 = 130$

12. If $\sin = -\frac{3}{5}$. Where $\pi < x < \frac{3\pi}{2}$, then $80(\tan^2 x - \cos x)$ is equal to

(1) 108 (2) 109 (3) 18 (4) 19

Sol. **(2)**

$\text{Sin } x = -\frac{3}{5}$ $\left(\pi, \frac{3\pi}{2}\right)$ IIIrd quadrant

$p = 3 \quad H = 5 \quad B = 4$

$\cos x = -\frac{4}{5} \qquad \tan = \frac{3}{4}$

$80\ (\tan^2 x - \cos x) = 80\left(\frac{9}{16} + \frac{4}{5}\right) = 45 + 64 = 109$

13. The set of all α, for which the vectors $\vec{a} = \alpha t\hat{i} + 6\hat{j} - 3\hat{k}$ and $\vec{b} = t\hat{i} - 2\hat{j} - 2\alpha t\,\hat{k}$ are inclined at an obtuse angle for all $t \in \mathbb{R}$, is

(1) [0, 1) (2) $\left(-\frac{4}{3}, 0\right]$ (3) (–2, 0) (4) $\left(-\frac{4}{3}, 1\right)$

Sol. **(2)**

$\vec{a} \cdot \vec{b} < 0$

$\alpha t^2 - 12 + 6\alpha t < 0 \qquad t \in R$

$\alpha t^2 + 6\alpha t - 12 < 0$

$\alpha < 0$ and $D < 0$

$36\alpha^2 + 48\alpha < 0$

$\alpha\,(3\alpha + 4) < 0$

$\alpha \in \left(-\frac{4}{3}, 0\right)$

also for $\alpha = 0$
a.b = –12
so $\alpha = 0$ also include
so final answer is $\left(-\frac{4}{3}, 0\right]$ Ans.

14. For the function $f(x) = (\cos x) - x + 1,\ x \in \mathbb{R}$, between the following two statements

(S1) $f(x) = 0$ for only one value of x in $[0, \pi]$.

(S2) $f(x)$ is decreasing in $\left[0, \frac{\pi}{2}\right]$ and increasing in $\left[\frac{\pi}{2}, \pi\right]$.

(1) Only (S1) is correct.
(2) Only (S2) is correct.
(3) Both (S1) and (S2) are incorrect.
(4) Both (S1) and (S2) are correct.

Sol. **(1)**

$f'(x) = -\sin x - 1$

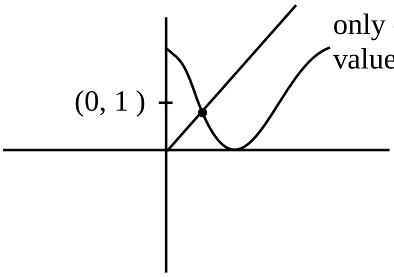
$= f'(x) < 0 \qquad x \in [0, \pi]$

f is decreasing $\forall\ x \in [0, \pi]$

$f(x) = 0$

$\cos x + 1 = x$

(only one solution)

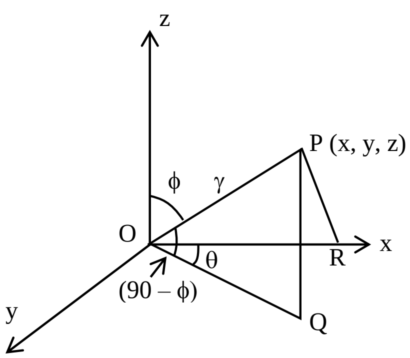


15. Let P(x, y, z) be a point in the first octant, whose projection in the xy-plane is the point Q. Let OP = γ; the angle between OQ and the positive x-axis be θ; and the angle between OP and the positive z-axis be ϕ, where O is the origin. Then the distance of P from the x-axis is

(1) $\gamma\sqrt{1-\sin^2\phi\cos^2\theta}$ (2) $\gamma\sqrt{1+\cos^2\phi\sin^2\theta}$ (3) $\gamma\sqrt{1+\cos^2\theta\sin^2\phi}$ (4) $\gamma\sqrt{1-\sin^2\theta\cos^2\phi}$

Sol. **(1)**

OQ = OP cos (90 – ϕ)



OR = OQ cos θ

OR = OP sin ϕ cos θ

OR = γ sin ϕ cos θ

PR = $\sqrt{\gamma^2 - OR^2}$

PR $= \gamma\sqrt{1-\sin^2\phi\cos^2\theta}$

16. Let $H: \frac{-x^2}{a^2} + \frac{y^2}{b^2} = 1$ be the hyperbola, whose eccentricity is $\sqrt{3}$ and the length of the latus rectum is $4\sqrt{3}$. Suppose the point (α, 6), α > 0 lies on H. If β is the product of the focal distances of the point (α, 6) then $\alpha^2 + \beta$ is equal to

(1) 171 (2) 169 (3) 172 (4) 170

Sol. **(1)**

$$-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$e = \sqrt{1+\frac{a^2}{b^2}} = \sqrt{3}$$

$a^2 = 2b^2$

$$L.R. = \frac{2a^2}{b} = 4\sqrt{3}$$

$a^2 = 2\sqrt{3}\,b$

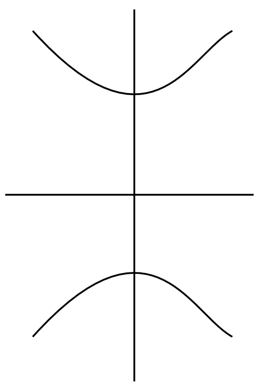
$\Rightarrow 2b^2 = 2\sqrt{3}\,b$

$\sqrt{3}\,b = b^2$

$b = \sqrt{3}$

$\boxed{b^2 = 3}$

a = 6



$$-\frac{x^2}{6} + \frac{y^2}{3} = 1$$

P(α, 6)

$-\frac{\alpha^2}{6}+\frac{36}{3}=1$

$\boxed{\alpha^2 = 66}$

Product of focal distance

$\beta = (ey_1 - b)\,(ey_1 + b)$

$\beta = e^2y_1^2 - b^2$

$= 3 \times 36 - 3$

$\beta = 105$

$\alpha^2 + \beta = 66 + 105 = 171$

17. Let the sum of two positive integers be 24. If the probability, that their product is not less than $\frac{3}{4}$ times their greatest possible product, is $\frac{m}{n}$, where gcd(m, n) = 1, then n – m equals

(1) 10 (2) 8 (3) 9 (4) 11

Sol. **(1)**

$a + b = 24$

$a \cdot b > \frac{3}{4}$ (g.p.p.) Probability = $\frac{m}{n}$

$\frac{a+b}{2} \geq (ab)^{\frac{1}{2}}$

$12 \geq (ab)^{\frac{1}{2}}$

$ab \leq 144$

maximum possible product

$a \cdot b > \frac{3}{4} \times 144$

$\boxed{a \cdot b > 108}$

favorable pair of (a, b)

$$\left.\begin{matrix}(18,6)\\(17,7)\\(16,8)\\(15,9)\\(14,10)\\(13,11)\end{matrix}\right] \times 2$$

(12,12)

favorable = 13

total case of x + y = 24 is 23

probability = $\frac{13}{23} = \frac{m}{n}$

n – m = **10 Ans.**

18. If the shortest distance between the lines

$L_1; \vec{r} = (2+\lambda)\hat{i} + (1-3\lambda)\hat{j} + (3+4\lambda)\hat{k}, \quad \lambda \in \mathbb{R}$

$L_2; \vec{r} = 2(1+\mu)\hat{i} + 3(1+\mu)\hat{j} + (5+\mu)\hat{k}, \quad \mu \in \mathbb{R}$

is $\frac{m}{\sqrt{n}}$, where gcd(m, n) = 1, then the value of m + n equals

(1) 384 (2) 387 (3) 390 (4) 377

Sol. **(2)**

$L_1 = \vec{r} : (2, 1, 3) + \lambda(1, -3, 4)$

$L_2 = \vec{r} : (2, 3, 5) + \mu(2, 3, 1)$

$d = \left|\frac{(a_1 - a_2)\cdot(p \times q)}{|p \times q|}\right|$

$d = \left|\frac{(0\hat{i} + 2\hat{j} + 2\hat{k})\cdot(-15\hat{i} + 7\hat{j} + 9\hat{k})}{\sqrt{355}}\right|$

$d = \left|\frac{14+18}{\sqrt{355}}\right| \Rightarrow \frac{32}{\sqrt{355}} = \frac{m}{\sqrt{n}}$

m + n = 32 + 355 = 387

19. Let [t] be the greatest integer less than or equal to t. Let A be the set of all prime factors of 2310 and $f : A \rightarrow Z$ be the function $f(x) = \left[\log_2\left(x^2 + \left[\frac{x^3}{5}\right]\right)\right]$. The number of one-to-one functions from *A* to the range of *f* is

(1) 24 (2) 120 (3) 20 (4) 25

Sol. **(2)**

2310 = 2 × 3 × 5 × 7 × 11

A = {2, 3, 5, 7, 11}

$F(x) = \left[\log 2\left(x^2 + \left[\frac{x^3}{5}\right]\right)\right]$

	Range
x	f(x)
x = 2	2
x = 3	3
x = 5	5
x = 7	7
x = 11	11

Range ⇒ {2, 3, 5, 7, 11}

No. of one-one function = 5! = **120 Ans.**

20. Let $A = \begin{bmatrix} 2 & a & 0 \\ 1 & 3 & 1 \\ 0 & 5 & b \end{bmatrix}$. If $A^3 = 4A^2 - A - 21I$, where I is the identity matrix of order 3 × 3, then 2a + 3b is equal to.

(1) –9 (2) –10 (3) –12 (4) –13

Sol. **(4)**

$$\begin{vmatrix} 2-\lambda & a & 0 \\ 1 & 3-\lambda & 1 \\ 0 & 5 & b-\lambda \end{vmatrix} = 0$$

$\Rightarrow (2-\lambda)(3b - 3\lambda - \lambda b + \lambda^2 - 5) - ab + a\lambda = 0$

$\Rightarrow 6b - 6\lambda - 2\lambda b + 2\lambda^2 - 10 - 3\lambda b + 3\lambda^2 + \lambda^2 b - \lambda^3 + 5\lambda - ab + a\lambda = 0$

$\Rightarrow -\lambda^3 + \lambda^2(2 + 2 + b) + \lambda(-6 - 2b - 3b + 5 + a) + 6b - 10 - ab = 0$

$\Rightarrow -\lambda^3 + \lambda^2(b + 5) + \lambda(a - 5b - 1) + 6b - 10 - ab = 0$

$\Rightarrow -A^3 + 4A^2 - A - 21I = 0$

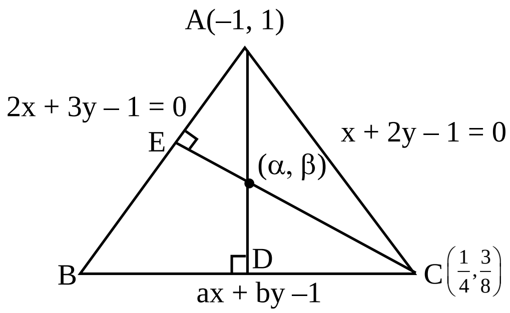
$b + 5 = 4$ $a - 5b - 1 = -1$ $2a + 3b$

$\boxed{b = -1}$ $a = -5$ $\Rightarrow 2(-5) + 3(-1)$

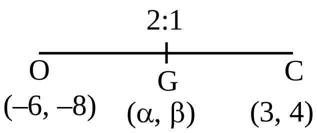
$\Rightarrow$ **– 13 Ans.**

21. If the orthocentre of the triangle formed by the lines 2x + 3y – 1 = 0, x + 2y – 1 = 0 and ax + by – 1 = 0, is the centroid of another triangle, whose circumcentre and orthocentre respectively are (3,4) and (–6, –8), then the value of |a-b| is _______.

Sol. **16**



For another Δ



$\alpha = \frac{2 \times 3 - 6}{3} = 0$ $\beta = \frac{2 \times 4 - 8}{3} = 0$

$(\alpha, \beta) \equiv (0, 0)$

$$x + 2y - 1 = 0$$
$$2x + 3y - 1 = 0$$
$$-\quad -\quad +$$
$$y - 1 = 0$$

$y = 1$

$x = -1$

$A(-1, 1)$

Slope of AD = $\frac{0-1}{0+1} = -1$

and

$M_{AD} \times M_{BC} = (-1)$

$(-1)M_{BC} = -1$

$\boxed{M_{BC} = 1}$

Now for point C

find equation of CE

$M_{CE} = \frac{3}{2}$

$\boxed{y = \frac{3}{2}x}$

and equation of AC

$x + 2y - 1 = 0$

$-3x + 2y = 0$

$4x - 1 = 0$

$\boxed{x = \frac{1}{4}}$

$\boxed{y = \frac{3}{2} \times \frac{1}{4} = \frac{3}{8}}$

Eq. of BC

$C\left(\frac{1}{4}, \frac{3}{8}\right)$

$m_{BC} = 1$

$y - \frac{3}{8} = 1\left(x - \frac{1}{4}\right)$

$\frac{8y-3}{8} = \frac{4x-1}{4}$

$8y - 8x + 2 - 3 = 0$

$-8x + 8y - 1 = 0$

$ax + by - 1 = 0$

$a = -8$

$b = 8$

Value of $|a - b| \Rightarrow |-8 - 8| = $ **16 Ans.**

22. The value of $\lim_{x\to 0} 2\left(\frac{1-\cos x\sqrt{\cos 2x}\sqrt[3]{\cos 3x}\ldots\ldots\sqrt[10]{\cos 10x}}{x^2}\right)$ is ______.

Sol. **55**

$$\lim_{x\to 0} 2\left(\frac{1-\cos x\sqrt{\cos 2x}\sqrt[3]{\cos 3x}\ldots\ldots\sqrt[10]{\cos 10x}}{x^2}\right)$$

using L'Hospital

$$\lim_{x\to 0} 2\left(\frac{-2\sin x(\ldots\ldots)+\frac{2\sin 2x(\ldots\ldots)}{2\sqrt{\cos 2x}}+\ldots\ldots}{2x}\right)$$

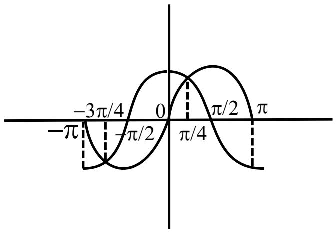
Now use standard limit

$$= (1 + 2 + \ldots\ldots + 10) = \frac{10(11)}{2} = 55$$

23. Let the area of the region enclosed by the curve y = min{sin x, cos x} and the x-axis between x = – π to x = π be A. Then A^2 is equal to _______.

Sol. **16**

y = min {sin x, cos x}   x = –π to x = π



$$A = \left|\int_{-\pi}^{-\frac{3\pi}{4}} \cos x\,dx\right| + \left|\int_{-\frac{3\pi}{4}}^{0} \sin x\,dx\right| + \left|\int_{0}^{\frac{\pi}{4}} \sin x\,dx\right| + \left|\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x\,dx\right| + \left|\int_{\frac{\pi}{2}}^{\pi} \cos x\,dx\right|$$

$$A = |\sin x|_{-\pi}^{-\frac{3\pi}{4}} + |-\cos x|_{-\frac{3\pi}{4}}^{0} + |-\cos x|_{0}^{\frac{\pi}{4}} + |\sin x|_{\frac{\pi}{4}}^{\frac{\pi}{2}} + |\sin x|_{\frac{\pi}{2}}^{\pi}$$

$$A = \left|-\frac{1}{\sqrt{2}}+0\right| + \left|-(1+\frac{1}{\sqrt{2}})\right| + \left|-\left(\frac{1}{\sqrt{2}}-1\right)\right| + \left|\left(1-\frac{1}{\sqrt{2}}\right)\right| + |0-1|$$

$$A = \frac{1}{\sqrt{2}} + 1 + \frac{1}{\sqrt{2}} + 1 - \frac{1}{\sqrt{2}} + 1 - \frac{1}{\sqrt{2}} + 1$$

A = 4

$A^2 = 16$

24. Let $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$. If the sum of the diagonal elements of A^{13} is 3^n, then n is equal to _______.

Sol. **7**

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & -3 \\ 3 & 0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 3 & -6 \\ 6 & -3 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 0 & -9 \\ 9 & -9 \end{bmatrix}$$

$$A^5 = \begin{bmatrix} -9 & -9 \\ 9 & -18 \end{bmatrix}$$

$$A^8 = \begin{bmatrix} 0 & -9 \\ 9 & -9 \end{bmatrix}\begin{bmatrix} 0 & -9 \\ 9 & -9 \end{bmatrix}$$

$$A^{13} = A^8 \times A^5 = \begin{bmatrix} 81 & 81 \\ -81 & 0 \end{bmatrix}\begin{bmatrix} -9 & -9 \\ 9 & -18 \end{bmatrix}$$

$$A^{13} = \begin{bmatrix} (-81)(-9)+(81\times 9) & - \\ - & (-81)(-9) \end{bmatrix}$$

Sum of diagonal = $(81 \times 27) = 3^4 \times 3^3 = 3^7$

$\Rightarrow$ **n = 7 Ans.**

25. Three balls are drawn at random from a bag containing 5 blue and 4 yellow balls. Let the random variables X and Y respectively denote the number of blue and yellow balls. If $\bar{X}$ and $\bar{Y}$ are the means of X and Y respectively, then $7\bar{X}+4\bar{Y}$ is equal to _______.

Sol. **17**

X = 1 $\quad \dfrac{{}^5C_1 \times {}^4C_2}{{}^9C_3} = \dfrac{5\times 6}{84} = \dfrac{5}{14}$

X = 2 $\quad \dfrac{{}^5C_2 \times {}^4C_1}{{}^9C_3} = \dfrac{10\times 4}{84} = \dfrac{10}{21}$

X = 3 $\quad \dfrac{{}^5C_3}{{}^9C_3} = \dfrac{5\times 4\times 3}{9\times 8\times 7} = \dfrac{5}{42}$

$Y = 1 = \dfrac{10}{21}$

$Y = 2 = \dfrac{5}{14}$

$Y = 3 = \dfrac{{}^4C_3}{{}^9C_3} = \dfrac{4\times 3\times 2}{9\times 8\times 7} = \dfrac{1}{21}$

$\bar{X} = p(x=1)x_1 + p(x=2)x_2 + p(x=3)x_3$

$\bar{X} = \frac{5}{14} + \frac{20}{21} + \frac{15}{42}$

$\bar{X} = \frac{30+80+30}{84} = \frac{140}{84}$

$\bar{Y} = p(y=1)y_1 + p(y=2)y_2 + p(y=3)y_3$

$\bar{Y} = \frac{20+30+6}{42} = \frac{20+30+6}{42} = \frac{56}{42}$

$= 7\bar{X} + 4\bar{Y}$

$= \frac{140}{12} + \frac{56}{21} \times 2 = \frac{980+448}{84} = \frac{1428}{84} =$ **17 Ans.**

26. Let $\alpha = \sum_{r=0}^{n}\left(4r^2+2r+1\right)^n C_r$ and $\beta = \left(\sum_{r=0}^{n}\frac{^nC_r}{r+1}\right) + \frac{1}{n+1}$. If $140 < \frac{2\alpha}{\beta} < 281$ then the value of n is _______.

Sol. **5**

$\alpha = \sum_{r=0}^{n}(4r^2+2r+1)^n C_r$

$\alpha = \sum_{r=0}^{n} 4r^{2n}C_r + 2r\,^nC_r + ^n C_r$

$(1 + x)^n = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + \ldots. + C_n x^n$

$n(1 + x)^{n-1} = C_1 + 2 C_2 x + 3 C_3 x^2 + 4 C_4 x^3 + \ldots. + n C_n x^{n-1}$ ……….. (i)

$\boxed{n(2)^{n-1} \Rightarrow r\,^nC_r}$

or (i) multiply by x

$nx(1 + x)^{n-1} = C_1x + 2C_2 x^2 + 3C_3 x^3 + 4 C_4x^4 + \ldots. + n C_n x^n$

$n(1 + x)^{n-1} + nx(n-1)(1 + x)^{n-2} = C_1 + 2^2 C_2 x + 3^2 C_3 x^2 + 4^2 C_{n^{n3}} + \ldots\ldots$

$\boxed{x = 1}$

$n(2)^{n-1} + n(n-1)2^{n-2} = r^2\,^nC_r$

$\alpha = 4\cdot\sum_{r=0}^{n} n2^{n-1} + n(n-1)2^{n-2} + 2\sum_{r=0}^{n} n2^{n-1} + \sum_{r=0}^{n} {}^nC_r$

$\alpha = 4\sum r^{2n}C_r + 2\sum r^nC_r + \sum {}^nC_r$

$\alpha = 4\left(n(2)^{n-1} + n(n-1)2^{n-2}\right) + 2\left(n(2^{n-1})\right) + 2^n$

$\alpha = 2n\cdot 2^n + n(n-1)2^n + n2^n + 2^n$

$\alpha = 2^n(2n + n^2 - n + n + 1)$

$\alpha = 2^n(n+1)^2$

$\beta = \left(\sum_{r=0}^{n}\frac{^nC_r}{r+1}\right) + \frac{1}{n+1}$

$$\Rightarrow \beta = \left(\sum_{r=0}^{n} \frac{n+1}{(r+1)} \frac{{}^nC_r}{(n+1)}\right) + \frac{1}{n+1}$$

$$\Rightarrow \sum_{r=0}^{n} \frac{{}^{n+1}C_{r+1}}{n+1} + \frac{1}{n+1}$$

$$\beta = \frac{2^{n+1}-1}{n+1} + \frac{1}{n+1} \Rightarrow \frac{2^{n+1}}{n+1}$$

$$140 < \frac{2 \cdot 2^n (n+1)^2 (n+1)}{2^{n+1}} < 281$$

$$140 < (n+1)^3 < 281$$

$\boxed{n = 5}$

27. Let the positive integers be written in the form :

1
2 3
4 5 6
7 8 9 10

If the k^{th} row contains exactly k numbers for every natural number k, then the row in which the number 5310 will be, is ________.

Sol. **103**

1
2 3
4 5 6
7 8 9 10

$S = 1 + 3 + 6 + 10, \ldots\ldots\ldots\ldots, T_n$

$S = \quad 1 + 3 + 6 + \ldots\ldots + T_{n-1} + T_n$

$0 = 1 + 2 + 3 + 4 + \ldots\ldots\ldots + n - T_n$

$$T_n = \frac{n(n+1)}{2}$$

For n = 102

$$T_n = \frac{102(103)}{2} \Rightarrow 5253$$

For n = 103

$$T_n = \frac{103 \times 104}{2} = 5336$$

$\boxed{K = 103}$

28. Let $\vec{a} = 9\hat{i} - 13\hat{j} + 25\hat{k}$, $\vec{b} = 3\hat{i} + 7\hat{j} - 13\hat{k}$ and $\vec{c} = 17\hat{i} - 2\hat{j} + \hat{k}$ be three given vectors. If $\vec{r}$ is vector such that $\vec{r} \times \vec{a} = (\vec{b} + \vec{c}) \times \vec{a}$ and $\vec{r} \cdot (\vec{b} - \vec{c}) = 0$, then $\frac{|593\vec{r} + 67\vec{a}|^2}{(593)^2}$ is equal to _______.

Sol. **569**

$\vec{a} = 9\hat{i} - 13\hat{j} + 25\hat{k}$

$\vec{b} = 3\hat{i} + 7\hat{j} - 13\hat{k}$

$\vec{b} - \vec{c} = -14\hat{i} + 9\hat{j} - 14\hat{k}$

$r - (\vec{b} + \vec{c}) = \lambda\vec{a}$

$\vec{r} = \lambda\vec{a} + \vec{b} + \vec{c}$

$\Rightarrow (\lambda\vec{a} + \vec{b} + \vec{c}).(\vec{b} + \vec{c}) = 0$

$\Rightarrow \lambda\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{b} + \vec{c}\cdot\vec{b} - \lambda\vec{a}\cdot\vec{c} - \vec{b}\cdot\vec{c} - \vec{c}\cdot\vec{c} = 0$

$$\lambda = \frac{\vec{c}\cdot\vec{c} - \vec{b}\cdot\vec{b}}{a\cdot\vec{b} - a\cdot\vec{c}} = \frac{294 - 227}{-389 - 204} = \frac{-67}{593}$$

$\therefore\ \vec{r} = \vec{b} + \vec{c} - \frac{67}{593}\vec{a}$

$\Rightarrow 593\vec{r} + 67\vec{a} = 593(\vec{b} + \vec{c})$

$\Rightarrow (\vec{b} + \vec{c}) = 569$

29. If the range of $f(\theta) = \frac{\sin^4\theta + 3\cos^2\theta}{\sin^4\theta + \cos^2\theta}$, $\theta \in R$ is $[\alpha, \beta]$, then the sum of the infinite G. P., whose first term is 64 and the common ratio is $\frac{\alpha}{\beta}$ equal to _______.

Sol. **96**

$$f(\theta) = \frac{\sin^4\theta + 3\cos^2\theta}{\sin^4\theta + \cos^2\theta}$$

$$f(\theta) = \frac{1 + 2\cos^2\theta}{\sin^4\theta + \cos^2\theta}$$

$$f(\theta) = 1 + \frac{2\cos^2\theta}{(1 - \cos^2\theta)^2 + \cos^2\theta}$$

$$f(\theta) = 1 + \frac{2\cos^2\theta}{1 + \cos^4\theta - \cos^2\theta}$$

$$f(\theta) = 1 + \frac{2}{\cos^2\theta + \frac{1}{\cos^2\theta} - 1}$$

$f(\theta)_{max} = 3$

$f(\theta)_{min} = 1$

$[\alpha, \beta] \equiv [1, 3]$

GP $\quad a = 64$

$$r = \frac{\alpha}{\beta} = \frac{1}{3}$$

$$\text{sum} = \frac{a}{1-r}$$

$$= \frac{64}{1-\frac{1}{3}}$$

$$= \frac{64 \times 3}{2} = 96 \text{ Ans.}$$

30. The number of 3-digit numbers, formed using the digits 2,3,4,5 and 7, when the repetition of digits is not allowed, and which are not divisible by 3, is equal to ____.

Sol. **36**

2, 3, 4, 5, 7

Number divisible by 3

(2, 3, 4) $\rightarrow 6$ (2, 3, 7) $\rightarrow 6$

(3, 4, 5) $\rightarrow 6$ (3, 5, 7) $\rightarrow 6$

Divisible by 3 = 6 × 4 = 24

total number = $^5C_3 \, \lfloor 3 = 60$

Not divisible by 3 = 60 – 24 = **36 Ans.**

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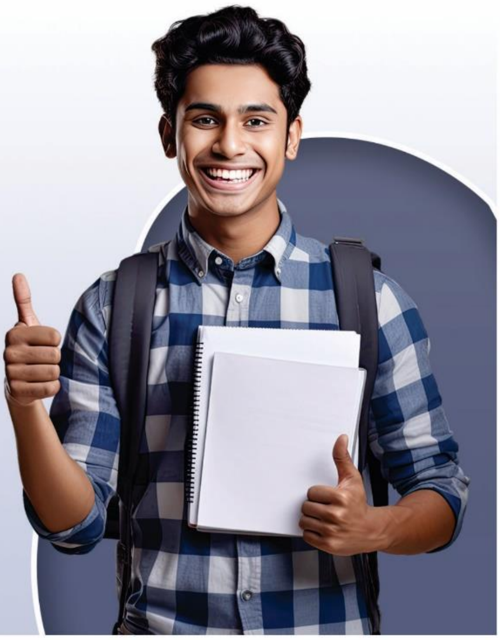

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