JEE MAIN 2024 SESSION-2

Paper with Solution

MATHS | 09th April 2024 _ Shift-2

SECTION – A

1. $\lim_{x\to 0}\frac{e-(1+2x)^{\frac{1}{2x}}}{x}$ is equal to

(1) $\frac{-2}{e}$ (2) 0 (3) $e-e^2$ (4) e

Sol. 4

$$\lim_{x\to 0}\frac{e-e^{\frac{\ln(1+2x)}{2x}}}{x}$$

$$\lim_{x\to 0}\frac{-e\left[e^{\frac{\ln(1+2x)}{2x}-1}-1\right]}{x}$$

$$\lim_{x\to 0}-\frac{e\left[e^{\frac{\ln(1+2x)}{2x}-1}-1\right]}{x\left(\frac{\ln(1+2x)}{2x}-1\right)}\times\left(\frac{\ln(1+2x)}{2x}-1\right)$$

$$\lim_{x\to 0}-e\left(\frac{\ln(1+2x)-2x}{2x^2}\right)\cdot(1)$$

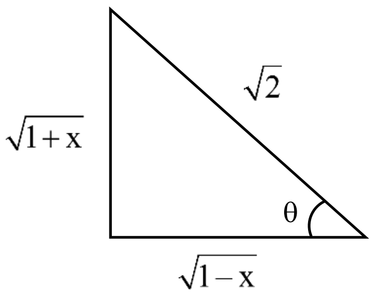
$$\lim_{x\to 0}\frac{-e\left[\left(2x-\frac{(2x)^2}{2}+\cdots\right)-2x\right]}{2x^2}$$

$$\Rightarrow\frac{e\cdot 4x^2}{4x^2}\Rightarrow e$$

2. The integer $\int_{1/4}^{3/4}\cos\left(2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right)dx$ is equal to

(1) 1/4 (2) 1/2 (3) –1/2 (4) –1/4

Sol. 4



$$\int_{\frac{1}{4}}^{\frac{3}{4}} \cos\left(2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right)\cdot dx$$

Let $\cot^{-1}\sqrt{\frac{1-x}{1+x}} = \theta$

$$\cot\theta = \frac{\sqrt{1-x}}{\sqrt{1+x}}$$

$$\therefore \cos\theta = \sqrt{\frac{1-x}{2}}$$

Now, $\int_{\frac{1}{4}}^{\frac{3}{4}} \cos 2\theta \cdot dx$

$$\int_{\frac{1}{4}}^{\frac{3}{4}} \left(2\cos^2\theta - 1\right)dx$$

$$\int_{\frac{1}{4}}^{\frac{3}{4}} \left(2\frac{(1-x)}{2} - 1\right)dx$$

$$\int_{\frac{1}{4}}^{\frac{3}{4}} [(1-x)-1]\,dx$$

$$\Rightarrow \int_{\frac{1}{4}}^{\frac{3}{4}} -x\,dx \Rightarrow \left[-\frac{x^2}{2}\right]_{1/4}^{3/4} \Rightarrow -\frac{1}{2}\left[\frac{9}{16}-\frac{1}{16}\right] \Rightarrow -\frac{1}{2}\times\frac{8}{16} \Rightarrow -\frac{1}{4}$$

3. Let $B = \begin{bmatrix} 1 & 3 \\ 1 & 5 \end{bmatrix}$ and A be a 2 × 2 matrix such that $AB^{-1} = A^{-1}$. If $BCB^{-1} = A$ and $C^4 + \alpha C^2 + \beta I = O$, then $2\beta - \alpha$ is equal to

(1) 8 (2) 16 (3) 10 (4) 2

Sol. **3**

$BCB^{-1} = A$

$BCB^{-1}\ BCB^{-1} = A.A$

$BCICB^{-1} = A\ .\ A$

$B^{-1}(BC^2B^{-1})B = B^{-1}A \cdot A\,B$

$C^2 = B^{-1}A\,A\,B$ ...(1)

Now,

$AB^{-1} = A^{-1}$

$AB^{-1}A = A^{-1} \cdot A = I$

$A^{-1}AB^{-1}A = A^{-1} \cdot I$

$B^{-1}A = A^{-1}$ ...(2)

From equation (1) & (2)

$C^2 = A^{-1} \cdot AB$

$C^2 = B$

Now, characteristic equation of c^2 is,

$|C^2 - \lambda I| = 0$

$|B - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 3 \\ 1 & 5-\lambda \end{vmatrix} = 0$$

$\lambda^2 - 6\lambda + 2 = 0$

$B^2 - 6B + 2 = 0$

$C^4 - 6C^2 + 2I = 0$

$\therefore \alpha = -6, \beta = 2$

So, $2\beta - \alpha = 10$

4. If an unbiased dice is rolled thrice, then the probability of getting a greater number in the i^{th} roll than the number obtained in the $(i-1)^{th}$ roll, I = 2,3, is equal to

(1) 2/54 (2) 5/54 (3) 1/54 (4) 3/54

Sol. 2

Let x_1, x_2, x_3 be the numbers on 1st, 2nd & 3rd throw respectively.

Given: $x_1 < x_2 < x_3$

$\therefore$ Favorable cases = ${}^6C_3$

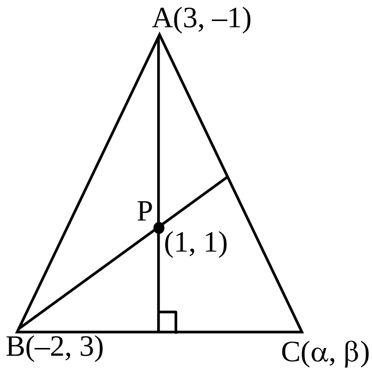
Total cases = $6^3 = 216$

Prob = $\frac{{}^6C_3}{216} \Rightarrow \frac{20}{216} = \frac{5}{54}$

5. Two vertices of a triangle ABC are A(3, –1) and B(–2, 3), and its orthocentre is P(1, 1). If the coordinates of the point C are (α,β) and the centre of the of the circle circumscribing the triangle PAB is (h, k), then the value of (α + β) + 2(h + k) equals

(1) 51 (2) 81 (3) 5 (4) 15

Sol. **3**



Equation of PC: $y-1=\frac{5}{4}(x-1) \quad \left(\because m_{AB}=-\frac{4}{5}\right)$

4y – 4 = 5x – 5

5x – 4y = 1 …(1)

Equation of BC: $y-3=1(x+2) \quad (\because m_{AP}=-1)$.

x – y = –5 …(2)

Solving equation (1) & (2), x = 21, y = 26

$\therefore \alpha=21, \beta=26 \Rightarrow (\alpha+\beta)=47$

Now, equation of ⊥ bisector of AP: y – 0 = 1(x – 2)

x – y = 2 …(3)

Equation of ⊥ bisector of $AB: y-1=\frac{5}{4}\left(x-\frac{1}{2}\right)$ …(4)

Solving equation (3) & (4),

$x=-\frac{19}{2}, y=-\frac{23}{2}$

i.e 2h = –19,2 k = –23

∴ 2(h + k) = –42

So, (α + β) + 2(h + k) = 47 – 42 = 5

6. Let the foci of a hyperbola H coincide with the foci of the ellipse $E:\frac{(x-1)^2}{100}+\frac{(y-1)^2}{75}=1$ and the eccentricity of the hyperbola H be the reciprocal of the eccentricity of the ellipse E. If the length of the transverse axis of H is α and the length of its conjugate axis is β, then $3\alpha^2 + 2\beta^2$ is equal to

(1) 242 (2) 225 (3) 237 (4) 205

Sol. **2**

$$E:\frac{(x-1)^2}{100}+\frac{(y-1)^2}{75}=1$$

Given, $e_H=\frac{1}{e_E}$

$$e_E^2=1-\frac{3}{4}=\frac{1}{4}$$

$$e_E=\frac{1}{2}$$

$\therefore$ $e_H = 2$

Transverse axis = $\alpha \Rightarrow a = \frac{\alpha}{2}$

conjugate axis = $\beta \Rightarrow b = \frac{\beta}{2}$

$$e_H^2=1+\frac{\beta^2}{\alpha^2}$$

$$4\alpha^2=\alpha^2+\beta^2 \Rightarrow 3\alpha^2=\beta^2 \qquad \ldots(1)$$

ae = 5

$$\frac{\alpha}{2}\times 2=5 \Rightarrow \alpha=5$$

From equation (1), $\beta^2 = 75$

$\therefore$ $3\alpha^2 + 2\beta^2 = 75 + 150 = 225$

7. Let a, ar, ar^2, …….. be an infinite G.P. If $\sum_{n=0}^{\infty} ar^n=57$ and $\sum_{n=0}^{\infty} a^3r^{3n}=9747$, then a + 18r is equal to

(1) 38 (2) 31 (3) 27 (4) 46

Sol. **2**

$$\sum_{n=0}^{\infty} ar^n=57$$

$\frac{a}{1-r}=57$ …(1)

also,

$\sum_{n=0}^{\infty} a^3 \cdot r^{3n} = 9747$

$\frac{a^3}{1-r^3} = 9747$ …(2)

Cubing equation (1)

$\frac{a^3}{(1-r)^3}=(57)^3$ …(3)

Now, dividing eq (2) & (3)

$$\frac{\frac{a^3}{(1-r)\left(1+r+r^2\right)}}{\frac{a^3}{(1-r)^{32}}}=\frac{9747}{57\times 57\times 57}$$

$$\frac{1+r^2-2r}{1+r+r^2}=\frac{1}{19}$$

$19 + 19r^2 - 38r = 1 + r + r^2$

$18r^2 - 39r + 18 = 0$

$6r^2 - 13r + 6 = 0$

$6r^2 - 9r - 4r + 6 = 0$

$3r(2r - 3) - 2(2r - 3) = 0$

$(2r - 3)(3r - 2) = 0$

$r = \frac{3}{2}, \frac{2}{3}$

Taking $r = \frac{2}{3}$

From equation (1) $a = 57\times\frac{1}{3} \Rightarrow 19$

$\therefore\ a + 18r = 19 + 18 \times \frac{2}{3} \Rightarrow 19 + 12 = 31$

8. Let z be a complex number such that the real part of $\frac{z-2i}{z+2i}$ is zero. Then, the maximum value of $|z-(6+8i)|$ is equal to

(1) 8 (2) 12 (3) ∞ (4) 10

Sol. **2**

$$\text{Re}\left(\frac{z-2i}{z+2i}\right)=0$$

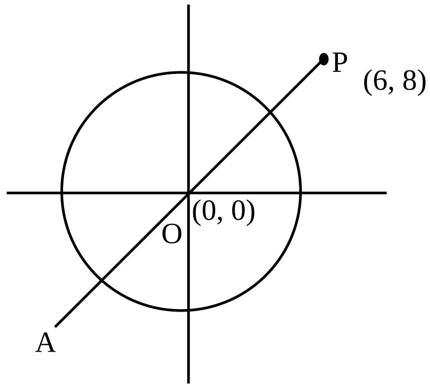
Let z = x + iy

$$\frac{x+iy-2i}{x+iy+2i} \Rightarrow \frac{x+i(y-2)}{x+i(y+2)}$$

Rationalizing

$$\Rightarrow \frac{x+i(y-2)}{x+i(y+2)} \times \frac{x-i(y+2)}{x-i(y+2)}$$

$$\Rightarrow \frac{x^2+\left(y^2-4\right)+i(xy-2x-xy-2x)}{x^2+(y+2)^2}$$



$$\text{Now, } \text{Re}\left(\frac{z-2i}{z+2i}\right)=0$$

$$\therefore \frac{x^2+y^2-4}{x^2+(y+2)^2}=0$$

$x^2 + y^2 = 4$ (circle)

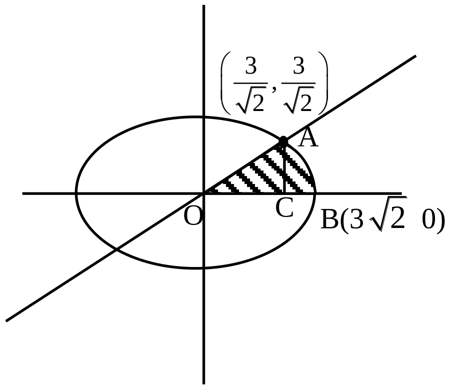
Maximum value of $|z-(6+8i)|$ means Maximum distance of point (6, 8) from the circle $x^2 + y^2 = 4$

Max. distance = OP + r = 10 + 2 = 12

9. The area (in square units) of the region enclosed by the ellipse $x^2 + 3y^2 = 18$ in the first quadrant below the line y = x is

(1) $\sqrt{3}\pi+1$ (2) $\sqrt{3}\pi-\frac{3}{4}$ (3) $\sqrt{3}\pi$ (4) $\sqrt{3}\pi+\frac{3}{4}$

Sol. **3**



E : $x^2 + 3y^2 = 18$ & $y = x$

$$\frac{x^2}{(3\sqrt{2})^2}+\frac{y^2}{(\sqrt{6})^2}=1$$

Solving above two equation

$$x=y=\frac{3}{\sqrt{2}}$$

$$A\left(\frac{3}{\sqrt{2}},\frac{3}{\sqrt{2}}\right)$$

Now, Area of region OABCO is = Area of ΔOAC+ Area of region CABC,

$$\text{Area} \Rightarrow \frac{1}{2}\times\frac{3}{\sqrt{2}}\times\frac{3}{\sqrt{2}}+\int_{3/\sqrt{2}}^{3\sqrt{2}}\frac{\sqrt{18-x^2}}{\sqrt{3}}\cdot dx$$

$$\Rightarrow \frac{9}{4}+\frac{1}{\sqrt{3}}\left[\frac{x}{2}\sqrt{18-x^2}+\frac{18}{2}\sin^{-1}\left(\frac{x}{3\sqrt{2}}\right)\right]_{3/\sqrt{2}}^{3\sqrt{2}}$$

$$\Rightarrow \frac{9}{4}+\frac{1}{\sqrt{3}}\left[0+9\times\frac{\pi}{2}-\frac{3}{2\sqrt{2}}\times\frac{3\sqrt{3}}{\sqrt{2}}-9\times\frac{\pi}{6}\right]$$

$$\Rightarrow \frac{9}{4}+\frac{1}{\sqrt{3}}\left[\frac{9\pi}{2}-\frac{3\pi}{2}-\frac{9\sqrt{3}}{4}\right]$$

$$\Rightarrow \frac{9}{4}+\frac{1}{\sqrt{3}}\left[3\pi-\frac{9\sqrt{3}}{4}\right] \Rightarrow \frac{9}{4}+\sqrt{3}\pi-\frac{9}{4} \Rightarrow \sqrt{3}\pi$$

10. Between the following two statements :

Statement I : Let $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=2\hat{i}+\hat{j}-\hat{k}$. Then the vector $\vec{r}$ satisfying $\vec{a}\times\vec{r}=\vec{a}\times\vec{b}$ and $\vec{a}\cdot\vec{r}=0$ is of magnitude $\sqrt{10}$.

Statement II : In a triangle ABC, $\cos 2A + \cos 2B + \cos 2C \geq -\frac{3}{2}$.

(1) Statement I is incorrect but Statement II is correct.
(2) Both Statement I and Statement II are incorrect.
(3) Both Statement I and Statement II are correct.
(4) Statement I is correct but Statement II is incorrect.

Sol. **1**

Given,

$$\vec{a}\times\vec{r} = \vec{a}\times\vec{b}$$

$\vec{a}.(\vec{r}-\vec{b})=0$

$\therefore \vec{r}-\vec{b}\ =\lambda\vec{a}$

$\vec{r}=\vec{b}+\lambda\vec{a}$

Now,

$\vec{a}\cdot\vec{r}=0$

$\vec{a}\cdot\vec{b}+\lambda|\vec{a}|^2=0$

$7+14\lambda=0$

$\lambda=-\frac{1}{2}$.

$\vec{r}=2\hat{i}+\hat{j}-\hat{k}-\frac{1}{2}(\hat{i}+2\hat{j}-3\hat{k})$

$\vec{r}=\frac{3}{2}\hat{i}+\frac{1}{2}\hat{k}$

$|\vec{r}|=\sqrt{\frac{9}{4}+\frac{1}{4}}=\frac{\sqrt{10}}{2}$

Statement - I is Incorrect

Statement -II $\cos 2A+\cos 2B+\cos 2C\le-\frac{3}{2}$.

When A + B + C = π

We know that,

cos 2 A + cos 2B + cos 2C = –1 –4 cos A cos B cos C

$\therefore$ cos 2A + cos 2B + cos 2C $\ge -1-4\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$ $\left(\text{When } A = B = C = \frac{\pi}{3}\right)$

$\geqslant-\frac{3}{2}$

Statement-II is correct.

11. Let the range of the function $f(x)=\frac{1}{2+\sin 3x+\cos 3x}, x\in\mathbb{R}$ be [a, b]. If α and β are respectively the A.M. and the G.M. of a and b, then $\frac{\alpha}{\beta}$ is equal to

(1) π (2) 2 (3) $\sqrt{\pi}$ (4) $\sqrt{2}$

Sol. 4

$$f(x)=\frac{1}{2+\sin 3x+\cos 3x}$$

We know that,

$$-\sqrt{2}\le \sin 3x+\cos 3x\le \sqrt{2}$$

$$2-\sqrt{2}\le 2+\sin 3x+\cos 3x\le 2+\sqrt{2}$$

$$\frac{1}{2+\sqrt{2}}\le \frac{1}{2+\sin 3x+\cos 3x}\le \frac{1}{2-\sqrt{2}}$$

$$\therefore\ a=\frac{1}{2+\sqrt{2}}\ \text{or}\ \frac{2-\sqrt{2}}{2}$$

$$b=\frac{1}{2-\sqrt{2}}\ \text{or}\ \frac{2+\sqrt{2}}{2}$$

$$\alpha\ = \text{A. M of a \& b} = \frac{\frac{2-\sqrt{2}}{2}+\frac{2+\sqrt{2}}{2}}{2}=1$$

$$\beta\ = \text{G. M of a \& b} = \sqrt{\frac{2-\sqrt{2}}{2}\times\frac{2+\sqrt{2}}{2}}=\frac{1}{\sqrt{2}}$$

$$\therefore \frac{\alpha}{\beta}=\frac{1}{\frac{1}{\sqrt{2}}}=\sqrt{2}$$

12. If $\log_e y = 3\sin^{-1}x$, then $(1-x^2)\,y''-x\,y'$ at $x=\frac{1}{2}$ is equal to

(1) $9e^{\pi/6}$ (2) $3e^{\pi/6}$ (3) $9e^{\pi/2}$ (4) $3e^{\pi/2}$

Sol. 3

$$\log_e y=3\sin^{-1}x$$

$$y=e^{3\sin^{-1}x}$$

differentiating w.r.t

$$y' = e^{3\sin^{-1}x}\cdot\frac{3}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}\cdot y'=3e^{3\sin^{-1}x}$$

Again differentiating w.r.t x

$$\sqrt{1-x^2}\cdot y''-\frac{xy'}{\sqrt{1-x^2}}=\frac{9e^{3\sin^{-1}x}}{\sqrt{1-x^2}}$$

$(1-x^2)\,y''-xy'=9y$ (From equation (1))

Now, from equation (1), at $x=\frac{1}{2}$

$y=e^{\pi/2}$

$\therefore\ (1-x^2)\,y''-xy'=9e^{\pi/2}$

13. The sum of the coefficient of $x^{2/3}$ and $x^{-2/5}$ in the binomial expansion of $\left(x^{2/3}+\frac{1}{2}x^{-2/5}\right)^9$ is

(1) 63/16 (2) 21/4 (3) 19/4 (4) 69/16

Sol. **2**

$$\left(x^{2/3}+\frac{1}{2}x^{-2/5}\right)^9$$

$$T_{r+1}={}^9C_r\left(x^{2/3}\right)^{9-r}\cdot\left(\frac{1}{2}x^{-2/5}\right)^r$$

$$={}^9C_r\left(\frac{1}{2}\right)^r\cdot x^{\frac{18-2r}{3}-\frac{2r}{5}}$$

For coeff. of $x^{2/3}$, $\frac{90-16r}{15}=\frac{2}{3}$

$16r=80$

$r=5$

For coeffi of $x^{-2/5}$, $\frac{90-16r}{15}=\frac{-2}{5}$

$16r=96$

r = 6

Sum of coff $\Rightarrow {}^9C_5\times\frac{1}{32}+{}^9C_6\frac{1}{64}$

$\Rightarrow\frac{63}{16}+\frac{21}{16}\Rightarrow\frac{84}{16}$

$\Rightarrow\frac{21}{4}$

14. If the variance of the frequency distribution

x	c	2c	3c	4c	5c	6c
f	2	1	1	1	1	1

is 160, then the value of $c \in \mathbb{N}$ is

(1) 7 (2) 6 (3) 8 (4) 5

Sol. **1**

x	c	2c	3c	4c	5c	6c
f	2	1	1	1	1	1

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{22c}{7}$$

$$\text{Now, } \operatorname{var}(x) = \frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i}$$

$$(x_i - \bar{x})^2 : \frac{225c^2}{49} \quad \frac{64c^2}{49} \quad \frac{c^2}{49} \quad \frac{36c^2}{49} \quad \frac{169c^2}{49} \quad \frac{400c^2}{49}$$

f : 2 1 1 1 1 1

$$\operatorname{var}(x) = \frac{c^2}{49}\left[\frac{450+64+1+36+169+400}{7}\right]$$

$$= \frac{1120c^2}{49 \times 7} = 160$$

$c^2 = 49$

$c = \pm 7$

$\therefore c = 7$ ($\because c \in N$)

15. Let $\vec{a} = 2\hat{i} + \alpha\hat{j} + \hat{k}, \vec{b} = -\hat{i} + \hat{k}, \vec{c} = \beta\hat{j} - \hat{k}$, where α and β are integers and $\alpha\beta = -6$. Let the values of the ordered pair (α, β) for which the area of the parallelogram of diagonals $\vec{a} + \vec{b}$ and $\vec{b} + \vec{c}$ is $\frac{\sqrt{21}}{2}$, be (α_1, β_1) and (α_2, β_2). Then $\alpha_1^2 + \beta_1^2 - \alpha_2\beta_2$ is equal to:

(1) 17 (2) 21 (3) 19 (4) 24

Sol. **3**

$$\text{Area of Parallelogram} = \frac{1}{2}|(\vec{a} + \vec{b}) \times (\vec{b} + \vec{c})| = \frac{\sqrt{21}}{2}$$

$$(\vec{a}+\vec{b})\times(\vec{b}+\vec{c}) = \begin{vmatrix} i & j & k \\ 1 & \alpha & 2 \\ -1 & \beta & 0 \end{vmatrix}$$

$\Rightarrow -2\beta\hat{i} - 2\hat{j} + (\beta+\alpha)\hat{k}$

$|(\vec{a}+\vec{b})\times(\vec{b}+\vec{c})| = \sqrt{4\beta^2+4+\beta^2+\alpha^2+2\alpha\beta} = \sqrt{21}$

$5\beta^2 + \alpha^2 + 4 - 12 = 21$

$\alpha^2 + 5\beta^2 = 29$

Also,

$\alpha\beta = -6$

$\beta = \dfrac{-6}{\alpha}$.

$\alpha^2 + \dfrac{180}{\alpha^2} = 29$

$\alpha^4 - 29\alpha^2 + 180 = 0$

$\alpha^4 - 20\alpha^2 - 9\alpha^2 + 180 = 0$

$(\alpha^2 - 20)\ (\alpha^2 - 9) = 0$

$\alpha = 3, -3$ as $\alpha \in I$

$\beta = -2, 2$

Now, $\alpha_1^2 + \beta_1^2 - \alpha_2\beta_2$

$\Rightarrow 9 + 4 + 6 \Rightarrow 19$

16. The value of the integral $\int_{-1}^{2} \log_e\left(x+\sqrt{x^2+1}\right)dx$ is

(1) $\sqrt{5}-\sqrt{2}+\log_e\left(\dfrac{9+4\sqrt{5}}{1+\sqrt{2}}\right)$

(2) $\sqrt{5}-\sqrt{2}+\log_e\left(\dfrac{7+4\sqrt{5}}{1+\sqrt{2}}\right)$

(3) $\sqrt{2}-\sqrt{5}+\log_e\left(\dfrac{9+4\sqrt{5}}{1+\sqrt{2}}\right)$

(4) $\sqrt{2}-\sqrt{5}+\log_e\left(\dfrac{7+4\sqrt{5}}{1+\sqrt{2}}\right)$

Sol. **3**

$\int_{-1}^{2} \log_e\left(x+\sqrt{x^2+1}\right)\cdot dx$

$$\int_{-1}^{2}\log_e\left(x+\sqrt{x^2+1}\right)\cdot 1dx$$

$\downarrow$ I $\quad\quad \downarrow$ II

Using by parts,

$$\left[\log_e\left(x+\sqrt{x^2+1}\right)\cdot x\right]_{-1}^{2}-\int_{-1}^{2}\frac{1+\frac{2x}{2\sqrt{x^2+1}}}{x+\sqrt{x^2+1}}\cdot x\cdot dx$$

$$2\log_e(2+\sqrt{5})+\log_e(\sqrt{2}-1)-\int_{-1}^{2}\frac{\left(\sqrt{x^2+1}+x\right)x}{\left(\sqrt{x^2+1}\right)\cdot\left(x+\sqrt{x^2+1}\right)}dx$$

$$\log_e\left[\frac{(2+\sqrt{5})^2}{\sqrt{2}+1}\right]-\int_{-1}^{2}\frac{x\cdot dx}{\sqrt{1+x^2}}$$

$$\log_e\left(\frac{9+4\sqrt{5}}{\sqrt{2}+1}\right)-\int_{2}^{5}\frac{dt}{2\sqrt{t}} \quad \left(\begin{array}{l}\text{put } 1+x^2=t\\ x\cdot dx=\frac{dt}{2}\end{array}\right)$$

$$\log_e\left(\frac{9+4\sqrt{5}}{\sqrt{2}+1}\right)-(\sqrt{t})_2^5$$

$$\log_e\left(\frac{9+4\sqrt{5}}{\sqrt{2}+1}\right)-\sqrt{5}+\sqrt{2}$$

$$\therefore \sqrt{2}-\sqrt{5}+\log_e\left(\frac{9+4\sqrt{5}}{\sqrt{2}+1}\right)$$

17. Let $\alpha, \beta; \alpha>\beta$, be the roots of the equation $x^2-\sqrt{2}x-\sqrt{3}=0$. Let $P_n=\alpha^n-\beta^n, n\in\mathbb{N}$. Then $\left(11\sqrt{3}-10\sqrt{2}\right)P_{10}+\left(11\sqrt{2}+10\right)P_{11}-11P_{12}$ is equal to

(1) $11\sqrt{2}\,P_9$ (2) $10\sqrt{3}\,P_9$ (3) $10\sqrt{2}\,P_9$ (4) $11\sqrt{3}\,P_9$

Sol. **2**

$$x^2-\sqrt{2}x-\sqrt{3}=0$$

$$P_n=\alpha^n-\beta^n$$

Now,

$$\alpha^{n-2}\left(\alpha^2-\sqrt{2}\alpha-\sqrt{3}\right)=0 \quad \ldots (1)$$

$$\beta^{n-2}\left(\beta^2-\sqrt{2}\beta-\sqrt{3}\right)=0 \qquad \ldots (2)$$

Subtracting equation (1) & (2)

$$P_n=\sqrt{3}P_{n-2}+\sqrt{2}P_{n-1}$$

$$P_{12}=\sqrt{3}P_{10}+\sqrt{2}P_{11}$$

$$P_{11}=\sqrt{3}P_{9}+\sqrt{2}P_{10}$$

Now, Given expression,

$$11\sqrt{3}P_{10}-10\sqrt{2}P_{10}+11\sqrt{2}P_{11}+10P_{11}-11P_{12}$$

$$11\sqrt{3}P_{10}-10\sqrt{2}P_{10}+11\sqrt{2}P_{11}+10\sqrt{3}P_9+10\sqrt{2}P_{10}-11\sqrt{3}P_{10}-11\sqrt{2}P_{11}$$

$$\Rightarrow 10\sqrt{3}P_9$$

18. $\lim_{x\to\frac{\pi}{2}}\left(\dfrac{\int_{x^3}^{(\pi/2)^3}\left(\sin\left(2t^{1/3}\right)+\cos\left(t^{1/3}\right)\right)dt}{\left(x-\frac{\pi}{2}\right)^2}\right)$ is equal to

(1) $\frac{5\pi^2}{9}$ (2) $\frac{3\pi^2}{2}$ (3) $\frac{9\pi^2}{8}$ (4) $\frac{11\pi^2}{10}$

Sol. **3**

$$\lim_{x\to\frac{\pi}{2}}\frac{\int_{x^3}^{\left(\frac{\pi}{3}\right)^3}\left(\sin(2t^{\frac{1}{3}})+\cos(t^{\frac{1}{3}})\right).dt}{\left(x-\frac{\pi}{2}\right)^2}$$

Using ('Hospital Rule)

$$\lim_{x\to\frac{\pi}{2}}\frac{-(\sin 2x+\cos x)\cdot 3x^2}{2\left(x-\frac{\pi}{2}\right)} \qquad \text{(Using lebuitz rule)}$$

Again using L'Hospitale rule,

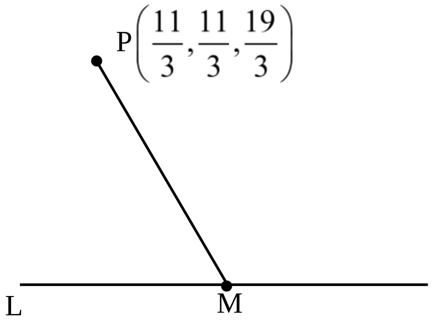
$$\lim_{x\to\frac{\pi}{2}}\frac{(-2\cos 2x+\sin x)3x^2+6x(-\sin 2x-\cos x)}{2}$$

$$\Rightarrow\frac{(-2\cos\pi+\sin\pi/2)3\frac{\pi^2}{4}+6\times\frac{\pi}{2}\left(-\sin\pi-\cos\frac{\pi}{2}\right)}{2}\Rightarrow\frac{(2+1)3\pi^2+0}{8}\Rightarrow\frac{9\pi^2}{8}$$

19. Consider the line L passing through the points (1, 2, 3) and (2, 3, 5). The distance of the point $\left(\frac{11}{3},\frac{11}{3},\frac{19}{3}\right)$ from the line L along the line $\frac{3x-11}{2}=\frac{3y-11}{1}=\frac{3z-19}{2}$ is equal to

(1) 4 (2) 3 (3) 5 (4) 6

Sol. **2**



Equation in of tine L passing through (1,2,3) & (2,3,5)

$$\frac{x-1}{1}=\frac{y-2}{1}=\frac{z-3}{2}=\lambda$$

M(λ + 1, λ + 2, 2 λ + 3)

DR's of PM: $\frac{3\lambda-8}{3},\frac{3\lambda-5}{3},\frac{6\lambda-10}{3}$

DR's of given line: $\frac{2}{3},\frac{1}{3},\frac{2}{3}$

$$\therefore \quad \frac{3\lambda-8}{2}=\frac{3\lambda-5}{1}=\frac{6\lambda-10}{2}$$

$3\lambda - 8 = 6\lambda - 10$

$\lambda=\frac{2}{3}.$

$B\left(\frac{5}{3},\frac{8}{3},\frac{13}{3}\right)$

$$AB=\sqrt{\frac{36+9+36}{9}}=3$$

20. Let $\int_0^x \sqrt{1-(y'(t))^2}\,dt=\int_0^x y(t)dt, 0\le x\le 3, y\ge 0, y(0)=0$. Then at x = 2, y'' + y + 1 is equal to

(1) $\sqrt{2}$ (2) 1 (3) 2 (4) 1/2

Sol. **2**

$$\int_0^x \sqrt{1-\left(y'(t)\right)^2}\cdot dt = \int_0^x y(t)\cdot dt$$

Differentiating both sides w.r.t x.

$$\sqrt{1-\left(y'(x)\right)^2}\cdot 1 = y(x)\cdot 1$$

$$1-\left(y'(x)\right)^2 = (y(x))^2$$

$$y'(x) = \sqrt{1-y^2(x)}$$

$$\frac{dy}{dx} = \sqrt{1-y^2}$$

$$\frac{dy}{\sqrt{1-y^2}} = dx$$

Integrating both sides,

$$\int \frac{dy}{\sqrt{1-y^2}} = \int dx$$

$\sin^{-1}y = x + c$

at $x = 0$, $y = 0$

$\sin^{-1}(0) = 0 + c \Rightarrow c = 0$

$y = \sin x$

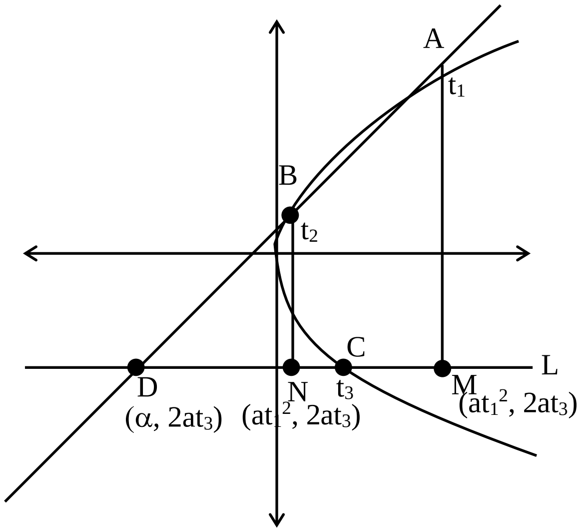
$y' = \cos x$

$y'' = -\sin x$

Now, $y'' + y + 1 \Rightarrow -\sin x + \sin x + 1 \Rightarrow 1$

21. Let A, B and C be three points on the parabola $y^2 = 6x$ and let the line segment AB meet the line L through C parallel to the x–axis at the point D. Let M and N respectively be the feet of the perpendiculars from A and B on L. Then $\left(\frac{AM \cdot BN}{CD}\right)^2$ is equal to ________.

Sol. **36**



$m_{AB} = m_{AD}$

$$\Rightarrow \frac{2}{t_1+t_2} = \frac{2a(t_1-t_3)}{at_1^2-\alpha}$$

$$\Rightarrow at_1^2 - \alpha = a\{t_1^2 - t_1t_3 + t_1t_2 - t_2t_3\}$$

$\Rightarrow \alpha = a(t_1t_3 + t_2t_3 - t_1t_2)$

AM = $[2a(t_1 - t_3)$, BN = $|2a(t_2 - t_3)|$,

CD = $|at_3^2 - \alpha|$

CD = $|at_3^2 - a(t_1t_3 + t_2t_3 - t_1t_2)|$

= $a|t_3^2 - t_1t_3 - t_2t_3 + t_1t_2|$

= $a|t_3(t_3 - t_1 - t_2(t_3 - t_1)|$

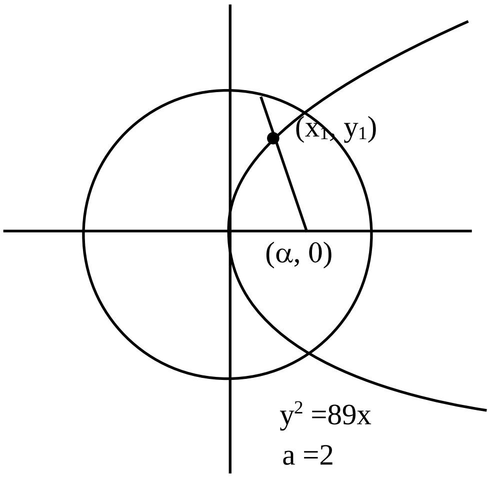
CD = $a|(t_3 - t_2)(t_3 - t_1)$

$$\left(\frac{AM\cdot BN}{CD}\right)^2 = \left\{\frac{2a(t_1-t_3)\cdot 2a(t_2-t_3)}{a(t_3-t_2)(t_3-t_1)}\right\}^2$$

$$16a^2 = 16\times\frac{9}{4} = 36$$

22. Consider the circle C : $x^2 + y^2 = 4$ and the parabola P : $y^2 = 8x$. If the set of all values of α, for which three chords of the circle C on three distinct lines passing through the point $(\alpha, 0)$ are bisected by the parabola P is the interval (p, q), then $(2q - p)^2$ is equal to _________.

Sol. **80**



$T = S_1$

$xx_1 + yy_1 = x_1^2 + y_1^2$

It passes through $(\alpha, 0)$

$\therefore\ \alpha x_1 = x_1^2 + y_1^2$

$\alpha(2t^2) = 4t^4 + 16\,t^2 \qquad (x_1 = 2t^2, y_1 = 4t)$

$\alpha = 2t^2 + 8$

$t^2 = \dfrac{\alpha - 8}{2}$

$\Rightarrow \alpha > 8$

Also,

$4t^4 + 16\,t^2 - 4 < 0$ (point lies inside the circle)

$t^2 = -2 + \sqrt{5}$

$\alpha = 4 + 2\sqrt{5}$

$\therefore \alpha \in (8, 4 + 2\sqrt{5})$

$\therefore\ (2q - p)^2 = 80$

23. Let $A = \{(x, y): 2x + 3y = 23, x, y \in \mathbb{N}\}$ and $B = \{x : (x, y) \in A\}$. Then the number of one-one functions from A to B is equal to ____.

Sol. **24**

$2x + 3y = 23, \qquad x, y \in N$

$A = \{(1,7), (4, 5), (7, 3), (10, 1)\}$

$B = \{1, 4, 7, 10\}$

then no. of one-one $f^n = 4! = 24$

24. The square of the distance of the image of the point (6, 1, 5) in the line $\frac{x-1}{3}=\frac{y}{2}=\frac{z-2}{4}$, from the origin is ___.

Sol. **62**

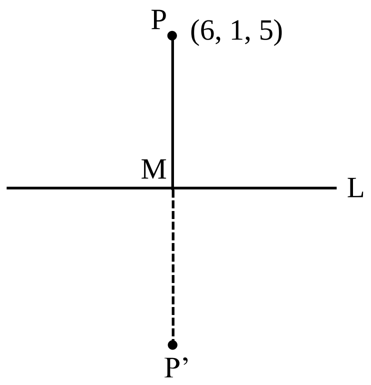


Image of point (6,1,5) in the line

$\frac{x-1}{3}=\frac{y}{2}=\frac{z-2}{4}=\lambda$

$x = 3\lambda + 1,\ y = 2\lambda,\ z = 4\lambda + 2$

$M(3\lambda + 1, 2\lambda, 4\lambda + 2)$

$\overrightarrow{PM}=(3\lambda-5)\hat{i}++(2\lambda-1)\hat{j}+(4\lambda-3)\hat{k}$

Now, $\overrightarrow{PM}$ $\perp$ to line L

$\therefore\quad \overrightarrow{PM}\cdot(3\hat{i}+2\hat{j}+4\hat{k})=0$

$9\lambda - 15 + 4\lambda - 2 + 16\lambda - 12 = 0$

$29\lambda = 29 \Rightarrow \lambda = 1$

M(4,2,6)

P'(2,3,7) $\rightarrow$ Image of P

Square of distance of P' from origin is equal to

$\left(\sqrt{(2-0)^2+(3-0)^2+(7-0)^2}\right)^2$

$\Rightarrow 4 + 9 + 49 \Rightarrow 62$

25. The number of integers, between 100 and 1000 having the sum of their digits equals to 14, is __________.

Sol. **70**

$x_1 + x_2 + x_3 = 14$, where $1 \le x_1 \le 9,\ 0 \le x_2, x_3 \le 9$

Coefficient of x^{14} in $\Rightarrow (x^1 + x^2 + \dots x^9)(x^0 + x^1 + x^2 + \dots + x^9)^2$

$\Rightarrow x(1 + x + \dots x^8)(1 + x + x^2 + \dots x^9)^2$

$$\Rightarrow x\left(\frac{1-x^9}{1-x}\right)\cdot\left(\frac{1-x^{10}}{1-x}\right)^2$$

$\Rightarrow x.(1-x^9)(1-x^{10})^2 . (1-x)^{-3}$

$\Rightarrow x.(1-x^9)(1-2x^{10}+x^{20})(1-x)^{-3}$

$\Rightarrow x.(1-2x^{10}+x^{20}-x^9+2x^{19}-x^{29})(1-x)^{-3}$

$\Rightarrow x(1-2x^{10}-x^9+\dots)(1-x)^{-3}$ {ignoring higher power}

No. of integer $\Rightarrow {}^{3+13-1}C_{13} - 2({}^{3+3-1}C_3) - ({}^{3+4-1}C_4)$

$\Rightarrow {}^{15}C_{13} - 2({}^5C_3) - {}^6C_4 \Rightarrow 105 - 20 - 15 \Rightarrow 70$

26. If $\left(\frac{1}{\alpha+1}+\frac{1}{\alpha+2}+\dots\dots+\frac{1}{\alpha+1012}\right)-\left(\frac{1}{2\cdot1}+\frac{1}{4\cdot3}+\frac{1}{6\cdot5}+\dots\dots+\frac{1}{2024\cdot2023}\right)=\frac{1}{2024}$, then α is equal to __________.

Sol. 1011

$$\left(\frac{1}{\alpha+1}+\frac{1}{\alpha+2}+\dots\dots+\frac{1}{\alpha+1012}\right)-\left(\frac{1}{2\cdot1}+\frac{1}{4\cdot3}+\frac{1}{6\cdot5}+\dots\dots+\frac{1}{2024\cdot2023}\right)=\frac{1}{2024}$$

↓

Let it be z

Considering,

$$Z = \frac{1}{2.1} + \frac{1}{4.3} + \frac{1}{6.5} + \dots \frac{1}{2024.2023}$$

$$Z = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots \frac{1}{2023} - \frac{1}{2024}$$

$$Z = \left(1+\frac{1}{3}+\frac{1}{5}+\dots\dots\frac{1}{2023}\right)-\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\dots\frac{1}{2024}\right)$$

$$Z = \left(1+\frac{1}{3}+\frac{1}{5}+\dots\frac{1}{2023}\right)-\frac{1}{2}\left(1+\frac{1}{2}+\frac{1}{3}+\dots\frac{1}{2012}\right)$$

Adding & subtracting $\left(\frac{1}{2}+\frac{1}{4}+\dots\frac{1}{2022}\right)$

$$Z = \left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\dots\frac{1}{2023}\right)-\frac{1}{2}\left(1+\frac{1}{2}+\frac{1}{3}+\dots\frac{1}{2012}\right)-\frac{1}{2}\left(1+\frac{1}{2}+\frac{1}{3}+\dots\frac{1}{2011}\right)$$

$$Z = \left(1+\frac{1}{2}+\frac{1}{3}+....\frac{1}{2023}\right)-\left(1+\frac{1}{2}+\frac{1}{3}+...\frac{1}{2011}\right)-\frac{1}{2024}$$

$$Z = \frac{1}{1012}+\frac{1}{1013}....\frac{1}{2023}-\frac{1}{2024}$$

Now, put z' in equation (1)

$$\left(\frac{1}{\alpha+1}+\frac{1}{\alpha+2}+...\frac{1}{\alpha+10/2}\right)-\left(\frac{1}{1012}+\frac{1}{1013}+....\frac{1}{2023}\right)+\frac{1}{2024}$$

Now, comparing above equation with equation (1)

$\alpha = 1011$

27. Consider the matrices : $A=\begin{bmatrix}2 & -5\\ 3 & m\end{bmatrix}, B=\begin{bmatrix}20\\ m\end{bmatrix}$ and $X=\begin{bmatrix}x\\ y\end{bmatrix}$. Let the set of all m, for which the system of equation AX = B has a negative solution (i.e., $x < 0$ and $y < 0$), be the interval (a, b). Then $8\int_a^b |A|\,dm$ is equal to ___.

Ans. **450**

Sol. $Ax = B$

$x = A^{-1}B$

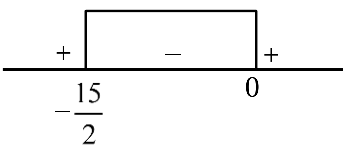
$$x = \frac{1}{2m+15}\begin{bmatrix}m & 5\\ -3 & 2\end{bmatrix}\begin{bmatrix}20\\ m\end{bmatrix}$$

$$x = \frac{1}{2m+15}\begin{bmatrix}25m\\ 2m-60\end{bmatrix}$$

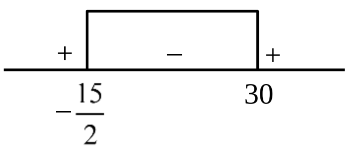
$$x = \frac{25m}{2m+15},\ y = \frac{2m-60}{2m+15}$$

Now, $x < 0$ and $y < 0$

$$\frac{25m}{2m+15} < 0 \text{ and } \frac{2(m-30)}{2m+15} < 0$$



and



$$M \in \left(-\frac{15}{2}, 0\right) \qquad M \in \left(-\frac{15}{2}, 30\right)$$

$\therefore\ m \in \left(-\frac{15}{2}, 0\right)$

$a = -\frac{15}{2}$, $b = 0$

Now, $8\int_a^b |A|.dm$

$8\int_{-\frac{15}{2}}^{0} (2m+15).dm$

$8\left[m^2 + 15m\right]_{-15/2}^{0}$

$-8\left[\frac{225}{4} - \frac{225}{2}\right] \Rightarrow -8 \times \left(-\frac{225}{4}\right) = 450$

28. Let the set of all values of p, for which $f(x) = (p^2 - 6p + 8)(\sin^2 2x - \cos^2 2x) + 2(2 - p)x + 7$ does not have any critical point, be the interval (a, b). Then 16 ab is equal to ___________.

Sol. **252**

$f(x) = (p^2 - 6p + 8)(\sin^2 2x - \cos^2 2x) + 2(2-p)x + 7$

$f(X) = (P^2 - 6P + 8)(-\cos 4x) + 2(2-p)x + 7$

$f'(x) = 4\sin 4x(p^2 - 6p + 8) + 2(2-p) \neq 0$

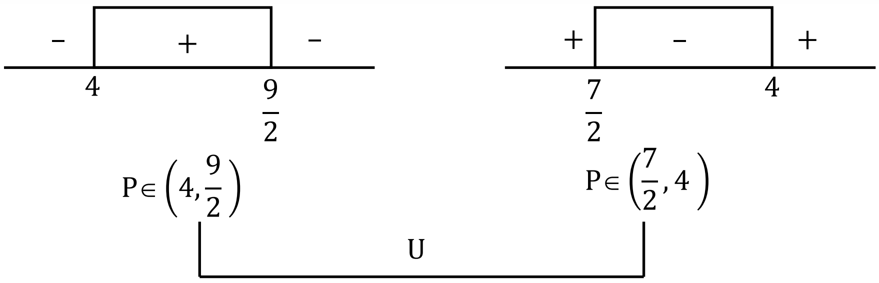
$\sin 4x \neq \frac{2(p-2)}{4(p-2)(p-4)}$

$\frac{(p-2)}{2(p-2)(p-4)} > 1 \text{or} \frac{p-2}{2(p-2)(p-4)} < -1 \qquad ,(p\neq 2)$

$\frac{1}{2p-8} - 1 > 0 \text{ or } \frac{1}{2p-8} + 1 < 0$

$\frac{1-2p+8}{2(p-4)} > 0 \text{ or } \frac{1+2p-8}{2(p-4)} < 0$

$\frac{a-2p}{2(p-4)} > 0 \text{ or } \frac{2p-7}{2(p-4)} < 0$



$P \in \left(\frac{7}{2}, \frac{9}{2}\right)$

$a = \frac{7}{2}, b = \frac{9}{2}$

$\therefore 16ab = 16 \times \frac{7}{2} \times \frac{9}{2} \Rightarrow 252$

29. Let the inverse trigonometric functions take principal values. The number of real solutions of the equation $2\sin^{-1} x + 3\cos^{-1} x = \frac{2\pi}{5}$, is __________.

Sol. **0**

$2\sin^{-1} x + 2\cos^{-1} x + \cos^{-1} x = \frac{2\pi}{5}$

$2\left(\frac{\pi}{2}\right) + \cos^{-1} x = \frac{2\pi}{5}$

$\cos^{-1} x = \frac{2\pi}{5} - \pi$

$\cos^{-1} x = \frac{-3\pi}{5}$

But $\cos^{-1} x \in [0, \pi]$

$\therefore$ No real value of x

No solution

30. For a differentiable function $f\ \mathbb{R} \to \mathbb{R}$, suppose $f'(x) = 3f(x) + \alpha$, where $\alpha \in \mathbb{R}$, $f(0) = 1$ and $\lim_{x \to -\infty} f(x) = 7$. Then $9f(-\log_e 3)$ is equal to ____.

Sol. **61**

$f'(x) = 3f(x) + \alpha$

$\frac{dy}{dx} = 3y + \alpha$

$$\frac{dy}{3y+\alpha}=dx$$

Integrating both the sides

$$\int\frac{dy}{3y+\alpha}=\int dx$$

$$\frac{1}{3}\ln|3y+\alpha|=x+c$$

$$\ln|3y+\alpha|=3x+3c$$

$$|3y+\alpha|=e^{3x+3c}$$

When $x\to-\infty \quad y=7$

$$3y+\alpha=0$$

$$\boxed{\alpha=-21}$$

$$\frac{1}{3}\ln|3y-21|=x+c$$

Now, when $x=0, y=1$

$$\frac{1}{3}\ln 18=c$$

$$\frac{1}{3}\ln|3y-21|=x+\frac{1}{3}\ln 18$$

$$\frac{1}{3}\ln\left(\frac{7-y}{6}\right)=x$$

Now, f(– ln 3)

$$\left[\ln\left(\frac{7-y}{6}\right)=-3\ln 3\right]$$

$$\left[\frac{7-y}{6}=\frac{1}{27}\right]$$

$$7-y=\frac{2}{9}$$

$$y=7-\frac{2}{9}\Rightarrow\frac{61}{9}$$

$$\therefore 9f(-\ln 3)=9\times\frac{61}{9}=61$$

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