JEE MAIN 2024 SESSION-2

Paper with Solution

MATHS | 04th April 2024 _ Shift-1

SECTION – A

1. One of the points of intersection of the curves $y = 1 + 3x - 2x^2$ and $y = \frac{1}{x}$ is $\left(\frac{1}{2}, 2\right)$. Let the area of the region enclosed by these curves be $\frac{1}{24}(\ell\sqrt{5} + m) - n\log_e(1+\sqrt{5})$, where ℓ, m, n $\in$ N. Then ℓ + m + n is equal to:

(1) 30 (2) 31 (3) 32 (4) 29

Sol. **(1)**

$1 = x + 3x^2 - 2x^3$

$2x^3 - 3x^2 - x + 1 = 0$

$(2x - 1)[x^2 - x - 1] = 0$

$x = \frac{1 \pm \sqrt{5}}{2} = \frac{1+\sqrt{5}}{2}$

$A = \int_{1/2}^{2}\left((1+3x-2x^2) - \frac{1}{x}\right)dx$

$= \left(x + \frac{3x^2}{2} - \frac{2x^3}{3} - \ell n\, x\right)_{1/2}^{\frac{1+\sqrt{5}}{2}}$

$= \left[\frac{1+\sqrt{5}}{2} + \frac{3}{2}\left[\frac{6+2\sqrt{5}}{4}\right] - \frac{2}{3}\frac{(6+2\sqrt{5})(1+\sqrt{5})}{8} - \ell n\left(\frac{1+\sqrt{5}}{2}\right)\right] - \left[\frac{1}{2} + \frac{3}{8} - \frac{1}{12} - \ell n\frac{1}{2}\right]$

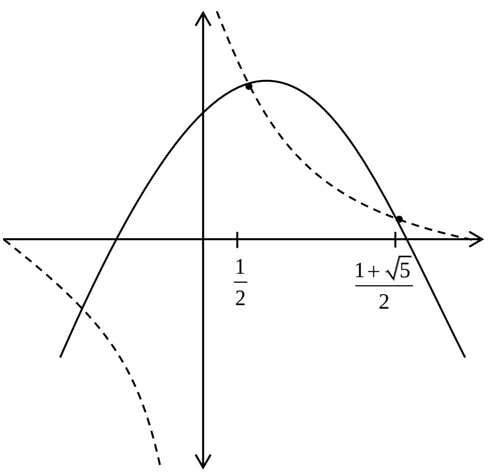
$= \left[\frac{1+\sqrt{5}}{2} + \frac{9+3\sqrt{5}}{4} - \frac{2}{3}\left(\frac{16+8\sqrt{5}}{8}\right) - \ell n(1+\sqrt{5}) + \ell n_2\right] - \left[\frac{12+9-2}{24} - \ell n_2\right]$

$= \left\{\frac{19+5\sqrt{5}}{4} - \frac{32}{24} - \frac{16\sqrt{5}}{24} - \ell n(1+\sqrt{5})\right\} - \frac{19}{24}$

$= \frac{66+30\sqrt{5}-32-16\sqrt{5}}{24} - \frac{19}{24} = \frac{15+14\sqrt{5}}{24} - \ell n(1+\sqrt{5})$

$\ell = 14$, m = 150, n = 1

ℓ + m + n = 30



2. The sum of all rational terms in the expansion of $\left(2^{\frac{1}{5}}+5^{\frac{1}{3}}\right)^{15}$ is equal to:

(1) 3133 (2) 633 (3) 6131 (4) 931

Sol. **(1)**

$= \left(2^{\frac{1}{5}}+5^{\frac{1}{3}}\right)^{15}$

$= T_{rH} = {}^{15}C_r (2)^{\frac{15-r}{5}} (5)^{\frac{r}{3}}$

r = 0 3, 6, 9, 12, 15 $\therefore$ r = 0 or 15

$T_1 = {}^{15}C_0 2^3$ $T_{16} = {}^{15}C_{15} 5^5$

Sum = 8 + 5^5 = 8 + 3125 = 3133

3. If the domain of the function $\sin^{-1}\left(\frac{3x-22}{2x-19}\right)+\log_e\left(\frac{3x^2-8x+5}{x^2-3x-10}\right)$ is (α, β], then 3α + 10β is equal to:

(1) 97 (2) 95 (3) 98 (4) 100

Sol. **(1)**

$y = \sin^{-1}\left(\frac{3x-22}{2x-19}\right)+\log_e\left(\frac{3x^2-8x+5}{x^2-3x-10}\right)$

(i) $-1 \le \frac{3x-22}{2x-19} \le 1$

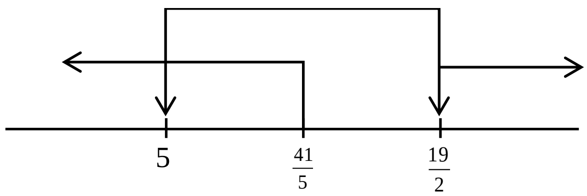
(a) $\frac{3x-22-2x+19}{2x-19} \le 0$

$= \frac{x-3}{2x-19} \le 0$

$\therefore x_{\in}\left[3, \frac{41}{5}\right]$

(b) $\frac{3x-22+2x+19}{2x-19} \ge 0$

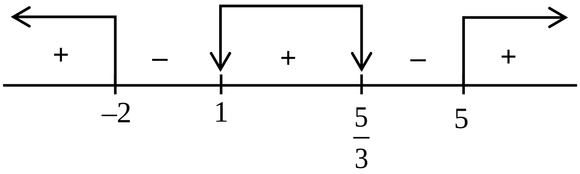
$= \frac{5x-41}{2x-19} \ge 0$



(ii) $\frac{3x^2-8x+5}{x^2-3x-10} > 0$

$= \frac{3x^2-3x-5x+5}{x^2-5x+2x-10} > 0$

$= \frac{(3x-5)(x-1)}{(x-5)(x+2)} > 0 = x_{\in}\left[5, \frac{41}{5}\right]$



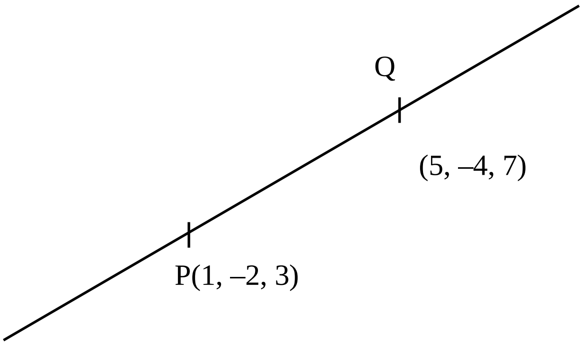
taking common of (i) and (ii)

$= \alpha = 5, \beta = \frac{41}{5} = 3\alpha + 10\beta = 15 + 82 = 97$

4. Let the point, on the line passing through the points P(1,-2,3) and Q(5,–4,7) farther from the origin and at a distance of 9 units from the point P be (α, β, γ). Then $\alpha^2 + \beta^2 + \gamma^2$ is equal to :

(1) 165 (2) 160 (3) 150 (4) 155

Sol. **(4)**



$= \frac{x-1}{4} = \frac{y+2}{-2} = \frac{z-3}{4} = \lambda$

Any point R $(4\lambda+1, -2\lambda -2, 4\lambda + 3)$

$PR = \sqrt{(4\lambda)^2 + (-2\lambda)^2 + (4\lambda)^2} = \sqrt{36\lambda^2} = 6|\lambda|$

$= 6|\lambda| = 9$

$\lambda = \frac{3}{2}, \frac{-3}{2}$

$\therefore$ P(7, –5, 9) or P(–5, 1, –3)

This is farther from (0, 0, 0)

$\therefore \alpha^2 + \beta^2 + \gamma^2 = 49 + 25 + 81 = 155$

5. Let $f : R \to R$ be a function given by

$$f(x) = \begin{cases} \frac{1-\cos 2x}{x^2}, & x < 0 \\ \alpha, & x = 0, \\ \frac{\beta\sqrt{1-\cos x}}{x}, & x > 0 \end{cases}$$

where $\alpha, \beta \in R$. If f is continous at x = 0, then $\alpha^2 + \beta^2$ is equal to:

(1) 48 (2) 6 (3) 3 (4) 12

Sol. **(4)**

$\lim_{x\to 0^-}\left(\frac{1-\cos 2x}{4x^2}\right)4 = \boxed{2 = \alpha}$

$= \lim_{x\to 0^+} \beta\sqrt{\frac{1-\cos 2x}{x^2}} = \frac{\beta}{\sqrt{2}} = 2 \Rightarrow \beta = 2\sqrt{2}$

$\alpha^2 + \beta^2 = 4 + 8 = 12$

6. Let the sum of the maximum and the minimum values of the function $f(x) = \frac{2x^2 - 3x + 8}{2x^2 + 3x + 8}$ be $\frac{m}{n}$, where gcd(m, n) = 1. Then m + n is equal to:

(1) 217 (2) 182 (3) 201 (4) 195

Sol. **(3)**

$y = \frac{2x^2 - 3x + 8}{2x^2 + 3x + 8}$

$= 2x^2(y - 1) + 3x(y + 1) + 8(y - 1) = 0$

$= D \geq 0 \qquad y \neq 1$

$= 9(y + 1)^2 - 64(y - 1)^2 \geq 0$

$= [3(y + 1) - 8(y - 1)]\,[3(y + 1) + 8(y - 1)] \geq 0$

$= (-5y + 11)(11y - 5) \geq 0$

$= (5y - 11)(11y - 5) \leq 0$

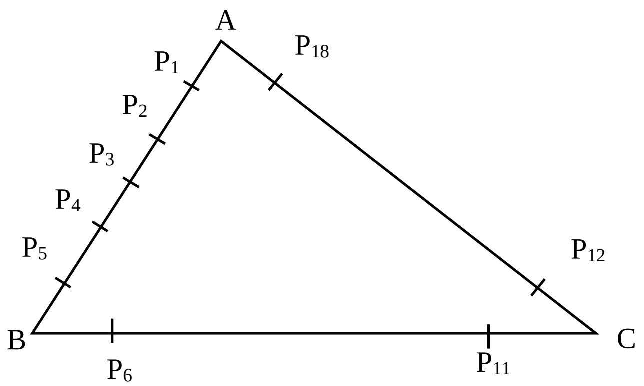
$= y = \left[\frac{5}{11}, \frac{11}{5}\right]$

$= m + n = \frac{25+121}{55} = \frac{146}{55} = m + n = 201$

7. There are 5 points P_1, P_2, P_3, P_4, P_5 on the side AB, excluding A and B, of a triangle ABC. Similarly there are 6 points $P_6, P_7, \ldots P_{11}$ on the side BC and 7 points $P_{12}, P_{13}, \ldots P_{18}$ on the side CA of the triangle. The number of triangles, that can be formed using the points $P_1, P_2, \ldots P_{18}$ as vertices, is:

(1) 751 (2) 771 (3) 776 (4) 796

Sol. **(1)**



No. of triangles = ${}^{18}C_3 - {}^{5}C_3 - {}^{6}C_3 - {}^{7}C_3$

$= 816 - 10 - 20 - 35 = 751$

8. If 2 and 6 are the roots of the equation $ax^2 + bx + 1 = 0$, then the quadratic equation, whose roots are $\frac{1}{2a+b}$ and $\frac{1}{6a+b}$, is:

(1) $x^2 + 8x + 12 = 0$ (2) $4x^2 + 14x + 12 = 0$

(3) $x^2 + 10x + 16 = 0$ (4) $2x^2 + 11x + 12 = 0$

Sol. **(1)**

$(x-2)(x-6) = x^2 - 8x + 12$

$\Rightarrow \frac{1}{12}x^2 - \frac{2}{3}x + 1 = 0$

$a = \frac{1}{12},\ b = \frac{-2}{3}$

$$\frac{1}{2a+b} = \frac{1}{\frac{1}{6}-\frac{2}{3}} = \frac{1}{\frac{1-4}{6}} = -2$$

$$\frac{1}{6a+b} = \frac{1}{\frac{1}{2}-\frac{2}{3}} = \frac{1}{\frac{3-4}{6}} = -6$$

$\therefore x^2 + 8x + 12 = 0$

9. Three urns A, B and C contain 7 red, 5 black; 5 red, 7 black and 6 red, 6 black balls, respectively. One of the urn is selected at random and a ball is drawn from it. If the ball drawn is black, then the probability that it is drawn from urn A is :

(1) $\frac{7}{18}$ (2) $\frac{5}{16}$ (3) $\frac{4}{17}$ (4) $\frac{5}{18}$

Sol. **(4)**

A	B	C
7R, 5B	5R, 7B	6R, 6B

$$P\left(\frac{A}{B}\right) = \frac{\frac{5}{12}}{\frac{5}{12}+\frac{7}{12}+\frac{6}{12}} = \frac{5}{18}$$

10. Let α and β be the sum and the product of all the non-zero solutions of the equation $(\bar{z})^2 + |z| = 0$, $z \in C$. Then $4(\alpha^2 + \beta^2)$ is equal to:

(1) 2 (2) 4 (3) 8 (4) 6

Sol. **(2)**

$= (\bar{z})^2 + |z| = 0$

$= (x - iy)^2 + \sqrt{x^2 + y^2} = 0$

$= x^2 - y^2 - 2xyi + \sqrt{x^2 + y^2} = 0$

(1) $2xy = 0$

$\therefore x = 0$ or $y = 0$

(2) $x^2 - y^2 + \sqrt{x^2 + y^2} = 0$

$\rightarrow$ if $x = 0$ $\quad -y^2 + |y| = 0$ $\rightarrow y = 1$, $\rightarrow y = -1$

$\rightarrow$ if $y = 0$ $\quad x^2 + |x| = 0$

$\rightarrow$ if $x = 0,\ y = 0$ $\qquad \therefore z = i, z = -i, z = 0$

$\Rightarrow \alpha = 0, \beta = 1$

$4(\alpha^2 + \beta^2) = 4$

11. Let $\alpha, \beta \in R$. Let the mean and the variance of 6 observations –3,4,7,–6, α, β be 2 and 23, respectively. The mean deviation about the mean of these 6 observations is :

(1) $\frac{14}{3}$ (2) $\frac{13}{3}$ (3) $\frac{16}{3}$ (4) $\frac{11}{3}$

Sol. **(2)**

Mean $\Rightarrow \alpha + \beta = 10$

$$\text{Var}(\sigma^2) = \frac{\sum x_i^2}{n} - (\bar{x})^2 = 23$$

$\therefore \alpha^2 + \beta^2 = 52$

Solve and get

$\alpha = 4, \beta = 6$

$$\therefore \text{MD}(\bar{x}) = \frac{13}{3}$$

12. If the solution $y = y(x)$ of the differential equation $(x^4 + 2x^3 + 3x^2 + 2x + 2)dy - (2x^2 + 2x + 3)dx = 0$ satisfies $y(-1) = -\frac{\pi}{4}$, then $y(0)$ is equal to:

(1) $-\frac{\pi}{12}$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{4}$ (4) 0

Sol. **(3)**

$$\int dy = \int \frac{(2x^2 + 2x + 3)}{(x^2 + 1)(x^2 + 2x + 2)} dx$$

$$y = \int \frac{dx}{x^2 + 2x + 2} + \int \frac{dx}{1 + x^2}$$

$$y = \tan^{-1}(x + 1) + \tan^{-1} x + c$$

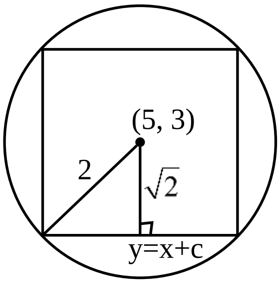
$y(-1) = -\frac{\pi}{4} \Rightarrow c = 0$

$y(0) = \frac{\pi}{4}$

13. A square is inscribed in the circle $x^2 + y^2 - 10x - 6y + 30 = 0$. One side of this square is parallel to $y = x + 3$. If (x_i, y_i) are the vertices of the square, then $\Sigma\left(x_i^2 + y_i^2\right)$ is equal to:

(1) 152 (2) 156 (3) 148 (4) 160

Sol. **(1)**



$\perp^{rl}$ from (5,3) to $x - y + c = 0$ is $\sqrt{2}$

$\therefore$ C = 0 or C = –4.

2 Lines are y = x and y = x – 4.

Solve with circle.

y = x	y = x – 4
(i) $x^2 - 8x + 15 = 0$	(ii) $x^2 - 12x + 35 = 0$
x = 3, y = 3	x = 5, y = 1
x = 5, y = 5	x = 7, y = 3

$\therefore \sum x_i^2 + y_i^2 = 152$

14. Let $f(x) = x^5 + 2e^{x/4}$ for all $x \in R$. Consider a function g(x) such that (gof)(x) = x for all $x \in R$. Then the value of 8g'(2) is:

(1) 2 (2) 8 (3) 4 (4) 16

Sol. **(4)**

$f(x) = x^5 + 2e^{x/4}$

g is inverse of f.

$$g'(y) = \frac{1}{f'(x)} = \frac{1}{5x^4 + \frac{1}{2}e^{x/4}}$$

y = 2 at x = 0 $\quad \therefore \; g'(2) = \frac{1}{\frac{1}{2}} = 2 \Rightarrow 8 \times g'(2) = 16$

15. Let $\alpha \in (0, \infty)$ and $A = \begin{bmatrix} 1 & 2 & \alpha \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$. If $\det\left(\text{adj}\left(2A - A^T\right)\cdot \text{adj}\left(A - 2A^T\right)\right) = 2^8$, then $(\det(A))^2$ is equal to:

(1) 1 (2) 36 (3) 49 (4) 16

Sol. **(4)**

$P = A - 2A^T$, $Q = 2A - A^T$

$\therefore Q^T = 2A^T - A = -P$

$|Q^T| = |-P|$

$\Rightarrow |Q| = -|P|$

Now $|\text{adj } P.\ \text{adj}Q|$

$\Rightarrow |\text{adj } P|.\ |\text{adj}Q|$

$\Rightarrow |P|^2.\ |Q|^2$

$= |P|^4 = 2^8$

$\therefore |P| = 4, |Q| = -4$

$$A - 2A^T = \begin{bmatrix} -1 & 0 & \alpha \\ -3 & 0 & -1 \\ -2\alpha & -1 & -2 \end{bmatrix}$$

$|A - 2A^T| = 1 + 3\alpha = 4$

$\alpha = 1$

$\therefore |A| = -4$

$|A|^2 = 16$

16. If the system of equations

$$x + (\sqrt{2}\sin\alpha)y + (\sqrt{2}\cos\alpha)z = 0$$
$$x + (\cos\alpha)y + (\sin\alpha)z = 0$$
$$x + (\sin\alpha)y - (\cos\alpha)z = 0$$

has a non-trivial solution, then $\alpha \in \left(0, \frac{\pi}{2}\right)$ is equal to:

(1) $\frac{7\pi}{24}$ (2) $\frac{3\pi}{4}$ (3) $\frac{11\pi}{24}$ (4) $\frac{5\pi}{24}$

Sol. **(4)**

For non-trivial Solution

$$\begin{vmatrix} 1 & \sqrt{2}\sin\alpha & \sqrt{2}\cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ 1 & \sin\alpha & -\cos\alpha \end{vmatrix} = 0$$

On solving

$$\sqrt{2}\cos 2\alpha - \sqrt{2}\sin 2\alpha = -1$$

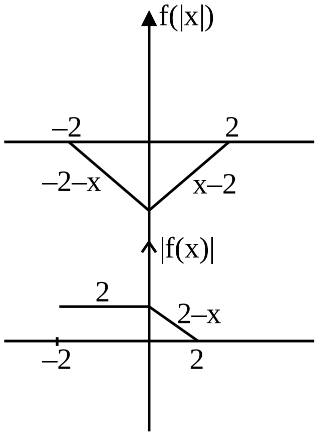
$\cos\left(2\alpha+\frac{\pi}{4}\right)=\frac{-1}{2}$

$\alpha=\frac{5\pi}{24}$.

17. Let $f(x)=\begin{cases}-2, & -2\le x\le 0\\ x-2, & 0<x\le 2\end{cases}$ and $h(x)=f(|x|)+|f(x)|$. Then $\int_{-2}^{2}h(x)dx$ is equal to:

(1) 6 (2) 4 (3) 1 (4) 2

Sol. **(4)**



$\therefore h(x)=\begin{cases}-x & x\in[-2,0]\\ 0 & x\in[0,2]\end{cases}$

$\int_{-2}^{2}h(x)dx=\int_{-2}^{0}(-x)dx$

$=\left(-\frac{x^2}{2}\right)_{-2}^{0}$

$= -(-2) = 2$

18. Let a unit vector which makes an angle of 60° with $2\hat{i}+2\hat{j}-\hat{k}$ and an angle of 45° with $\hat{i}-\hat{k}$ be $\vec{C}$. Then $\vec{C}+\left(-\frac{1}{2}\hat{i}+\frac{1}{3\sqrt{2}}\hat{j}-\frac{\sqrt{2}}{3}\hat{k}\right)$ is:

(1) $\frac{\sqrt{2}}{3}\hat{i}-\frac{1}{2}\hat{k}$

(2) $-\frac{\sqrt{2}}{3}\hat{i}+\frac{\sqrt{2}}{3}\hat{j}+\left(\frac{1}{2}+\frac{2\sqrt{2}}{3}\right)\hat{k}$

(3) $\left(\frac{1}{\sqrt{3}}+\frac{1}{2}\right)\hat{i}+\left(\frac{1}{\sqrt{3}}-\frac{1}{3\sqrt{2}}\right)\hat{j}+\left(\frac{1}{\sqrt{3}}+\frac{\sqrt{2}}{3}\right)\hat{k}$

(4) $\frac{\sqrt{2}}{3}\hat{i}+\frac{1}{3\sqrt{2}}\hat{j}-\frac{1}{2}\hat{k}$

Sol. **(1)**

Let $\vec{c}=p\hat{i}+q\hat{j}+rk$

$p^2+q^2+r^2=1$ ...(1)

$p-r=1$ ...(2)

$4p + 4q - 2r = 3$ ...(3)

$4(1 + r) + 4q - 2r = 3$

$2r + 4q = -1$

$4q = -(2r + 1)$

$q = -\frac{(2r+1)}{4}$

$1+r^2+2r+\frac{4r^2+1+4r}{16}+r^2=1$

$\Rightarrow 36r^2+17+36r=16$

$\Rightarrow \quad 36r^2+36r+1=0$

$\Rightarrow r\frac{-36\pm\sqrt{1296-144}}{2\times 36}$

$\Rightarrow r=\frac{-36\pm 24\sqrt{2}}{2\times 36}$

$\Rightarrow r=\frac{-6\pm 4\sqrt{2}}{2\times 6}$

$\Rightarrow r=-\frac{1}{2}\pm\frac{\sqrt{2}}{3}$

$r=-\frac{1}{2}+\frac{\sqrt{2}}{3}$

$p=\frac{1}{2}+\frac{\sqrt{2}}{3}$

$q=-\frac{1}{4}\left(\frac{2\sqrt{2}}{3}\right)$

$q=\frac{-1}{3\sqrt{2}}$

$\vec{c}=\left(\frac{1}{2}+\frac{\sqrt{2}}{3}\right)\hat{i}-\frac{1}{3\sqrt{2}}\hat{j}+\left(-\frac{1}{2}+\frac{\sqrt{2}}{5}\right)k$

$\frac{\sqrt{2}}{3}\hat{i}+0\hat{j}-\frac{1}{2}\hat{k}$

19. Let the first three terms 2, p and q, with $q\neq 2$, of a G.P. be respectively the 7^{th}, 8^{th} and 13^{th} terms of an A.P. If the 5^{th} term of the G.P. is the n^{th} term of the A.P., then n is equal to:

(1) 169 (2) 163 (3) 177 (4) 151

Sol. **(2)**

$2,\ p = 2r,\ q = 2r^2$

$a + 6d = 2$
$a + 7d = 2r$ ⟹ $d = 2(r - 1)$
$a + 12d = 2r^2$ ⟹ $5d = 2r(r - 1)$

$\frac{1}{5} = \frac{1}{r}$

r = 5, ∴ d = 8

a = –46

p = 10, q = 50

$T_5 = 2[r]^4 = 2\left[5^4\right] = 1250.$ = a + (n – 1)8

= 1250 = –46 + (n – 1)8

n = 163

20. The vertices of a triangle are A(–1,3), B(–2, 2) and C(3, –1). A new triangle is formed by shifting the sides of the triangle by one unit inwards. Then the equation of the side of the new triangle nearest to origin is :

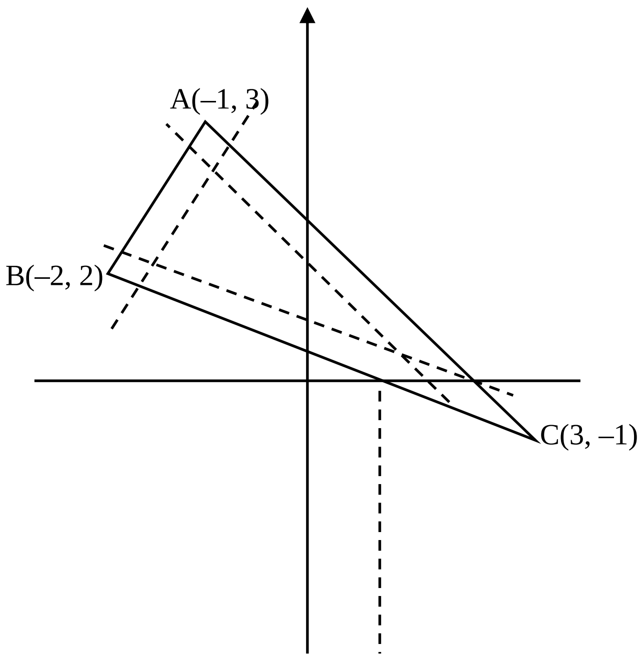
(1) $x - y - (2 + \sqrt{2}) = 0$

(2) $x + y - (2 - \sqrt{2}) = 0$

(3) $-x + y - (2 - \sqrt{2}) = 0$

(4) $x + y + (2 - \sqrt{2}) = 0$

Sol. **(2)**



$AB : y - 2 = \frac{1}{1}(x + 2)$

x – y – 4 = 0. x – y + c = 0

$\left|\frac{c + 4}{\sqrt{2}}\right| = 1$

$C = \sqrt{2} - 4, -\sqrt{2} - 4$

$BC :\ y + 1 = \frac{3}{-5}(x - 3)$

– 5y – 5 = 3x – 9

3x + 5y = 4

$\left|\frac{c+4}{\sqrt{34}}\right| = 1$

AC : y – 3 = –1 (x + 1)

x + y – 2

$\left|\frac{c+2}{\sqrt{2}}\right| = 1$

C = $\sqrt{2}-2, -\sqrt{2}-2$

Nearest : x + y – 2 + $\sqrt{2}$ = 0

21. Let A be a square matrix of order 2 such that |A| = 2 and the sum of its diagonal elements is –3. If the points (x, y) satisfying $A^2 + xA + yI = O$ lie on a hyperbola, whose transverse axis is parallel to the x-axis, eccentricity is e and the length of the latus rectum is ℓ, then $e^4 + \ell^4$ is equal to____.;

Sol. **Bonus**

22. If the shortest distance between the lines $\frac{x+2}{2}=\frac{y+3}{3}=\frac{z-5}{4}$ and $\frac{x-3}{1}=\frac{y-2}{-3}=\frac{z+4}{2}$ is $\frac{38}{3\sqrt{5}}$ k, and $\int_0^k \left[x^2\right]dx = \alpha-\sqrt{\alpha}$, where [x] denotes the greatest integer function, then $6\alpha^3$ is equal to____.

Sol. **48**

$$\vec{p}\times\vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 1 & -3 & 2 \end{vmatrix}$$

$= \hat{i}(18) - \hat{j}(0) + \hat{k}(-9)$

$= 18\hat{i} - 9\hat{k}$

= (2, 0, –1) = dr's

A(–2, –3, 5) B(3, 2, –4)

$$S_d = \left|\frac{(5,5,-9)\cdot(2,0,-1)}{\sqrt{5}}\right| = \frac{10+9}{\sqrt{5}} = \frac{19}{\sqrt{5}} = \frac{38}{3\sqrt{5}}\cdot K$$

$K = \frac{3}{2}$

$$\therefore \int_0^{\frac{3}{2}}\left[x^2\right]dx = \int_0^1 0dx + \int_1^{\sqrt{2}} 1dx + \int_{\sqrt{2}}^{\frac{3}{2}} 2dx$$

$= \sqrt{2} - 1 + 2\left(\frac{3}{2} - \sqrt{2}\right)$

$= \sqrt{2} - 1 + 3 - 2\sqrt{2}$

$= 2 - \sqrt{2}$ $\quad\quad \therefore 6\alpha^3 = 48$

23. If $\lim_{x\to 1}\frac{(5x+1)^{1/3}-(x+5)^{1/3}}{(2x+3)^{1/2}-(x+4)^{1/2}} = \frac{m\sqrt{5}}{n(2n)^{2/3}}$, where gcd(m, n) = 1, then 8m + 12n is equal to

Sol. **100**

$$\lim_{x\to 1}\frac{(5x+1)^{\frac{1}{3}}-(x+5)^{\frac{1}{3}}}{(2x+3)^{\frac{1}{2}}-(x+4)^{\frac{1}{2}}}=\frac{m\sqrt{3}}{n(2n)^{2/3}}$$

$$\lim_{x\to 1}\frac{\frac{1}{3}(5x+1)^{-\frac{2}{3}}\cdot 5-\frac{1}{3}(x+5)^{-\frac{2}{3}}}{\frac{1}{2}(2x+3)^{-\frac{1}{2}}\cdot 2-\frac{1}{2}(x+4)^{-\frac{1}{2}}}$$

$$=\frac{\frac{5}{3}(6)^{-\frac{2}{3}}-\frac{1}{3}(6)^{-\frac{2}{3}}}{(5)^{-\frac{1}{2}}-\frac{1}{2}(5)^{-\frac{1}{2}}}$$

$$=\frac{(6)^{-\frac{2}{3}}\left(\frac{4}{3}\right)}{\frac{1}{2}(5)^{-\frac{1}{2}}}$$

$$=\frac{8}{3}\cdot\frac{(5)^{\frac{1}{2}}}{(2\cdot 3)^{2/3}}$$

$$=\frac{8\sqrt{5}}{(3)(2\cdot 3)^{\frac{2}{3}}}=\frac{m\sqrt{5}}{n(2n)^{\frac{2}{3}}}$$

$\Rightarrow m = 8, n = 3$

$8m + 12n = 8 \times 8 + 12 \times 3$

$= 64 + 36 = 100$

24. If $\int_0^{\frac{\pi}{4}}\frac{\sin^2 x}{1+\sin x\cos x}dx=\frac{1}{a}\log_e\left(\frac{a}{3}\right)+\frac{\pi}{b\sqrt{3}}$, where a, b $\in$ N, then a + b is equal to _____.

Sol. **8**

$$I=\int_0^{\frac{\pi}{4}}\frac{\sin^2 x}{1+\sin x\cos x}dx \qquad \text{divide by } \cos^2 x$$

$$I=\int_0^{\frac{\pi}{4}}\frac{\tan^2 x}{\sec^2 x+\tan x}dx \qquad ;\ \tan x = t$$

$;\ \sec^2 x\,dx = dt$

$$I=\int_0^{1}\frac{t^2}{\left(t^2+t+1\right)}\frac{dt}{\left(1+t^2\right)}$$

$$I=\frac{1}{2}\int_0^1\frac{\left(t^2+1\right)+\left(t^2-1\right)}{(t^2+1)\left(t^2+1\right)}dt$$

$$1=\frac{1}{2}\int_0^1\left(\frac{1}{t^2+t+1}+\frac{t^2-1}{(t^2+1)(t+t+1)}\right)dt$$

$$I=\frac{1}{2}\int_0^1\frac{1}{t^2+t+1}dt+\frac{1}{2}\int_0^1\frac{\left(1-\frac{1}{t^2}\right)}{\left(t+\frac{1}{t}\right)\left(t+\frac{1}{t}+1\right)}dt$$

$\downarrow$ I_1 $\qquad$ $\downarrow$ I_2

$$I_1=\int_0^1\frac{1}{t^2+t+1}dt$$

$$I_1=\int_0^1\frac{1}{\left(t+\frac{1}{2}\right)^2+\frac{3}{4}}dt=\int_0^1\frac{1}{\left(t+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}dt$$

$$I_1=\left[\frac{2\sqrt{3}}{3}\tan^{-1}\left(\frac{2t+1}{\sqrt{3}}\right)\right]_0^1$$

$$I_1=\frac{2}{\sqrt{3}}\left[\frac{\pi}{3}-\frac{\pi}{6}\right]=\frac{\pi}{3\sqrt{3}}$$

$$I_2=\int_\infty^2\frac{du}{u(u+1)}=[\log u-\log\left(u+1\right)]_\infty^2$$

$$=\log\left(\frac{u}{u+1}\right)\Big]_\infty^2$$

$$I_2=\log\left(\frac{2}{3}\right)$$

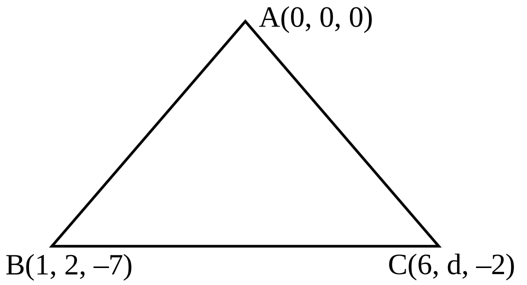
$$I=\frac{1}{2}\cdot\frac{\pi}{3\sqrt{3}}+\frac{1}{2}\log\frac{2}{3}$$

a = 2 ; b = 6

a + b = 8

25. Let ABC be a triangle of area $15\sqrt{2}$ and the vectors $\overrightarrow{AB}=\hat{i}+2\hat{j}-7\hat{k},\overrightarrow{BC}=a\hat{i}+b\hat{j}+c\hat{k}$ and $\overrightarrow{AC}=6\hat{i}+d\hat{j}-2\hat{k},d>0$. Then the square of the length of the largest side of the triangle ABC is___.

Sol. **54**



$$\frac{1}{2}\begin{vmatrix} i & j & k \\ 1 & 2 & -7 \\ 6 & d & -2 \end{vmatrix} = \pm 30\sqrt{2}$$

$$\hat{i}(-4+7d) - \hat{j}(-2+42) + \hat{k}(d-12) = \pm 30\sqrt{2}$$

$$\Rightarrow (7d-4)^2 + 40^2 + (d-12)^2 = 1800$$

$$49d^2 + 16 - 56d + 1600 + d^2 + 144 - 24d = 1800$$

$$50d^2 - 80d - 40 = 0$$

$$5d^2 - 8d - 4 = 0$$

$$5d^2 - 10d + 2d - 4 = 0$$

$$(5d+2)(d-2) = 0$$

$$d = 2$$

$$AB = \sqrt{54}$$

$$AC = \sqrt{36+4+4} = \sqrt{44}$$

$$BC = \sqrt{25+0+25}$$

26. Let $a = 1 + \frac{^2C_2}{3!} + \frac{^3C_2}{4!} + \frac{^4C_2}{5!} + \ldots$, $b = 1 + \frac{^1C_0 + ^1C_1}{1!} + \frac{^2C_0 + ^2C_1 + ^2C_2}{2!} + \frac{^3C_0 + ^3C_1 + ^3C_2 + ^3C_3}{3!} + \ldots$ Then $\frac{2b}{a^2}$ is equal to____.

Sol. **8**

$$a = 1 + \frac{2c_2}{\lfloor 3} + \frac{3c_2}{\lfloor 4} + \frac{4c_2}{\lfloor 5} + \cdots$$

$$a = 1 + \sum_{r=2}^{\infty} \frac{^rC_2}{(r+1)}$$

$$a = 1 + \sum_{r=2}^{\infty} \frac{r(r-1)}{2\lfloor r+1}$$

$$a = 1 + \frac{1}{2}\sum_{r=2}^{\infty}\left[\frac{r(r+1)-2r}{\lfloor (r+1)}\right.$$

$$a = 1 + \frac{1}{2}\left[\sum_{r=2}^{\infty}\frac{1}{\lfloor r-1} - \frac{2}{\lfloor r} + \frac{2}{\lfloor r+1}\right]$$

$$a = 1 + \frac{1}{2}\left[(e-1) - 2(e-2) + 2\left(e - 2 - \frac{1}{2}\right)\right]$$

$$a = 1 + \frac{1}{2}[e-2] = \frac{e}{2}$$

$$b = 1 + \frac{{}^1C_0 + {}^1C_1}{1!} + \frac{{}^2C_0 + {}^2C_1 + {}^2C_2}{2!} + \frac{{}^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3}{3!} + \dots$$

$b = e^2$

Now, $= \frac{2b}{a^2} = \frac{2 \times e^2}{e^2} \times 4 = 8$

27. Let A be 3×3 matrix of non-negative real elements such that $A\begin{bmatrix}1\\1\\1\end{bmatrix} = 3\begin{bmatrix}1\\1\\1\end{bmatrix}$. Then the maximum value of det(A) is ____.

Sol. **27**

A.B = 3B

(A – 3I). B = 0

Now |A – 3I| = 0

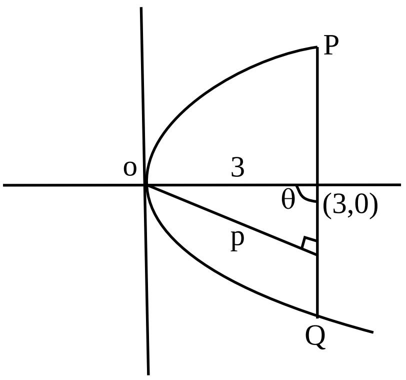
for maximum value of |A|

|A| = |3I|

$\therefore\ |A|_{max} = 27$

28. Let the length of the focal chord PQ of the parabola $y^2 = 12x$ be 15 units. If the distance of PQ from the origin is p, then $10\,p^2$ is equal to___.

Sol. **72**



|PQ| = 15

$4\,a\,\text{cosec}^2\theta = 15$

$\text{cosec}^2\theta = \frac{15}{12}$ ...(i)

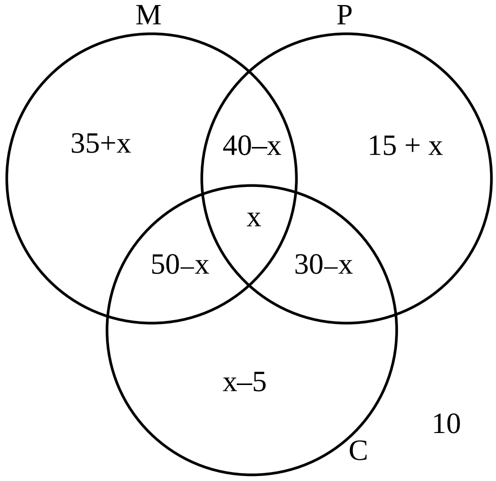
$p = 3\sin\theta$

$10p^2 = 9 \times \frac{1}{\text{cosec}^2\theta} \times 10$

$10p^2 = 9 \times \frac{12}{15} \times 10 = 72$

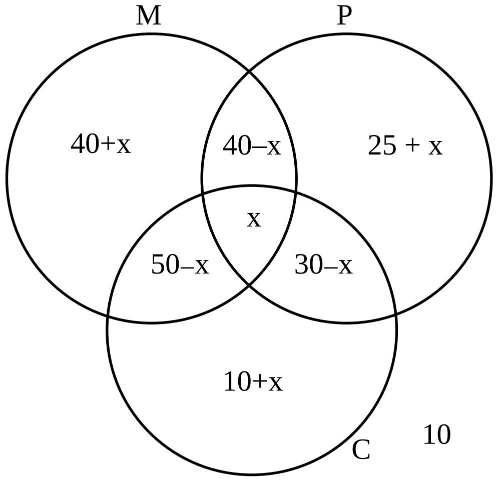
29. In a survey of 220 students of a higher secondary school, it was found that at least 125 and at most 130 students studied Mathematics; at least 85 and at most 95 studied Physics; at least 75 and at most 90 studied Chemistry; 30 studied both Physics and Chemistry; 50 studied both Chemistry and Mathematics; 40 studied both Mathematics and Physics and 10 studied none of these subjects. Let m and n respectively be the least and the most number of students who studied all the three subjects. Then m + n is equal to____.

Sol. **45**



$165 + x = 210$

$x = 45 \Rightarrow x = 30 = m$



$205 + x = 220$

$x = 15 = n$

$\Rightarrow m + n = 45$

30. Let the solution y = y(x) of the differential equation $\frac{dy}{dx} - y = 1 + 4\sin x$ satisfy y(π) = 1. Then $y\left(\frac{\pi}{2}\right) + 10$ is equal to ____.

Sol. **7**

$\frac{dy}{dx} - y = 1 + 4\sin x$

I.F. = $e^{\int -1dx} = e^{-x}$

$ye^{-x} = \int (e^{-x})(1 + 4\sin x)dx$

$= y \cdot e^{-x} = e^{-x} + 4\int e^{-x}\sin x dx + C$

$\int e^{ax}\sin bx\, dx = \frac{e^{ax}}{a^2 + b^2}(a\sin bx - b\cos bx)$

$\Rightarrow ye^{-x} = -e^{-x} - \frac{4}{2}e^{-x}(\sin x + \cos x) + C$

$\because y(\pi) = 1$

$\Rightarrow e^{-\pi} = -e^{-\pi} + 2e^{-\pi} + C$

$\Rightarrow C = 0$

$\Rightarrow y\, e^{-x} = -e^{-x} - 2\, e^{-x}(\sin x + \cos x)$

$\Rightarrow y = -1 - 2(\sin x + \cos x)$

$\Rightarrow y\left(\frac{\pi}{2}\right) + 10 = -1 - 2(1) + 10 = 7$

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