JEE MAIN 2024 SESSION-2
Paper with Solution

Maths | 05th April 2024 _ Shift-2

SECTION – A

1. The values of m, n, for which the system of equations

$x + y + z = 4,$

$2x + 5y + 5z = 17,$

$x + 2y + mz = n$

has infinitely many solutions, satisfy the equation :

(1) $m^2 + n^2 + mn = 68$ (2) $m^2 + n^2 - m - n = 46$

(3) $m^2 + n^2 + m + n = 64$ (4) $m^2 + n^2 - mn = 39$

Sol. **4**

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & 5 \\ 1 & 2 & m \end{vmatrix} = 0 \Rightarrow m = 2$$

$\Rightarrow m^2 + n^2 - mn = 39$

$$D_2 = \begin{vmatrix} 1 & 4 & 1 \\ 2 & 17 & 5 \\ 1 & n & 2 \end{vmatrix} = 0 \Rightarrow n = 7$$

2. The differential equation of the family of circles passing through the origin and having centre at the line y = x is :

(1) $(x^2 + y^2 + 2xy)\,dx = (x^2 + y^2 - 2xy)\,dy$

(2) $(x^2 + y^2 - 2xy)\,dx = (x^2 + y^2 + 2xy)\,dy$

(3) $(x^2 - y^2 + 2xy)\,dx = (x^2 - y^2 + 2xy)dy$

(4) $(x^2 - y^2 + 2xy)dx = (x^2 - y^2 - 2xy)dy$

Sol. **4**

Equation of circle $(x - \alpha)^2 + (y - \alpha)^2 = 2\alpha^2$

$\Rightarrow 2(x - \alpha) + 2(y - \alpha)\,y' = 0$

$$\Rightarrow \alpha = \frac{x + yy'}{1 + y'}$$

$\Rightarrow (x^2 - y^2 - 2xy)y' + (-x^2 + y^2 + 2xy) = 0$

$\Rightarrow (x^2 - y^2 + 2xy)dx = (x^2 - y^2 - 2xy)dy$

3. For $x \geq 0$, the least value of K, for which $4^{1+x}+4^{1-x}, \frac{K}{2}, 16^{x}+16^{-x}$ are three consecutive terms of an A.P. is equal to :

(1) 10 (2) 4 (3) 16 (4) 8

Sol. **1**

$$K=4\left(4^{x}+\frac{1}{4^{x}}\right)+\left(16^{x}+\frac{1}{16^{x}}\right)$$

$K \geq 10 \Rightarrow K_{min} = 10$

4. Let $\beta(m,n)=\int_0^1 x^{m-1}(1-x)^{n-1}\,dx,\ m,n>0$. If $\int_0^1\left(1-x^{10}\right)^{20}dx = a\times\beta(b,c)$, then $100(a+b+c)$ equals ________ .

(1) 1120 (2) 1021 (3) 2120 (4) 2012

Sol. **3**

$$I=\int_0^1(1-x^{10})^{20}dx$$

$$x^{10}=t\Rightarrow x=(t)^{1/10}$$

$$I=\frac{1}{10}\int_0^1(t)^{-9/10}(1-t)^{20}dt$$

$$a=\frac{1}{10},b=\frac{1}{10},c=21$$

$$100\left(\frac{1}{10}+\frac{1}{10}+21\right)$$

$\Rightarrow 10 + 10 + 2100 = 2120$

5. 60 words can be made using all the letters of the word BHBJO, with or without meaning. If these words are written as in a dictionary, then the 50^{th} word is :

(1) OBBJH (2) JBBOH (3) HBBJO (4) OBBHJ

Sol. **1**

B …….. = 4!, H……… $=\frac{4!}{2!}$, J……… $=\frac{4!}{2!}$

O, BBHJ, $\frac{\boxed{\text{OBBJH}}}{50^{th}}$

6. If $y(\theta)=\dfrac{2\cos\theta+\cos 2\theta}{\cos 3\theta+4\cos 2\theta+5\cos\theta+2}$, then at $\theta=\dfrac{\pi}{2}, y''+y'+y$ is equal to :

(1) $\dfrac{1}{2}$ (2) 2 (3) 1 (4) $\dfrac{3}{2}$

Sol. 2

$$y(\theta)=\frac{2\cos\theta+\cos 2\theta}{\cos 3\theta+4\cos 2\theta+5\cos\theta+2}=\frac{2\cos^2\theta+2\cos\theta-1}{4\cos^3\theta+8\cos^2\theta+2\cos\theta-2}=\frac{1}{2(1+\cos\theta)}=\frac{\sec^2\frac{\theta}{2}}{4}$$

$$y''=\frac{1}{4}\left(\sec^2\frac{\theta}{2}\tan^2\theta+\frac{\sec^4\frac{\theta}{2}}{2}\right),\ y'=\frac{\sec\frac{\theta}{2}\left(\sec\frac{\theta}{2}\tan\frac{\theta}{2}\right)}{4}$$

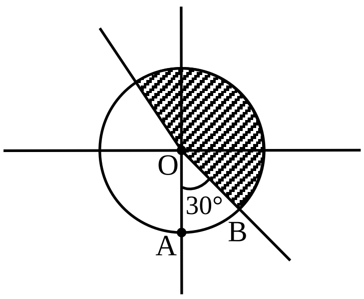
$\Rightarrow y''+y'+y=2$

7. Let $S_1=\{z\in\mathbf{C}:|z|\le 5\}, S_2=\left\{z\in\mathbf{C}:\text{Im}\left(\dfrac{z+1-\sqrt{3}i}{1-\sqrt{3}i}\right)\ge 0\right\}$ and $S_3=\{z\in\mathbf{C}:\text{Re}(z)\ge 0\}$. Then the area of the region $S_1\cap S_2\cap S_3$ is :

(1) $\dfrac{125\pi}{12}$ (2) $\dfrac{125\pi}{24}$ (3) $\dfrac{125\pi}{4}$ (4) 125π

Sol. 1

$S_1\rightarrow$ area inside circle with radius 5



$$S_2\quad I_m\left(\frac{(x+1)+i(y-\sqrt{3})}{1-\sqrt{3}i}\times\frac{1+\sqrt{3}i}{1+\sqrt{3}i}\right)$$

$$\Rightarrow I_m\left(\frac{\left[(x+1)+i(y-\sqrt{3})\right]\left(1+\sqrt{3}i\right)}{1+3}\right)$$

$$\Rightarrow S_2=\sqrt{3}x+\sqrt{3}+y-\sqrt{3}\ge 0$$

$\sqrt{3}x + y \geq 0$

$S_3 = x \geq 0$

Area of half circle – area of arc AB

$\frac{25\pi}{2} - \frac{1}{2} \times 25 \times \frac{\pi}{6}$

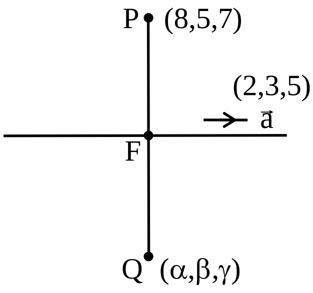
$\frac{25\pi}{2} - \frac{25\pi}{2}$

$\frac{125\pi}{12}$

8. Let (α, β, γ) be the image of the point (8,5,7) in the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{5}$. Then $\alpha + \beta + \gamma$ is equal to:

(1) 20 (2) 14 (3) 18 (4) 16

Sol. **2**



Coordinate of F <2λ + 1, 3λ – 1, 5λ + 2>

$\vec{PF} = <2\lambda - 7, 3\lambda - 6, 5\lambda - 5>$

$\vec{PF} \cdot \vec{a} = 0 \Rightarrow \lambda = \frac{3}{2}$

$\frac{\alpha+8}{2} = 2\lambda + 1, \frac{\beta+5}{2} = 3\lambda - 1, \frac{\gamma+7}{2} = 5\lambda + 2$

$\Rightarrow \alpha = 4\lambda - 6, \beta = 6\lambda - 7, \gamma = 10\lambda - 3$

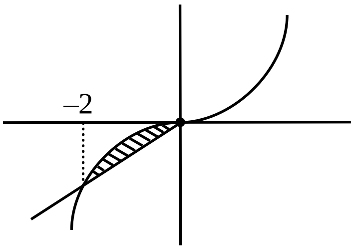
$\Rightarrow \alpha + \beta + \gamma = 20\lambda - 16 = 14$

9. The area enclosed between the curves y = x|x| and y = x – |x| is :

(1) 1 (2) $\frac{4}{3}$ (3) $\frac{2}{3}$ (4) $\frac{8}{3}$

Sol. **2**

$$y=\begin{cases} x^2 & x\ge 0 \\ -x^2 & x<0 \end{cases} \qquad y=\begin{cases} 0 & x\ge 0 \\ 2x & x<0 \end{cases}$$



$$\int_{-2}^{0}\left(-x^2-2x\right)dx=-\left(\frac{x^3}{3}+x^2\right)_{-2}^{0}$$

$$=-\frac{8}{3}+4=\boxed{\frac{4}{3}}$$

10. Let ABCD and AEFG be squares of side 4 and 2 units, respectively. The point E is on the line segment AB and the point F is on the diagonal AC. Then the radius r of the circle passing through the point F and touching the line segments BC and CD satisfies:

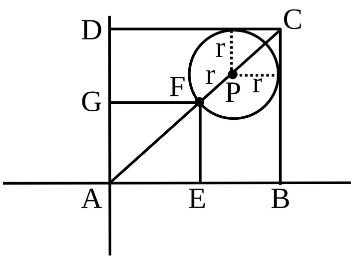
(1) $2r^2-8r+7=0$

(2) r = 1

(3) $r^2-8r+8=0$

(4) $2r^2-4r+1=0$

Sol. **3**



$AC=4\sqrt{2}$

AC = AF + PF + PC

$\Rightarrow\ 2\sqrt{2}+r+\sqrt{2}r=4\sqrt{2}$

$r=4-2\sqrt{2}$

$\Rightarrow r^2-8r+16=8\Rightarrow r^2-8r+8=0$

11. Consider three vectors $\vec{a}, \vec{b}, \vec{c}$. Let $|\vec{a}|=2, |\vec{b}|=3$ and $\vec{a}=\vec{b}\times\vec{c}$. If $\alpha \in \left[0, \frac{\pi}{3}\right]$ is the angle between the vectors $\vec{b}$ and $\vec{c}$, then the minimum value of $27|\vec{c}-\vec{a}|^2$ is equal to:

(1) 105 (2) 121 (3) 110 (4) 124

Sol. **4**

$|\vec{c}-\vec{a}|^2 = (\vec{c}-\vec{a}\cdot)\cdot(\vec{c}-\vec{a})$

$= |\vec{c}|^2 + |\vec{a}|^2 - 2\vec{a}\cdot\vec{c}$

$\Rightarrow \vec{a} = \vec{b}\times\vec{c}$ hence $\vec{a} \perp \vec{c}$

$|\vec{c}-\vec{a}|^2 = |\vec{c}|^2 + 4$

$|\vec{a}| = |\vec{b}\times\vec{c}| = |\vec{b}||\vec{c}|\sin\theta$

$\frac{2}{3} = |\vec{c}|\sin\theta$

$\Rightarrow 4 + \frac{4}{a\sin^2\theta} \Rightarrow$ is min

$\Rightarrow 4 + \frac{16}{27} = \frac{124}{27} \times 27 = 124$

12. Let the circle $C_1 : x^2 + y^2 - 2(x+y) + 1 = 0$ and C_2 be a circle having centre at (–1,0) and radius 2. If the line of the common chord of C_1 and C_2 intersects the y-axis at the point P, then the square of the distance of P from the centre of C_1 is :

(1) 2 (2) 1 (3) 6 (4) 4

Sol. **1**

$x^2 + y^2 - 2x - 2y + 1 = 0 \rightarrow C_1$

$x^2 + y^2 + 2x - 3 = 0 \rightarrow C_2$

$\Rightarrow$ common chord

$4x + 2y - 4 = 0$

$\Rightarrow 2x + y - 2 = 0$

$P \equiv (0, 2)$ center of $C_1 \equiv (1, 1)$

$1 + 1 = 2$

13. Let f, g : R → R be defined as :

$$f(x) = |x-1| \text{ and } g(x) = \begin{cases} e^x, & x \ge 0 \\ x+1, & x \le 0. \end{cases}$$

Then the function f(g(x)) is

(1) onto but not one-one.

(2) one-one but not onto.

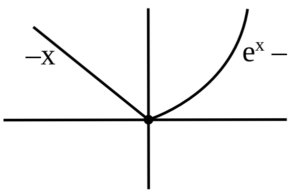
(3) neither one-one nor onto.

(4) both one-one and onto.

Sol. **3**

$$f(x) = \begin{cases} 1-x & x<1 \\ x-1 & x \ge 1 \end{cases} \qquad g(x) = \begin{cases} e^x & x \ge 0 \\ x+1 & x \le 0 \end{cases}$$

$$f(g(x)) = \begin{cases} 1-(x+1) & x<0 \\ e^x - 1 & x \ge 0 \end{cases}$$



neither one-one nor onto

14. Let the set S = {2,4,8,16, …, 512} be partitioned into 3 sets A, B, C with equal number of elements such that $A \cup B \cup C = S$ and $A \cap B = B \cap C = A \cap C = \phi$. The maximum number of such possible partitions of S is equal to:

(1) 1640

(2) 1710

(3) 1680

(4) 1520

Sol. **3**

$\{2, 2^2, 2^3 \ldots\ldots 2^9\}$

$$\Rightarrow \frac{9!}{3!3!3!3!} \times 3! = \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4}{6 \times 6 \times 2}$$

= 1680

15. If the constant term in the expansion of $\left(\frac{\sqrt[5]{3}}{x} + \frac{2x}{\sqrt[3]{5}}\right)^{12}$, $x \ne 0$, is $\alpha \times 2^8 \times \sqrt[5]{3}$, then 25α is equal to :

(1) 639

(2) 724

(3) 693

(4) 742

Sol. **3**

$$\left(\frac{3^{1/5}}{x}+\frac{2x}{5^{1/3}}\right)^{12} \Rightarrow T_{r+1} = {}^{12}C_r \frac{(3)^{\frac{12-r}{5}}\cdot 2^r}{5^{r/3}} x^{2r-12}$$

r = 6

$$T_7 = {}^{12}C_6 \frac{(3)^{\frac{6}{5}}\cdot 2^6}{5^2}$$

$$= \frac{12\times11\times10\times9\times8\times7}{6\times5\times4\times3\times2\times1}\times\frac{3\times3^{1/5}\times2^6}{25}$$

$$\Rightarrow 6\times11\times2\times7\times3\times3^{1/5}\times2^6 = 25\alpha$$

$$\Rightarrow 3^{1/5}\times2^8\times3\times3\times11\times7 = 693$$

16. Let $f:[-1,2]\rightarrow\mathbf{R}$ be given by $f(x)=2x^2+x+\left[x^2\right]-[x]$, where [t] denotes the greatest integer less than or equal to t. The number of points, where f is not continuous, is :

(1) 5 (2) 6 (3) 4 (4) 3

Sol. **3**

$f(x) = 2x^2 + [x^2] + x - [x]$

$$f(x)=\begin{cases} 2x^2+x+1 & -1\le x<0 \\ 2x^2+x & 0\le x<1 \\ 2x^2+x & 1\le x<\sqrt{2} \\ 2x^2+x+1 & \sqrt{2}\le x<\sqrt{3} \\ 2x^2+x+2 & \sqrt{3}\le x\le 2 \end{cases}$$

4 pts

17. Let $\vec{a}=2\hat{i}+5\hat{j}-\hat{k},\vec{b}=2\hat{i}-2\hat{j}+2\hat{k}$ and $\vec{c}$ be three vectors such that $(\vec{c}+\hat{i})\times(\vec{a}+\vec{b}+\hat{i})=\vec{a}\times(\vec{c}+\hat{i})$. If $\vec{a}\cdot\vec{c}=-29$, then $\vec{c}\cdot(-2\hat{i}+\hat{j}+\hat{k})$ is equal to :

(1) 10 (2) 12 (3) 15 (4) 5

Sol. **4**

$$(c+i)\times(\vec{a}+\vec{b}+i)-\vec{a}\times(\vec{c}+i)=0$$

$$\Rightarrow (c+\hat{i})\times(2\vec{a}+\vec{b}+i)=0 \Rightarrow (c+\hat{i})||(2\vec{a}+\vec{b}+i)$$

$\Rightarrow$ (x + 1, y, z) | | (7, 8, 0)

$$\Rightarrow \vec{c}=\frac{-9}{2}i-4j$$

$$\Rightarrow \vec{c}\cdot(-2i+\hat{j}+k)=5$$

18. Let $\alpha\beta \neq 0$ and $A=\begin{bmatrix} \beta & \alpha & 3 \\ \alpha & \alpha & \beta \\ -\beta & \alpha & 2\alpha \end{bmatrix}$. If $B=\begin{bmatrix} 3\alpha & -9 & 3\alpha \\ -\alpha & 7 & -2\alpha \\ -2\alpha & 5 & -2\beta \end{bmatrix}$ is the matrix of cofactors of the elements of A, then det(A B) is equal to :

(1) 64 (2) 216 (3) 125 (4) 343

Sol. **2**

$$A=\begin{bmatrix} \beta & \alpha & 3 \\ \alpha & \alpha & \beta \\ -\beta & \alpha & 2\alpha \end{bmatrix} \quad B=\begin{bmatrix} 3\alpha & -9 & 3\alpha \\ -\alpha & 7 & -2\alpha \\ -2\alpha & 5 & -2\beta \end{bmatrix}$$

Equating cofactors

$\Rightarrow (2\alpha^2 - 3\alpha) = \alpha$ $\qquad \alpha = 0 \; \alpha = 2$

$\Rightarrow 2\alpha^2 - \alpha\beta = 3\alpha$ $\qquad \alpha = 2 \; \beta = 1$

$|AB| = |A|^3 = 6^3 = 216$

19. The coefficients a, b, c in the quadratic equation $ax^2 + bx + c = 0$ are from the set {1,2,3,4,5,6}. If the probability of this equation having one real root bigger than the other is p, then 216p equals :

(1) 19 (2) 76 (3) 38 (4) 57

Sol. **3**

$D > 0 \Rightarrow b^2 > 4ac$

b = 3 (a, c) = (1,1) (1,2) (2,1)

b = 4 (a, c) = (1,1) (1,2) (2,1) (1,3) (3,1)

⋮

b = 6 (a, c) = (1,1) (1,2) …………

total case = 216

Fav. case = 38

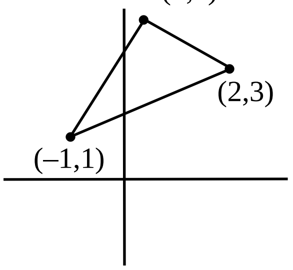
$$P = \frac{38}{216}$$

216P = 38

20. Let A(–1,1) and B(2,3) be two points and P be a variable point above the line AB such that the area of ΔPAB is 10 . If the locus of P is a + by = 15, then 5a + 2b is :

(1) $-\frac{12}{5}$ (2) 6 (3) $-\frac{6}{5}$ (4) 4

Sol. **1**



$$\frac{1}{2}\begin{vmatrix} -1, & 1 \\ 2 & 3 \\ h, & k \\ -1, & 1 \end{vmatrix} = 10$$

$(-3 + 2k + h) - (2 + 3h - k) = 20$

$-2h + 3k = 25$

$$\Rightarrow \frac{-6h}{5} + \frac{9k}{5} = 15$$

$$a = -\frac{6}{5}$$

$$b = \frac{9}{5} \quad 5a + 2b = -6 + \frac{18}{5} = -\frac{12}{5}$$

SECTION – B

21. If $f(t) = \int_0^{\pi} \frac{2x\,dx}{1-\cos^2 t \sin^2 x}, 0 < t < \pi$, then the value of $\int_0^{\frac{\pi}{2}} \frac{\pi^2 dt}{f(t)}$ equals ___ .

Sol. **1**

$$I = \int_0^{\pi} \frac{2xdx}{1-\alpha^2 \sin^2 x} \Rightarrow I = 2\pi \int_0^{\pi/2} \frac{\operatorname{cosec}^2 xdx}{\cot^2 x + (1-\alpha^2)} \Rightarrow I = \frac{\pi^2}{\sqrt{1-\alpha^2}}$$

$$\Rightarrow \int_0^{\pi/2} \frac{\pi^2}{\pi^2} \sqrt{1-\alpha^2} dt = \int_0^{\pi/2} \sin t\, dt = 1$$

22. Let a > 0 be a root of the equation $2x^2+x-2=0$. If $\lim_{x\to\frac{1}{a}}\frac{16\left(1-\cos\left(2+x-2x^2\right)\right)}{(1-ax)^2}=\alpha+\beta\sqrt{17}$, where $\alpha,\beta\in Z$, then $\alpha+\beta$ is equal to ____ .

Sol. **170**

$$\lim_{x\to\frac{1}{a}}\frac{16(1-\cos(2+x-2x^2))}{(1-ax)^2}$$

$$\lim_{x\to\frac{1}{a}}\frac{16(1-\cos(2+x-2x^2))}{(2+x-2x^2)^2}\times\frac{(2+x-2x^2)^2}{(1-ax)^2}$$

$$\Rightarrow 8\lim_{x\to\frac{1}{a}}\frac{(2+x-2x^2)^2}{(1-ax)^2}$$

$$\Rightarrow 8\lim_{t\to a}\frac{(2t^2+t-2)^2}{t^2(t-a)^2}$$

$$8\times4\lim_{t\to a}\frac{\left(t^2+\frac{t}{2}-1\right)^2}{t^2(t-a)^2}$$

$$\frac{32}{a^2}\times\left(a-\frac{1}{a}\right)^2\Rightarrow 170$$

23. Let a line perpendicular to the line 2x – y = 10 touch the parabola $y^2=4(x-9)$ at the point P. The distance of the point P from the centre of the circle $x^2+y^2-14x-8y+56=0$ is _____ .

Sol. **10**

slope of tangent is $-\frac{1}{2}$ hence P.O.C. is (13, –4) so distance is 10.

24. If $1+\frac{\sqrt{3}-\sqrt{2}}{2\sqrt{3}}+\frac{5-2\sqrt{6}}{18}+\frac{9\sqrt{3}-11\sqrt{2}}{36\sqrt{3}}+\frac{49-20\sqrt{6}}{180}+\ldots$ upto $\infty=2+\left(\sqrt{\frac{b}{a}}+1\right)\log_e\left(\frac{a}{b}\right)$, where a and b are integers with $\gcd(a,b)=1$, then 11a + 18b is equal to:

Sol. **76**

$$S=1+\frac{x}{2\sqrt{3}}+\frac{x^2}{18}+\frac{x^3}{36\sqrt{3}}+......\infty$$

$$x=\sqrt{3}-\sqrt{2}\qquad \frac{x}{\sqrt{3}}=t$$

$$S=1+\frac{t}{2}+\frac{t^2}{6}+\frac{t^3}{12}......\infty$$

$$S = \left(1 + t + \frac{t^2}{2} + ...\infty\right) - \left(\frac{t}{2} + \frac{t^3}{3}.......\right)$$

$$S = \left(t + \frac{t^2}{2} + ...\infty\right) - \frac{1}{t}\left(t + \frac{t^2}{2}.......\right) + 2$$

$$S = 2 + \left(1 - \frac{1}{t}\right)(-\log(1-t)) = \left(\frac{1}{t} - 1\right)\log(1-t) + 2$$

$$2 + \left(\sqrt{\frac{3}{2}} + 1\right)\ln\left(\frac{2}{3}\right)$$

a = 2, b = 3

11a + 18b = 76

25. Let the mean and the standard deviation of the probability distribution

X	α	1	0	–3
PX	$\frac{1}{3}$	K	$\frac{1}{6}$	$\frac{1}{4}$

be μ and σ, respectively. If $\sigma - \mu = 2$, then $\sigma + \mu$ is equal to _____ .

Sol. **5**

$$\frac{1}{3} + k + \frac{1}{6} + \frac{1}{4} = 1$$

$$\Rightarrow k = \frac{1}{4}$$

$$\mu = \frac{\alpha}{3} + \frac{1}{4} - \frac{3}{4} = \frac{\alpha}{3} - \frac{1}{2}$$

$$\sigma = \sqrt{\frac{2\alpha^2}{9} + \frac{\alpha}{3} + \frac{9}{4}} = \mu + 2$$

$\Rightarrow \alpha = 6$

$\sigma + \mu = 2\mu + 2 = 5$

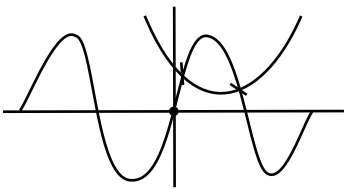
26. The number of solutions of $\sin^2 x + (2 + 2x - x^2)\sin x - 3(x-1)^2 = 0$, where $-\pi \le x \le \pi$, is ____ .

Sol. **2**

$\sin^2 x - [(x-1)^2 - 3]\sin x - 3(x-1)^2 = 0$

$\Rightarrow \sin x\,[\sin x - (x-1)^2] + 3[\sin x - (x-1)^2] = 0$

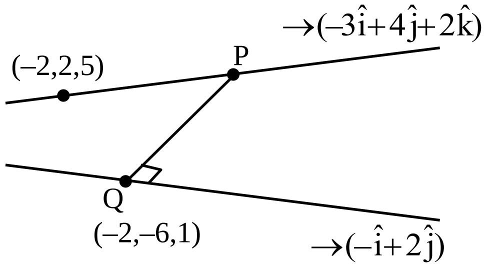
$\Rightarrow \sin x = 3, \sin x = (x-1)^2$



number of solution = 2

27. Let the point $(-1, \alpha, \beta)$ lie on the line of the shortest distance between the lines $\frac{x+2}{-3}=\frac{y-2}{4}=\frac{z-5}{2}$ and $\frac{x+2}{-1}=\frac{y+6}{2}=\frac{z-1}{0}$. Then $(\alpha-\beta)^2$ is equal to ____ .

Sol. **25**



$P(-3\lambda-2, 4\lambda+2, 2\lambda+5)$

$Q(-\mu-2, 2\mu-6, 1)$

$\overrightarrow{PQ} = <3\lambda-\mu, 2\mu-4\lambda-8, -2\lambda-4>$

$\overrightarrow{PQ} \parallel <4, 2, 2>$

$\Rightarrow \lambda=-1, \mu=1$

$\Rightarrow \alpha=-3, \beta=2$

$(\alpha-\beta)^2 = 25$

28. The number of real solutions of the equation $x|x+5|+2|x+7|-2=0$ is ____ .

Sol. **3**

$x<-7$	$-7 \le x < -5$	$-5 \le x$
$\Rightarrow -x^2-5x-2x-14-2=0$	$-x^2-5x+2x+14-2=0$	$x^2+5x+2x+14-2=0$
$\Rightarrow -x^2-7x-16=0$	$\Rightarrow -x^2-3x+12=0$	$x^2+7x+12=0$
$\Rightarrow x^2+7x+16=0$	$\Rightarrow x^2+3x-12=0$	$x=-4 \quad x=-3$

29. Let $y = y(x)$ be the solution of the differential equation

$$\frac{dy}{dx}+\frac{2x}{(1+x^2)^2}y = xe^{\frac{1}{(1+x^2)}}; y(0)=0.$$

Then the area enclosed by the curve $f(x)=y(x)e^{-\frac{1}{(1+x^2)}}$ and the line $y-x=4$ is

Sol. **18**

$$\frac{dy}{dx}+\frac{2x}{(1+x^2)^2}y=x\cdot e^{\frac{1}{1+x^2}}$$

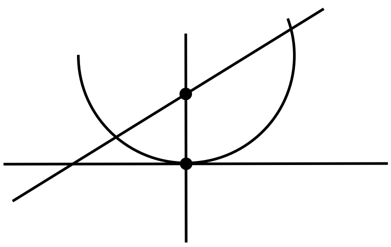
$$\text{I.F.} = e^{\int\frac{2x}{(1+x^2)^2}dx}=e^{-\frac{1}{1+x^2}}$$

$$y\cdot e^{-\frac{1}{1+x^2}}=\int x\,dx$$

$$y\cdot e^{-\frac{1}{1+x^2}}=\frac{x^2}{2}$$

$$y=\frac{x^2}{2}\cdot e^{\frac{1}{1+x^2}}$$

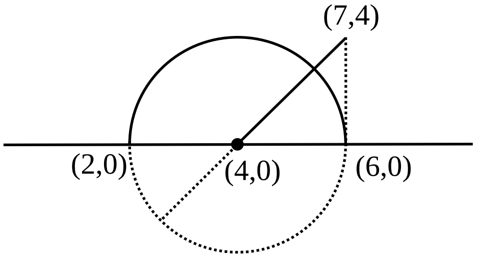
$$f(x)=\frac{x^2}{2} \text{ \& } y = x + 4$$



Area = 18

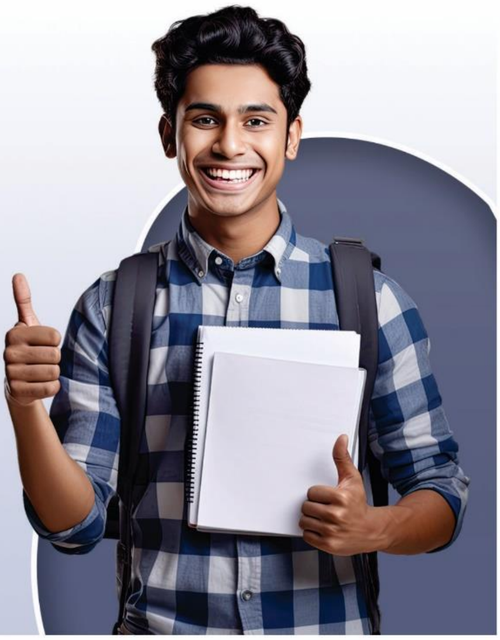
30. Let the maximum and minimum values of $\left(\sqrt{8x-x^2-12}-4\right)^2+(x-7)^2, x\in\mathbf{R}$ be M and m, respectively. Then $M^2 - m^2$ is equal to ___ .

Sol. **1600**



m = 9   M = 41

$M^2 - m^2 = 1600$



Continuing to keep the pledge

of **imparting education** for the **last 17 Years**

65136+ SELECTIONS SINCE 2007	JEE (Advanced) **12142**	JEE (Main) **32584**	NEET/AIIMS **17875** (Under 50000 Rank)	NTSE/OLYMPIADS **2535** (6th to 10th class)

Most Promising **RANKS**
Produced by MOTION Faculties

NEET / AIIMS

- **AIR-1 to 10** — 25 Times
- **AIR-11 to 50** — 84 Times
- **AIR-51 to 100** — 84 Times

JEE MAIN+ADVANCED

- **AIR-1 to 10** — 8 Times
- **AIR-11 to 50** — 37 Times
- **AIR-51 to 100** — 41 Times

Nation's Best **SELECTION**
Percentage (%) Ratio

Student Qualified in NEET

(2023)
6492/7084 = **91.64%**
(2022)
4837/5356 = **90.31%**

Student Qualified in JEE ADVANCED

(2023)
2747/5182 = **53.01%**
(2022)
1756/4818 = **36.45%**

Student Qualified in JEE MAIN

(2024-First Attemp)
6495/10592 = **61.31%**
(2023)
5993/8497 = **70.53%**
(2022)
4818/6653 = **72.41%**

NITIN VIIJAY (NV Sir)
Founder & CEO

MOTION