1 – FUNDAMENTAL OF MATHEMATICS

WORKSHEET

1. State weather the following collections is a set or not ?

(i) The collection of natural numbers between 2 and 20

(ii) The collection of numbers which satisfy the equation $x^2 - 5x + 6 = 0$

(iii) The collection of prime numbers between 1 and 100.

(iv) The collection of all intelligent women in Jalandhar.

2. Find the smallest set A such that $A \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$

3. In a town of 10,000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C, 5% families buy A and B, 3 % buy B and C and 4% buy A and C. If 2% families buy all the three newspapers, then find number of families which buy newspaper A only.

4. Solve the following Inequalities

$$\frac{x^2 + 4x + 4}{2x^2 - x - 1} > 0$$

5. Find the value of

$\log_{10}5.\log_{10}20 + (\log_{10}2)^2$

6. Solve the following inequalities

$$\log_{\frac{5}{8}}\left(2x^2 - x - \frac{3}{8}\right)$$

7. The set $A = \{x : x \in R, x^2 = 16 \text{ and } 2x = 6\}$ is

(A) Null set

(B) Singleton set

(C) Infinite set

(D) not a well defined collection

8. Let $A = \{x : x \in R, -1 < x < 1\}$, $B = \{x : x \in R, x \le 0 \text{ or } x \ge 2\}$ and $A \cup B = R - D$, then the set D is

(A) $\{x : 1 < x \le 2\}$

(B) $\{x : 1 \le x < 2\}$

(C) $\{x : 1 \le x \le 2\}$

(D) $\{x : 1 < x < 2\}$

9. A class has 175 students. The following data shows the number of students obtaining one or more subjects : Mathematics 100, Physics 70, Chemistry 40, Mathematics and Physics 30, Mathematics and Chemistry 28, Physics and Chemistry 23, Mathematics & Physics & Chemistry 18. How many students have offered Mathematics alone ?

(A) 35 (B) 48

(C) 60 (D) 22

10. The number of the integral solutions of $x^2 + 9 < (x + 3)^2 < 8x + 25$ is :

(A) 1 (B) 3

(C) 4 (D) 5

11. $(\log_2 10) \cdot (\log_2 80) - (\log_2 5) \cdot (\log_2 160)$ is equal to :

(A) $\log_2 5$ (B) $\log_2 20$

(C) $\log_2 10$ (D) $\log_2 16$

12. The set of all the solutions of the inequality $\log_{1-x}(x - 2) \geq -1$ is

(A) $(-\infty, 0)$ (B) $(2, \infty)$

(C) $(-\infty, 1)$ (D) ϕ

13. Write the following expression in appropriate intervals so that they are bereft of modulus sign

$|x^2 - 7x + 10|$

14. Solve the following inequalities :

(i) $|x - 3| \geq 2$

(ii) $||x - 2| - 3| \leq 0$

15. Write the following expression in appropriate intervals so that they are bereft of modulus sign

(i) $|\log_{10} x| + |2^{x-1} - 1|$

(ii) $|(\log_2 x)^2 - 3(\log_2 x) + 2|$

16. Solve the following inequalities :

(i) $\dfrac{\sqrt{2x - 1}}{x - 2} < 1$

(ii) $x - \sqrt{1 - |x|} < 0$

17. Find the set of values of λ for which the equation $|x^2 - 4| \times |-12| = \lambda$ has 6 distinct real roots.

18. Product of real roots of the equation $t^2x^2 + |x| + 9 = 0$

(1) is always positive

(2) is always negative

(3) does not exist

(4) none of these

19. The sum of the roots of the equation, $x^2 + |2x - 3| - 4 = 0$, is :

(1) $-\sqrt{2}$ (2) $\sqrt{2}$

(3) -2 (4) 2

20. Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \neq 0$. If p, q ,r are in the A.P. and $\dfrac{1}{\alpha} + \dfrac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is :

(1) $\dfrac{\sqrt{34}}{9}$ (2) $\dfrac{2\sqrt{13}}{9}$

(3) $\dfrac{\sqrt{61}}{9}$ (4) $\dfrac{2\sqrt{17}}{9}$

HINTS & SOLUTION

Sol.1 (i) Yes (ii) Yes (iii) Yes (iv) No

Sol.2 {3, 5, 9}

Sol.3 3300

Sol.4 $(-\infty, -2) \cup (-2, -1/2) \cup (1, \infty)$

Sol.5 1

Sol.6 $\left[-\frac{1}{2}, -\frac{1}{4}\right) \cup \left(\frac{3}{4}, 1\right]$

Sol.7 (A)

Sol.8 (B)

Sol.9 (C)

Sol.10 (D)

Sol.11 (D)

Sol.12 (D)

Sol.13 $x^2 - 7x + 10,\ x > 5 \text{ or } x \le 2$

$-(x^2 - 7x + 10),\ 2 < x < 5$

Sol.14 (i) $x \in (-\infty, 1] \in [5, \infty)$

(ii) $x = 5 \text{ or } x = -1$

Sol.15 (i) $\log_{10} x + 2^{x-1} - 1 \quad x \ge 1$

$-(\log_{10} x + 2^{x-1} - 1) \quad 0 < x < 1$

(ii) $(\log_2 x)^2 - 3(\log_2 x) + 2$

$x \in (0, 2] \cup [4, \infty)$

$-((\log_2 x)^2 - 3(\log_2 x) + 2)$

$x \in (2, 4)$

Sol.16 (i) $\left[\frac{1}{2}, 2\right) \cup (5, \infty)$

(ii) $[-1, (\sqrt{5} - 1)/2)$

Sol.17 $\lambda \in (12, 16)$

Sol.18 (3)

Sol.19 (2)

Sol.20 (2)