CLASSIFICATION OF NUMBERS

INTRODUCTION

Number System is a method of writing numerals to represent numbers.

- Ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are used to represent any number (however large it may be) in our number system.
- Each of the symbols 0,1, 2, 3, 4, 5, 6, 7, 8, 9 is called a digit or a figure.

CLASSIFICATION OF NUMBER

(i) **Natural numbers:** Counting numbers are known as natural numbers. N = { 1, 2, 3, 4, ... }.

(ii) **Whole numbers:** All natural numbers together with 0 form the collection of all whole numbers. W = { 0, 1, 2, 3, 4, ... }.

(iii) **Integers:** All natural numbers, 0 and negative of natural numbers form the collection of all integers. I or Z = { ..., – 3, – 2, – 1, 0, 1, 2, 3, ... }.

(a) **Positive integers**

The set I+ = {1, 2, 3, 4, ...} is the set of all positive integers, Clearly, positive integers and natural numbers are synonyms.

(b) **Negative integers**

The set I– = {–1, –2, –3, ...} is the set of all negative integers.

0 is neither positive nor negative.

(c) **Non-negative integers**

The set {0, 1, 2, 3, ...} is the set of all non-negative integers.

(d) **Non-positive integers**

The set {......., –3, –2, –1, 0} is the set of all non-positive integers.

(iv) **Even Numbers:** All integers which are divisible by 2 are called even numbers. Even numbers are denoted by the expression 2n, where n is any integer.

So, if E is a set of even numbers, then E = { ..., – 4, –2, 0, 2, 4,...}.

(v) **Odd Numbers:** All integers which are not divisible by 2 are called odd numbers.

Odd numbers are denoted by the general expression 2n –1 where n is any integer.

If O is a set of odd numbers, then O = {..., –5, –3, –1, 1, 3, 5,...}.

(vi) **Rational Number :**

The rational numbers are all the numbers that can be expressed in the form of $\frac{p}{q}$, where p and q are integers and co-prime and q ≠ 0.

e.g., $\frac{2}{3}$, – 3, 0, 4.33 etc.

Rational numbers between two numbers

One way to find a rational number between two rational numbers is to find their average, called mean.

To find a rational number between x and y, we will find the mean of x and y. i.e.

$\frac{x+y}{2}$ is a rational number lying between x and y. This number will be the mid-value of that two numbers.

Decimal expansion of rational numbers

Every rational number can be expressed as terminating decimal or non-terminating but repeating decimals.

Terminating decimal (The remainder becomes zero)

The word "terminate" means "end". A decimal that ends is a terminating decimal. A terminating decimal doesn't keep going. A terminating decimal will have finite number of digits after the decimal point.

$\frac{3}{4} = 0.75$, $\frac{8}{10} = 0.8$, $\frac{5}{4} = 1.25$, $\frac{25}{16} = 1.5625$

Method to convert non-terminating decimal to the form p/q.

In a non-terminating decimal. we have two types of decimal representations

(i) Pure recurring decimal

(ii) Mixed recurring decimal

(i) Pure recurring decimal

It is a decimal representation in which all the digits after the decimal point are repeated.

Following are the steps to convert it in the form p/q.

Step-1 : Denote pure recurring decimal as x.

Step-2 : Write the number in decimal form by removing bar from top of repeating digits.

Step-3 : Count the number of digits having bar on their heads.

Step-4 : Multiply the repeating decimal by 10, 100, 1000, ... depending upon 1 place repetition, 2 place repetition, 3 place repetition and so on present in decimal number.

Step-5 : Subtract the number obtained in step 2 from a number obtained in step 4.

Step-6 : Find the value of x in the form p/q.

(ii) Mixed recurring decimal

It is a decimal representation in which there are one or more digits present before the repeating digits after decimal point.

Following are the steps to convert it to the form p/q.

Step-1 : Denote mixed recurring decimal as x.

Step-2 : Count the number of digits after the decimal point which do not have bar on them. Let it be 'n'.

Step-3 : Multiply both sides of x by 10n to get only repeating decimal numbers on the right side of the decimal point.

Step-4: Further use the method of converting pure recurring decimal to the form p/q and get the value of x.

(vii) Irrational Numbers :

A number is called an irrational number, if it cannot be written in the form p/q, where p & q are integers and $q \neq 0$. All Non-terminating & Non-repeating decimal numbers are Irrational numbers.

E.g. $\sqrt{2}$, $\sqrt{3}$, $3\sqrt{2}$, $2+\sqrt{3}$, $2+\sqrt{2+\sqrt{3}}$ 3 , π , etc ....

Decimal expansion of irrational numbers

Every irrational number can be expressed as non-terminating and non-repeating decimal. e.g. $\sqrt{2}$ = 1.4142135 ....

> **Remark :** To find an irrational number between two numbers a and b is $\sqrt{ab}$.

(viii) Real numbers : Numbers which can represent actual physical quantities in a meaningful way are known as real numbers. These can be represented on the number line. Number line is geometrical straight line with arbitrarily defined zero (origin).

(ix) Prime numbers : All natural numbers that have one and itself only as their factors are called prime numbers i.e. prime numbers are exactly divisible by 1 and themselves. e.g. 2, 3, 5, 7, 11, 13, 17, 19, 23,...etc.

If P is the set of prime number then P = {2, 3, 5, 7,...}.

Table of prime Numbers (1-100):

2	11	23	31	41	53	61	71	83	97
3	13	29	37	43	59	67	73	89	
5	17			47			79		
7	19								

Test to find whether a given number is a prime

Step-1 : Select a least positive integer n such that n^2 > given number.

Step-2 : Test the divisibility of given number by every prime number less than n.

Step-3 : The given number is prime only if it is not divisible by any of these primes.

(x) Co-prime Numbers : If the H.C.F. of the given numbers (not necessarily prime) is 1 then they are known as co-prime numbers. e.g. 4, 9 are co-prime as H.C.F. of (4, 9) = 1.

➢ **Any two consecutive numbers will always be co-prime.**

(xi) Twin primes : Pairs of prime numbers which have only one composite number between them are called Twin primes. 3, 5 ; 5, 7 ; 11,13 ; 17, 19 ; 29, 31 ; 41, 43 ; 59, 61 and 71, 73 etc. are twin primes.

(xii) Composite numbers : All natural numbers, which are not prime are composite numbers.

If C is the set of composite number then C = {4, 6, 8, 9, 10, 12,...}.

➢ **1 is neither prime nor composite number.**

(xiii) Imaginary Numbers : All the numbers whose square is negative are called imaginary numbers. e.g. 3i, -4i, i, ... ; where $i = \sqrt{-1}$.

(xiv) Perfect numbers

If sum of proper divisors of a number is the number itself then the number is known as perfect number.

e.g., 6, 28 ..... etc.

SURDS AND EXPONENTS

SURDS

Let a be a rational number and n be a positive integer, then irrational number is of the form $\sqrt[n]{a}$ is given a special name surd, where 'a' is called radicand and it should always be a rational number.

Also the symbol $\sqrt[n]{\ }$ is called the radical sign and the index n is called order of the surd. $\sqrt[n]{a}$ is read as 'nth root of a' and can also be written as $a^{\frac{1}{n}}$.

LAW OF SURDS

(i) $\left(\sqrt[n]{a}\right)^n = \left(a^{\frac{1}{n}}\right)^n = a^{\frac{1}{n}\times n} = a.$

(ii) $\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab}$

(iii) $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$

[Important for changing order of surds]

or, $\sqrt[n]{a^m} = \sqrt[n\times p]{a^{m\times p}}$

(iv) $\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m} = a^{m/n}$

(v) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$

(vi) $\sqrt[n]{a} = \sqrt[n\times p]{a^p}$

Ex.1 Simplify :

$\sqrt[3]{2}.\sqrt[3]{4} = \sqrt[3]{2\times 4} = \sqrt[3]{2^3} = (2^3)^{\frac{1}{3}} = 2.$

OPERATION OF SURDS

(a) Addition and Subtraction of Surds : Addition and subtraction of surds are possible only when order and radicand are same i.e. only for like surds.

Ex.2 Simplify :

$5\sqrt[3]{250} + 7\sqrt[3]{16} - 14\sqrt[3]{54}$

Sol: $5\sqrt[3]{250} + 7\sqrt[3]{16} - 14\sqrt[3]{54}$

$= 5\sqrt[3]{125\times 2} + 7\sqrt[3]{8\times 2} - 14\sqrt[3]{37\times 2}$

$= 5\times 5\sqrt[3]{2} + 7\times 2\sqrt[3]{2} - 14\times 3\sqrt[3]{2}$

$= (25 + 14 - 42)\ \sqrt[3]{2} = -3\sqrt[3]{2}$.

(b) Multiplication and Division of Surds

Ex.3 Simplify $\sqrt{8a^5b}\times\sqrt[3]{4a^2b^2}$

Sol: $\sqrt{8a^5b}\times\sqrt[3]{4a^2b^2} = \sqrt[6]{8^3a^{15}b^3}\times\sqrt[6]{4^2a^4b^4}$

$= \sqrt[6]{2^{13}a^{19}b^7} = 2^2a^3b\sqrt[6]{2ab}$

$= 4a^3b\sqrt[6]{2ab}$.

Ex.4 Divide : $\sqrt{24}\div\sqrt[3]{200}$.

Sol : $\sqrt{24}\div\sqrt[3]{200} = \dfrac{\sqrt{24}}{\sqrt[3]{200}} = \dfrac{\sqrt[6]{(24)^3}}{\sqrt[6]{(200)^2}} = \dfrac{\sqrt[6]{216}}{\sqrt[6]{625}}$.

(c) Comparison of Surds

It is clear that if x > y > 0 and n > 1 is a positive integer then $\sqrt[n]{x} > \sqrt[n]{y}$.

Ex.5 Arrange $\sqrt{2}$, $\sqrt[3]{3}$ and $\sqrt[4]{5}$ in ascending order..

Sol: $\sqrt{2}$, $\sqrt[3]{3}$ and $\sqrt[4]{5}$

L.C.M. of 2, 3, 4 is 12.

$\therefore \quad \sqrt{2} = \sqrt[2\times6]{2^6} = \sqrt[12]{64}$

$\sqrt[3]{3} = \sqrt[3\times4]{3^4} = \sqrt[12]{81}$

$\sqrt[4]{5} = \sqrt[4\times3]{5^3} = \sqrt[12]{125}$

As,

$\therefore \quad \sqrt[12]{64} < \sqrt[12]{81} < \sqrt[12]{125} \Rightarrow$

$\sqrt{2} < \sqrt[3]{3} < \sqrt[4]{5}$.

RATIONALISATION OF SURDS

Rationalising factor : If the product of two surds is a rational number, then each surd is called a rationalising factor (RF) of the other.

Rationalisation of surds : The process of converting a surd into rational number by multiplying it with a suitable RF, is called the rationalisation of the surd.

Monomial surds and their RF : The general form of a monomial surd is $\sqrt[n]{a}$ and its RF is $a^{1-\frac{1}{n}}$

Ex.6 Find rationalisation factor of $\sqrt[3]{5}$.

Sol: Rationalisation factor of $\sqrt[3]{5}$ is

$= 5^{1-\frac{1}{3}} = 5^{\frac{2}{3}} = \sqrt[3]{5^2} = \sqrt[3]{25}$.

Binomial surds and their RF : The surds of the types : $a+\sqrt{b}$, $a-\sqrt{b}$, $\sqrt{a}+\sqrt{b}$ and $\sqrt{a}-\sqrt{b}$ are called binomial surds.

Conjugate Surds : The binomial surds which differ only in sign between the terms separating them are known as conjugate surds.

In binomial surds, the conjugate surds are RF of each other.

For example :

(i) RF of $\sqrt{a}+\sqrt{b}$ is $\sqrt{a}-\sqrt{b}$.

(ii) Rf of $\sqrt{a}-\sqrt{b}$ is $\sqrt{a}+\sqrt{b}$.

Ex.7 : Rationalize the denominator $\frac{1}{7+5\sqrt{3}}$.

Sol: $\frac{1}{7+5\sqrt{3}} = \frac{1}{7+5\sqrt{3}} \times \frac{7-5\sqrt{3}}{7-5\sqrt{3}} =$

$\frac{7-5\sqrt{3}}{49-75} = \frac{7-5\sqrt{3}}{-26} = \frac{5\sqrt{3}-7}{-26}$.

Trinomial surds : A surd which consists of three terms, atleast two of which are monomial surds, is called a trinomial surd.

Example : $7 + \sqrt{3} + \sqrt{5}$.

In order to rationalize $= \frac{x}{\sqrt{a}+\sqrt{b}+\sqrt{c}}$

(i) Multiply and divide by $\sqrt{a}+\sqrt{b}-\sqrt{c}$

(ii) Multiply and divide by $(a+b-c)2\sqrt{ab}$

Ex.8 : Rationalize : $\frac{1}{\sqrt{6}+\sqrt{3}+\sqrt{5}}$

Sol: $\frac{1}{\sqrt{6}+\sqrt{3}+\sqrt{5}} = \frac{1}{\left(\sqrt{6}+\sqrt{3}\right)+\sqrt{5}}$

$\times \frac{\left(\sqrt{6}+\sqrt{3}\right)-\sqrt{5}}{\left(\sqrt{6}+\sqrt{3}\right)+\sqrt{5}} = \frac{\left(\sqrt{6}+\sqrt{3}\right)-\sqrt{5}}{\left(\sqrt{6}+\sqrt{3}\right)^2+\left(\sqrt{5}\right)^2} =$

$\frac{\sqrt{6}+\sqrt{3}-\sqrt{5}}{6+3+2\sqrt{18}-5} = \frac{\sqrt{6}+\sqrt{3}-\sqrt{5}}{4+6\sqrt{2}}$

$$= \frac{\sqrt{6}+\sqrt{3}-\sqrt{5}}{(4+6\sqrt{2})} = \frac{4-6\sqrt{2}}{4-6\sqrt{2}}$$

$$= \frac{4\sqrt{6}+4\sqrt{3}-4\sqrt{5}-6\sqrt{12}-6\sqrt{6}+6\sqrt{10}}{16-72}$$

$$= \frac{-2\sqrt{6}+4\sqrt{3}-4\sqrt{5}-12\sqrt{3}+6\sqrt{10}+6\sqrt{10}}{-56}$$

$$= \frac{\sqrt{6}+4\sqrt{3}+2\sqrt{5}-3\sqrt{10}}{28}$$

EXPONENTS

The repeated multiplication of the same factor can be written in a more compact form, called exponential form. Laws of exponents : If a is any non – zero rational number and m, n are whole numbers, then

(i) On the same base in multiplication, powers are added. $a^m \times a^n = a^{m+n}$

For example : $3^2 \times 3^4 = 3^{2+4} = 3^6$.

(ii) On the same base in division, powers are subtracted. $\frac{a^m}{a^n} a^{m-n}$.

For example : $\frac{3^5}{3^2} = 3^{5-2} = 3^3$.

(iii) $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}, n > m$.

For example : $\frac{2^3}{2^4} = \frac{1}{2^{4-3}} = \frac{1}{2}$

(iv) $(a^m)^n = a^{mn}$

For example : $(2^2)^3 = 2^{2\times3} = 2^6$.

(v) $a^n \times a^{-n} = a^0 = 1$

(vi) $a^m \times b^m = (ab)^m$

For example :

$2^2 \times 3^2 = (2\times3)_2 = 6^2 = 36$.

(vii) $a^{bn} = a^{b+b+b\ldots n}$ times

where a, b are positive real numbers and m, n are rational numbers.

Ex.9 Simplify : $\frac{(25)^{3/2}\times(243)^{3/5}}{(16)^{5/4}\times(8)^{4/3}}$

Sol: We have, $\frac{(25)^{3/2}\times(243)^{3/5}}{(16)^{5/4}\times(8)^{4/3}}$

$$= \frac{(5^2)^{3/2}\times(3^5)^{3/5}}{(2^4)^{5/4}\times(2^3)^{4/3}} = \frac{5^{2\times3/2}\times3^{5\times3/5}}{2^{4\times5/4}\times2^{3\times4/3}}$$

$$= \frac{5^3\times3^3}{2^5\times2^4} = \frac{125\times27}{32\times16} = \frac{3375}{512}.$$

Ex.10 If $25^{x-1} = 5^{2x-1} - 100$, find the value of x.

Sol: We have,

$25^{x-1} = 5^{2x-1} - 100$,

$\Rightarrow (5^2)^{x-1} = 5^{2x-1} - 100$

$\Rightarrow 5^{2x-2} - 5^{2x-1} = -100$

$\Rightarrow 5^{2x-2} - 5^{2x-2}.5^1 = -100$

$\Rightarrow 5^{2x-2}(1-5) = -100$

$\Rightarrow 5^{2x-2} - (-4) = -100$

$\Rightarrow 5^{2x-2} = 25$

$\Rightarrow 5^{2x-2} = 5^2$

$\Rightarrow 2x - 2 = 2$

$\Rightarrow x = 4 \qquad \Rightarrow x = 2.$

Ex.11 Assuming that x is a positive real number and a, b, c are rational numbers, show that : $\left(\frac{x^b}{x^c}\right)^a\left(\frac{x^c}{x^a}\right)^b\left(\frac{x^a}{x^b}\right)^c = 1$

Sol: $\left(\frac{x^b}{x^c}\right)^a\left(\frac{x^c}{x^a}\right)^b\left(\frac{x^a}{x^b}\right)^c$

$= (x^{b-c}).(x^{c-a}).(x^{a-b})$

$= x^{ab-ac}\,x^{bc-ba}.\,x^{ac-bc}.$

$= x^{ab-ac+bc-ba+ac-bc} = x^{0=1}.$

WORKSHEET

1. Simply (make the denominator rational) $\frac{12}{3+\sqrt{5}-2\sqrt{2}}$.

2. Find the square root of $7 + 2\sqrt{10}$.

3. If $x = \frac{1}{2+\sqrt{3}}$, find the value of $x^3 - x^2 - 11x + 4$.

4. If $x = 3 - 2\sqrt{2}$, find $x^2 + \frac{1}{x^2}$.

5. Rationalise the denominator of $\frac{1}{\sqrt{3}-\sqrt{2}-1}$.

6. Evaluate the following :

(i) $\left(\sqrt[3]{64}\right)^{\frac{-1}{2}}$ **(ii)** $\left(\frac{121}{169}\right)^{\frac{-3}{2}}$

7. If $a^x = b$, $b^y = c$, $c^z = a$, prove that $xyz = 1$ where a, b, c are distinct numbers.

8. If $n^2 + 2n - 8$ is a prime number where $n \in N$, then n is

(A) also a prime number

(B) relatively prime to 10

(C) relatively prime to 6

(D) a composite number

9. Consider the equation $x^3 - 3x^2 + 2x = 0$, then

(A) Number of even integers satisfying the equation is 2

(B) Number of odd integers satisfying the equation is 1

(C) Number of odd prime natural numbers satisfying the equation is 1

(D) Number of composite natural numbers satisfying the equation is 1

10.

	Column I		Column II
(A)	{2, 3}	**(p)**	is a pair of primes
(B)	{11, 13}	**(q)**	is a pair of twin primes
(C)	{5, 11}	**(r)**	is a pair of co-primes
(D)	{2, 6}	**(s)**	is a pair of even number
(E)	{23, 81}		

11. The multiplication of a rational number 'x' and an irrational number 'y' is :

(A) always rational

(B) rational except when $y = \pi$

(C) always irrational

(D) irrational except when x = 0

12. $12^3 \times 3^4 \times 5^2$, find the total number of even factors of N.

13. Solve for $x \in R$ –

(i) $4^x - 10.2^{x-1} = 24$

(ii) $4.2^{2x} - 6^x = 18.3^{2x}$

(iii) $3^{2x-3} - 9^{x-1} + 27^{2x/3} = 675$.

(iv) $7^{x+2} - \frac{1}{7}.7x{+}l - 14.7^{x-1} + 2.7^x = 48$

(v) $\left(\frac{5}{3}\right)^{x+1} . \left(\frac{9}{25}\right)^{x^2+2x-11} = \left(\frac{5}{3}\right)^9$

(vi) $5^{2x} = 3^{2x} + 2.5^x + 2.3^x$

14. If $2\left(\sqrt{3+\sqrt{5-\sqrt{13+\sqrt{48}}}}\right) = \sqrt{4} + \sqrt{b}$ where a and b are natural number find (a + b) .

15. Which is greater ?

(i) $\sqrt[3]{3}$ or $\sqrt[4]{5}$

(ii) $\sqrt[8]{12}$ or $\sqrt[4]{16}$

(iii) $\sqrt{2}$ or $\sqrt[3]{3}$

16. Find all primes which can be represented both as sums and as differences of two primes.

17. Find three least positive integers n such that there are no primes between n and n + 10, and three least positive integers m such that there are no primes between 10m and 10(m + l).

18. Find four solutions of the equation $p^2 + 1 = q^2 + r^2$ with primes p, q, and r.

19. Find all primes p, q, and r such that the numbers p(p + l), q(q + l), r(r + 1) form an increasing arithmetic progression.

20. Find five least positive integers n for which $n^2 + 1$ is a product of three different primes, and find a positive integer n for which $n^2 + 1$ is a product of three different odd primes.

ANSWER SHEET

Sol.1 The expression $\frac{12(3+\sqrt{5}+2\sqrt{2})}{\left(3+\sqrt{5}\right)^2-\left(2\sqrt{2}\right)^2}$

$= \frac{12(3+\sqrt{5}+2\sqrt{2})}{6+6\sqrt{2}}$

$= \frac{2(3+\sqrt{5}+2\sqrt{2})(\sqrt{5}-1)}{(\sqrt{5}+1)\times(\sqrt{5}-1)}$

$= \frac{2(2+2\sqrt{5}+2\sqrt{10}-2\sqrt{2})}{4}$

$= 1 + \sqrt{5} + \sqrt{10} - \sqrt{2}$.

Sol.2 Let $\sqrt{7+2\sqrt{10}} = \sqrt{x}+\sqrt{y}$.

Squaring, $x + y + 2\sqrt{xy} = 7 + 2\sqrt{10}$

Hence $x + y = 7$ and $xy = 10$. These two relations give $x = 5$, $y = 2$.

Hence $\sqrt{7+2\sqrt{10}} = \sqrt{5}+\sqrt{2}$.

Remark: $\sqrt{\ }$ symbol stands for the positive square root only.

Sol.3 As $x = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$= \frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2}$

$x = \frac{2-\sqrt{3}}{4-3} = 2 - \sqrt{3}$

$x - 2 = -\sqrt{3}$ squaring both sides; we

$(x-2)^2 = (-\sqrt{3})^2$

$\Rightarrow x^2 + 4 - 4x = 3 \Rightarrow x^2 - 4x + 1 = 0$

Now, $x^3 - x^2 - 11x + 4$

$= x^3 - 4x^2 + x + 3x^2 - 12x + 4$

$= x(x^2\ 4x + 1) + 3(x^2 - 4x + 1) + 1$

$= x \times 0 + 3(0) + 1 = 0 + 0 = 0 = 1$

Sol.4 We have $x = 3 - 2\sqrt{2}$

$\therefore \quad \frac{1}{x} = \frac{1}{3-2\sqrt{2}} = \frac{1}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}}$

$= \frac{3+2\sqrt{2}}{(3)^2-(2\sqrt{2})^2} = \frac{3+2\sqrt{2}}{9-8} = 3 + 2\sqrt{2}$

Sol.5 $\frac{1}{\sqrt{3}-\sqrt{2}-1}$

$= \frac{1}{\sqrt{3}-(\sqrt{2}-1)} \times \frac{\sqrt{3}+\sqrt{2}+1}{\sqrt{3}+\sqrt{2}+1}$

$= \frac{\sqrt{3}+\sqrt{2}+1}{(\sqrt{3})^2-(\sqrt{2}+1)^2} = \frac{\sqrt{2}+\sqrt{3}+2}{-\sqrt{2}}$

$= -\left(\frac{\sqrt{2}+\sqrt{6}+2}{4}\right)$

Sol.6

(i) $\left(\sqrt[3]{64}\right)^{\frac{-1}{2}} = \left[(64)^{\frac{1}{3}}\right]^{\frac{-1}{2}} = (64)^{\frac{1}{3}\times\frac{-1}{2}}$

$= (64)^{\frac{-1}{6}} = \left(2^6\right)^{\frac{-1}{6}} = 2^{-1} = \frac{1}{2}$

(ii) $\left(\frac{11\times11}{13\times13}\right)^{\frac{-3}{2}} = \left(\frac{11^2}{13^2}\right)^{\frac{-3}{2}} = \left(\frac{11}{13}\right)^{2\times\frac{-3}{2}}$

$= \left(\frac{11}{13}\right)^{-3} = \left(\frac{13}{11}\right)^{3} = \frac{2197}{133}$

Sol.7 We have,

$a^{xyz} = (a^x)^{yz}$

$\Rightarrow \quad a^{xyz} = (b)^{yz}$ $[\because a^x = b] \Rightarrow a^{xyz} = (by)^z$

$\Rightarrow \quad a^{xyz} = c^z$ $[\because b^y = c] \Rightarrow a^{xyz} = a$ $[\because c^z = a]$

$\Rightarrow \quad a^{xyz} = a^1 \Rightarrow xyz = 1$

Sol.8 $n^2 + 2n - 8 = p \Rightarrow (n + 1)^2 = p + 9$

$\therefore$ $n \in N$ so p + 9 is a perfect square So p can only be 7 $\Rightarrow$ n = 3 **Ans. (A, B)**

Sol.9 $x \in x^2 - 3x + 2 = 0$

$x(x - 1)(x - 2) = 0$

x = 0, 1, 2 **Ans. (A, B)**

10.

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(C)	{5, 11}	**(r)**	is a pair of co-primes
(D)	{2, 6}	**(s)**	is a pair of even number
(E)	{23, 81}		

Sol : (A) → (p, r) ; (B) → (p, q, r) ; (C) → (p, r) ; (D) → (s) ; (E) → r

Sol.11 For if x = 0 then xy = 0 which is rational. **Ans. (D)**

Sol.12 The factorized form of N is $(2^2 \times 3^1)^3 \times 3^4 \times 5^2 \Rightarrow 2^6 \times 3^7 \times 5^2$. Hence, the total number of factors of N is (6 + 1) (7 + 1) (2 + 1) = 7 x 8 x 3 = 168. Some of these are odd multiples and some are even.
The odd multiples are formed only with the combination of 3s and 5s. So, the total number of odd factors is (7 + 1) (2 + 1) = 24. Therefore, the number of even factors is 168 – 24 = 144.

So.13

(i) $2^{2x} - 5.2^x - 24 = 0$

$(2^x - 8)(2^x + 3) = 0$

$2^x = 8 \;\; (2^x \neq -3 \because a^x > 0)$

x = 3

(ii) Divide by 2^{2x}

$$18\left(\frac{3}{2}\right)^{2x} + \left(\frac{3}{2}\right)^{x} - 4 = 0$$

$$18\left(\frac{3}{2}\right)^{2x} + 9.\left(\frac{3}{2}\right)^{x} - 8\left(\frac{3}{2}\right)^{x} - 4 = 0$$

$$9\left(\frac{3}{2}\right)^{x} + \left(2.\left(\frac{3}{2}\right)^{x} + 1\right) - 4\left(2.\left(\frac{3}{2}\right)^{x} + 1\right) = 0$$

$$\left(2\left(\frac{3}{2}\right)^{x} + 1\right)\left(9\left(\frac{3}{2}\right)^{x} - 4\right) = 0$$

$$\because \quad \left(\frac{3}{2}\right)^{x} \neq \frac{-1}{2}$$

$$\therefore \quad \left(\frac{3}{2}\right)^{x} = \frac{4}{9} = \left(\frac{3}{2}\right)^{-2}$$

$\Rightarrow x = -2$

(iii) $$\frac{3^{2x}}{24} - \frac{3^{2x}}{9} + 3^{2x} = 675$$

$$9^x\left(\frac{1}{27} - \frac{3}{27} + \frac{27}{27}\right) = 675$$

$$9^x = \frac{675 \times 27}{25} = 9^3$$

$\Rightarrow x = 3$

(iv) $49.^{7x} = -\ 7^x - 2.7^x + 2.7^x = 48$

$48.7^x = 48$

$7^x = 1 \Rightarrow x = 0$

(v) $$\left(\frac{3}{5}\right)^{2x^2+4x-22-x-1} = \left(\frac{3}{5}\right)^{-9}$$

$2x^2 + 3x - 23 = -9$

$2x^2 + 3x - 14 = 0$

$2x^2 + 3x - 4x = 0$

$2x^2 + 7x - 4x - 14 = 0$

$(2x + 7)(x - 2) = 0$

$$\Rightarrow x = \frac{-7}{2}, 2$$

(vi) $5^{2x} - 3^{2x} = 2(5^x + 3^x)$

$(5^x + 3^x)(5^x - 3^x) = 2(5^x + 3^x)$

$(5^x + 3^x)(5^x - 3^x - 2) = 0$

$\therefore\ 5^x + 3^x \neq 0$

$\because$ only solution $5^x - 3^x - 2 = 0$

$5^x = 3^x + 2 \Rightarrow x = 1.$

Sol.14 $2\left(\sqrt{3+\sqrt{5-\sqrt{13+\sqrt{48}}}}\right)$

$= 2\left(\sqrt{3+\sqrt{5-\sqrt{12}-1}}\right)$

$= 2\left(\sqrt{3+\sqrt{4-\sqrt{12}}}\right)$

$= 2\left(\sqrt{3+\sqrt{\left(\sqrt{3}-1\right)^2}}\right)$

$= 2\left(\sqrt{3+\sqrt{3}-1}\right)$

$= 2\left(\sqrt{2+\sqrt{3}}\right) = \sqrt{8+4\sqrt{3}}$

$= \sqrt{\left(\sqrt{2}+\sqrt{6}\right)^2} = \sqrt{2}+\sqrt{6} = \sqrt{a}+\sqrt{6}$

On comparing a + b = 2 + 6 = 8

Sol.15

(i) $\sqrt[3]{3}$ or $\sqrt[4]{5}$

$\left(\sqrt[3]{3}\right)^{12}$ or $\left(\sqrt[4]{12}\right)^{12} \Rightarrow 3^4$ pr 5^3

$\therefore$ 53 is greater $\Rightarrow \sqrt[4]{5}$ is greater

(ii) 12 or 6^2

6^2 is greater $\Rightarrow \sqrt[4]{6}$ is greater 2^3 or 3^2

3^2 is greater $\Rightarrow \sqrt[3]{3}$ is greater

Sol.16 There is only one such prime, namely 5. In fact, suppose that the prime r can be represented both as a sum and as a difference of two primes.

We must have obviously r > 2, hence r is an odd prime.

Being both a sum and a difference of two primes, one of them must be even, hence equal 2.

Thus we must have r = p + 2 = q – 2, where p and q are primes. In this case, however, p, r = p + 2, and q = r + 2 would be three consecutive odd primes, and there is only one such a triplet: 3, 5, and 7 (since out of every three consecutive odd numbers one must be divisible by 3).

We have therefore r = 5 = 3 + 2 = 7 – 2.

Sol.17 n = 113, 139, 181; m = 20, 51, 62.

Sol.18 $13^2 + 1 = 7^2 + 11^2$, $17^2 + 1 = 11^2 + 13^2$, $23^2 + 1 = 13^2 + 192$, $31^2 + 1 = 11^2 + 29^2$.

Remark : The identity $(5x + 13)^2 + 1 = (3x + 7)^2 + (4x + l)^2$ shows that if p = 5x + 13, q = 3x + 7, and, = 4x + l are primes, then $p^2 + 1 = q^2 + r^2$.

Sol.19 Such numbers are, for instance, p = 127, q = 3697, r = 5527.

It is easy to check (for instance, in the tables of prime numbers) that these numbers are primes, and that the numbers p(p + l), q(q + l), and r(r + l) form an arithmetic progression.

We shall present a method of finding such numbers. From the identity

$$n(n + l) + (41n + 20)(41n + 21) = 2(29n + 14)(29n + 15)$$

it follows that for a positive integer n, the numbers

$$n(n + 1),\ (29n + 14)(29n + 15),\ \text{and}\ (41n + 20)(41n + 21)$$

form an arithmetic progression. If for some positive integer n the numbers n, 29n + 14, and 41n + 20 were all primes, we would have found a solution. Thus, we ought to take consecutive odd primes for n and check whether the numbers 29n + 14 and 41n + 20 are primes.

The least such number is n = 127 which leads to the above solution. We cannot claim, however, that in this manner we obtain all triplets of primes with the required properties.

Sol.20 The five least positive integer n for which $n^2 + 1$ is a product of three different primes are n = 13, 17, 21, 23 and 27. We have $13^2 + 1 = 2.5.17$, $17^2 + 1 = 2.5.29$, $21^2 + 1 = 2.13.17$, $23^2 + 1 = 2.5.53$. For n = 112, we have $112^2 + 1 = 5.13.193$.