1 – LINES & ANGLES-I

POINT, LINE AND ANGLE

In earlier classes, you have studied about a point, a line, a plane and an angle. Let us quickly recall these concepts.

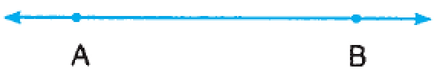
Point : If we press the tip of a pen or pencil on a piece of paper, we get a fine dot, which is called a point.

•
B   • A   • C

A point is used to show the location and is represented by capital letters A, B, C etc.

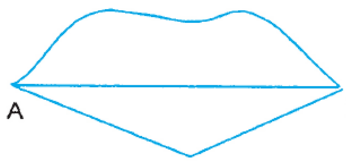
(1) **Line :** Now mark two points A and B on your note book. Join them with the help of a ruler or a scale and extend it on both sides. This gives us a straight line or simply a line. (see below figure)

In geometry, a line is extended infinitely on both sides and is marked with arrows to give this idea. A line is named using any two points on it, viz, AB or by a single small letter l, m etc. (see below figure)



The part of the line between two points A and B is called a line segment and will be named AB.

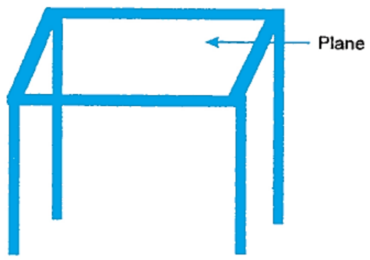
Observe that a line segment is the shortest path between two points A and B. (see below figure)



(2) **Ray :** If we mark a point X and draw a line, starting from it extending infinitely in one direction only, then we get a ray XY.

X is called the initial point of the ray XY.

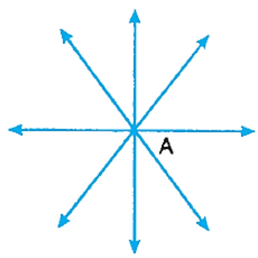
(3) **Plane:** If we move our palm on the top of a table, we get an idea of a plane.



Similarly, floor of a room also gives the idea of part of a plane. Plane also extends infintely lengthwise and breadthwise.

Mark a point A on a sheet of paper.

How many lines can you draw passing though this point? As many as you wish.

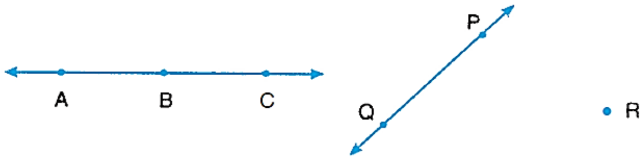


In fact, we can draw an infinite number of lines through a point.

Take another point B, at some distance from A. We can again draw an infinite number of lines passing through B.

Out of these lines, how many pass through both the points A and B? Out of all the lines passing through A, only one passes through B. Thus, only one line passes through both the points A and B. We conclude that **one and only one line can be drawn passing through two given points.**

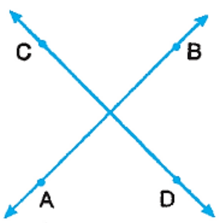
Now we take three points in plane.



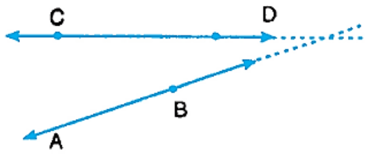
We observe that a line may or may not pass through the three given points. If a line can pass through three or more points, then these points are said to be **collinear**. For example the points A, B and C in the above figure are collinear points.

If a line **can not** be drawn passing through all three points (or more points), then they are said to be **non-collinear**. For example points P, Q and R, in the Fig. 10.9, are noncollinear points.

Since two points always lie on a line, we talk of collinear points only when their number is three or more. Let us now take two distinct lines AB and CD in a plane.



(a)



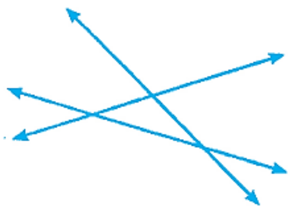
(b)

(c)

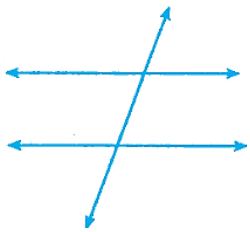
How many points can they have in common? We observe that these lines can have. either

(i) one point in common as in above three figures (a) and (b). [In such a case they are called intersecting lines] or (ii) no points in common as in above figure (c). In such a case they are called **parrallel lines**.

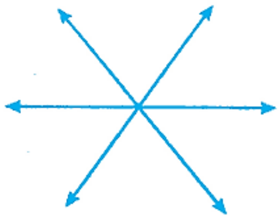
Now observe three (or more) distinct lines in plane.



(a)



(b)



(c)

(d)

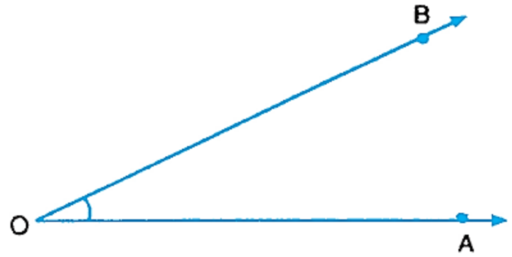
What are the possibilities ?

(i) They may interest in more than one point as in above fig. 10.11 (a) and 10.11 (b).

or (ii) They may intesect in one point only as in above fig. (c). In such a case they are called concurrent lines.

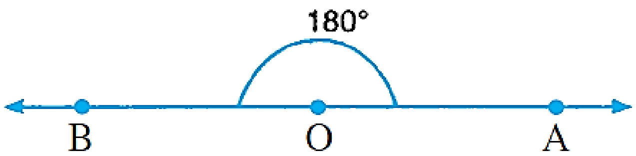
or(iii) They may be non intersecting lines parrallel to each other as in above fig. (d).

(4) **Angle :** Mark a point O and draw two rays OA and OB starting from O. The figure we get is called an angle. Thus, an angle is a figure consisting of two rays starting from a common point.



This angle may be named as angle AOB or angle BOA or simply angle O; and is written as ∠AOB or ∠BOA or ∠O. [see above fig.]

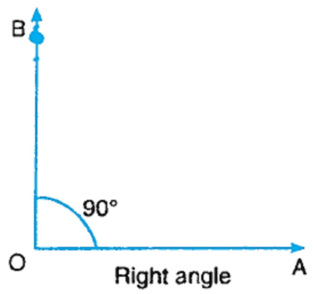
An angle is measured in degrees. If we take any point O and draw two rays starting from it in opposite directions then the measure of this angle is taken to be 180º degrees, written as 180º.

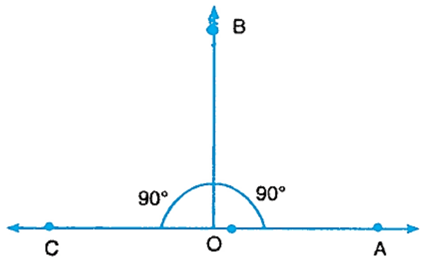


This measure divided into 180 equal parts is called one degree (written as 1º).

Angle obtained by two opposite rays is called a **straight angle**.

An angle of 90º is called a **right angle**, for example ∠BOA or ∠BOC is a right angle in below figures.

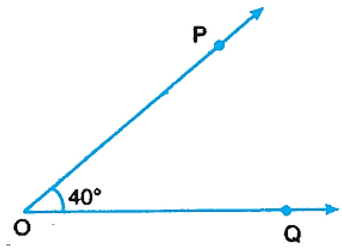


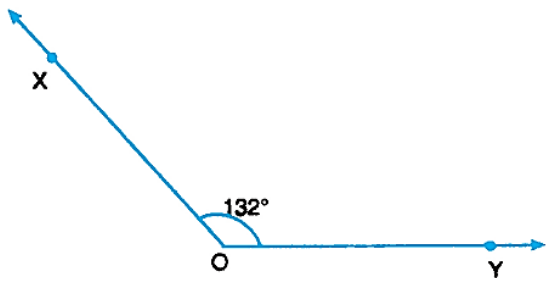


Two lines or rays making a right angle with each other are called **perpendicular lines**. In Fig. 10.13 we can say OA is perpendicular to OB or vice-versa.

An angle less than 900 is called an **acute angle**. For example ∠POQ is an acute angle in belos figure. (a).

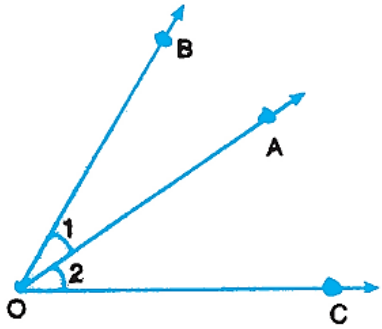
An angle greater than 900 but less than 1800 is called an **obtuse angle**. For example, ∠XOY is an obtuse angle in below figure (b).

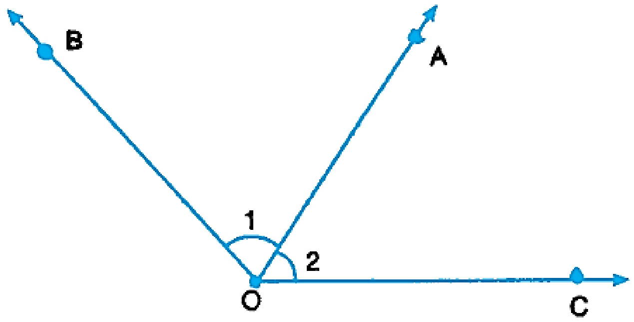




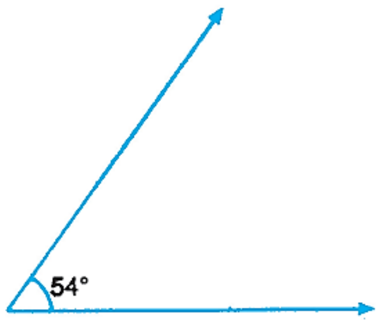
PAIRS OF ANGLES

Observe the two angles ∠1 and ∠2 in each of the figures in above figure. Each pair has a common vertex O and a common side OA in between OB and OC. Such a pair of angles is called a 'pair of adjacent angles'.

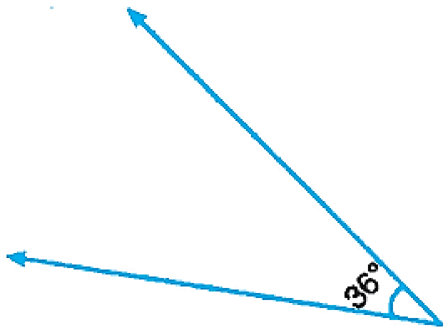


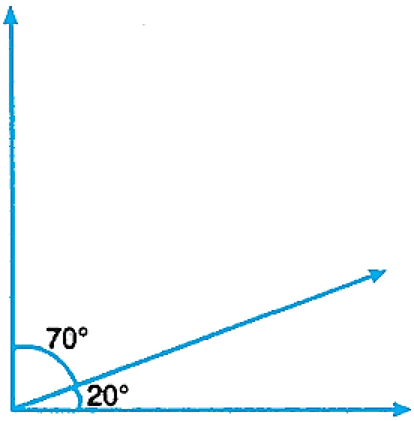


Observe the angles in each pair in below figure. [(a) and (b)]. They add up to make a total of 90º.



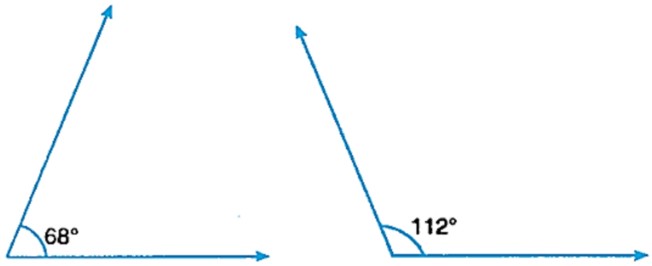
(a)



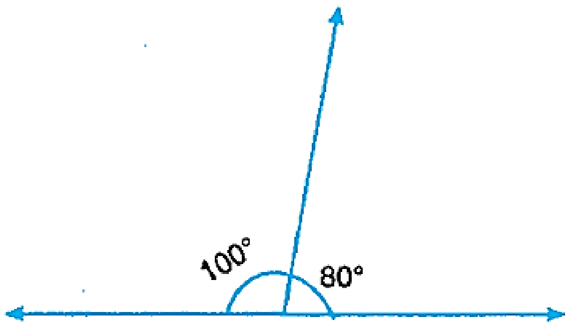


(b)

A pair of angles, whose sum is 90o, is called a pair of **complementary angles**. Each angle is called the **complement** of the other.



(a)



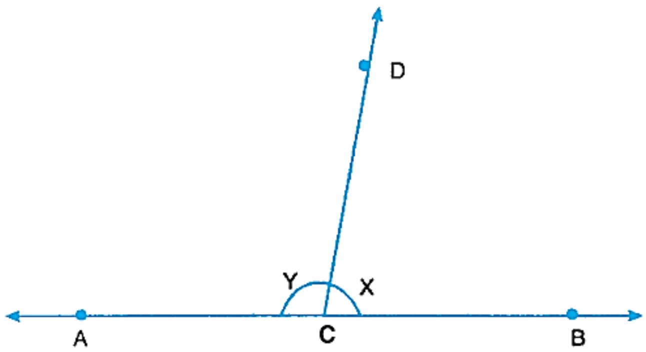
(b)

Again observe the angles in each pair in above figure [(a) and (b)]. These add up to make a total of 180º.

A pair of angles whose sum is 1800, is called a pair of supplementary angles.

Each such angle is called the **supplement** of the other.

Draw a line AB. From a point C on it draw a ray CD making two angles ∠X and ∠Y.

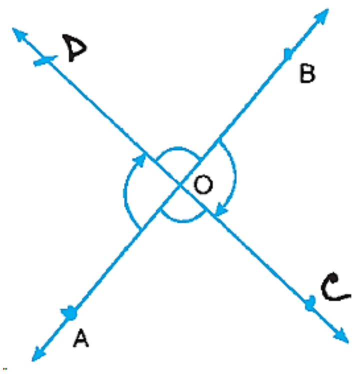


If we measure ∠X and ∠Y and add, we will always find the sum to be 180^0, whatever be the position of the ray CD. We conclude

If a ray stands on a line then the sum of the two adjacent angles so formed is 180º.

The pair of angles so formed as in above figure is called a **linear pair** of angles.

Note that they also make a pair of supplementary angles. Draw two intersecting lines AB and CD, intersecting each other at O.



∠AOC and ∠DOB are angles opposite to each other. These make a pair of **vertically oppposite angles**. Measure them. You will always find that ∠AOC = ∠DOB.

∠AOD and ∠BOC is another pair of vertically opposite angles. On measuring, you will again find that ∠AOD = ∠BOC

We conclude :

If two lines intersect each other, the pair of vertically opposite angles are equal.

WORKSHEET

1. Write the complement of each of the following angles:

(i) 20° (ii) 35° (iii) 90° (iv) 77

(v) 30°

Sol.1

(i) given angle is 20 since, the sum of an angle and its compliment is 90 Hence, its compliment will be (90-20=70)

(ii) Given angle is 35

Since, the sum of an angle and its compliment is 90

Hence, its compliment will be (90-35=55)

(iii) Given angle is 90

Since, the sum of an angle and its compliment is 90 Hence, its compliment will be (90-90 = 0)

(iv) Given angle is 77

Since, the sum of angle and its compliment is 90 Hence, its compliment will be (90 – 77 = 13)

(v) Given angle is 30

Since, the sum of an angle and its compliment is 90

Hence, its compliment will be (90-30=60)

2. Write the supplement of each of the following angles:

(i) 54° (ii) 132° (iii) 138°

Sol.2

(i) The given angle is 54,

Since the sum of an angle and its supplement is 180,

Hence, its supplement will be (180-54=126)

(ii) The given angle is 132,

Since the sum of an angle and its supplement is 180,

Hence, its supplement will be 180 – 132 = 48

(iii) The given angle is 138.

Since the sum of an angle and its supplement is 180,

Hence, its supplement will be 180-138 = 42

3. If an angle is 28° less than its complement, find its measure?

Sol.3 Let the angle measured be 'x' in degrees

Hence, its complement will be 90 – x°

Angle = Complement – 28

x = (90 – x) – 28

zx – 62

x = 31

Therefore, angle measured is 31°

4. If an angle is 30° more than half of its complement, find the meausre of the angle?

Sol.4 Let the measured angle be 'x'

Hence its complement will be (90 – x)

It is given that,

Angle = 30 + complement/2

$$x = 30 + \frac{(90 - x)}{2}$$

$$30\ \frac{x}{2} = 30 + 45$$

3x = 150

x = 50

Therefore the angle is 50°

5. Two supplementary angles are in the ratio 4:5. Find the angles?

Sol.5 Supplementary angles are in the ratio 4:5

Let the angles be 4x and 5x

It is given that they are supplementary angles

Hence 4x + 5x = 180

9x = 180

x = 20

Hence, 4x = 4(20) = 80

5(x) = 5(20) = 100

Hence, angles are 80 and 100

6. Two supplementary angles differ by 48°. Find the angles?

Sol.6 Given that two supplementary angles differ by 48°

Let the angles measured be x°

Therefore, its supplementary angle will be (180 – x)°

It is given that:

(180 – x) – x = 48

(180 – 48) =2x

2x = 132

$x = \frac{132}{2}$

x = 66

Hence, 180 – x = 114°

Therefore, the angles are 66 and 114.

7. An angle is equal to 8 times its complement. Determine its measure?

Sol.7 Let 'x' be the measured angle

angle = 8 times complement

angle = 9(90 – x)

x = 8(90 – x)

x = 720 – 8x

x + 8x = 720

9x = 720

x = 80

Therefore measured angle is 80.

8. If the angles (2x – 10)° and (x – 5)° are complementary, find x?

Sol.8 Given that (2x – 10)° and (x – 5)° are complementary.

Since angles are complementary, their sum will be 90

(2x – 10) + (x- 5) = 90

3x – 15 = 90

3x = 90 + 15

3x = 105

$x = \frac{105}{3}$

x = 35

Hence, the value of x = (35)°

9. If the compliment of an angle is equal to the supplement of Thrice of itself, find the measure of the angle?

Sol.9 Let the angle measured be 'x' say.

Its complementary angle is (90 – x) and

Its supplementary angle is (180 – 3x)

Given that, supplementary of 4 times the angle = (180 – 3x)

According to the given information;

(90 – x) = (180 – 3x)

3x – x = 180 – 90

2x = 90

$x = \frac{90}{2}$

x = 45

Therefore, the measured angle x = (45)°

10. If an angle differes from its complement by $(10)°$, find the angle?

Sol.10 Let the measured angle be 'x' say

given that,

The angles measured will differ by $(20)°$

$x - (90 + x) = 10$

$x - 90 + x = 10$

$2x = 90 + 10$

$2x = 100$

$x = \frac{100}{2}$

$x = 50$

Therefore the measure of the angle is $(50)°$

11. If the supplement of an angle is 3 times its complement, find its angle?

Sol.11 Let the angle in case be 'x'

Given that,

Supplement of an angle = 3 times its complementary angle

Supplementary angle = 180-x

Supplementary angle = 90 – x

Applying given data,

$180 - x = 3\ (90 - x)$

$3x - x = 270 - 180$

$2x = 90$

$x = \frac{90}{2}$

$x = 45$

Therefore, the angle in case in 45°

12. If the supplement of an angle is two third of itself. Determine the angle and its supplement?

Sol.12 Supplementary of an angle = $\frac{2}{3}$ angle

Let the angle in case be 'x'

Supplementary of angle x will be (180 – x)

It is given that

$180 - x = \frac{2}{3}x$

$(180 - x)\ 3 = 2x$

$540 - 3x = 2x$

$5x = 540$

$x = \frac{540}{5}$

$x = 108$

Hence, supplementary angle = 180 – 108 = 72

Therefore, angle = 180 – 108 = 72

There fore, angle in are 108° and supplementary angleis 72°

13. An angle is 14° more than its complementary angle. What is its measure?

Sol.13 Let the angle in case be 'x'

Complementary angle of 'x' is (90 – x)

From given data,

$x - (90 - x) = 14$

$x - 90 + x\ 14$

$2x = 104$

$x = \frac{104}{2}$

$x = 52$

Hence the angle in case is found to be 52°

14. The measure of an angle is twice the measure of its supplementary angle. Find the meausre of the angles?

Sol.14 Let the angle in case be 'x'

The supplementary of a angle x is (180– x)

Apppying given data:

$x = 2\,(180 - x)$

$x = 360 - 2x$

$3x = \frac{360}{3}$

$x = 120$

Therefore the value of the angle in case is 120°

15. How many pairs of adjacent angles are formed when two lines interesect at a point?

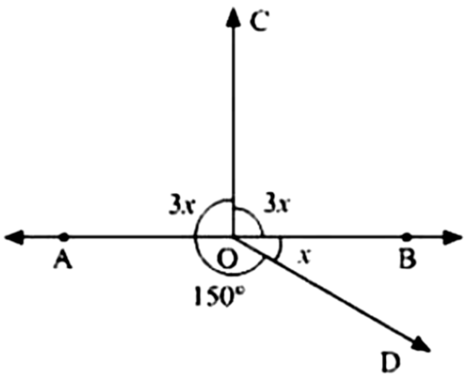
Sol.15 Four pairs of adjacent angles will be formed when two lines intersect at a point.

The 4 pairs are :

(∠AOD, ∠DOB), (∠DOB, ∠BOC), (∠COA, ∠AOD) and (∠BOC, ∠COA)

Hence, 4 pairs of adjacent angles are formed when two lines intersect at a point.

16. In the below figure, find value of x?



Sol.16 Since the sum of all the angles round a point is equal to 360°

$3x + 3x + 150 + x = 360$

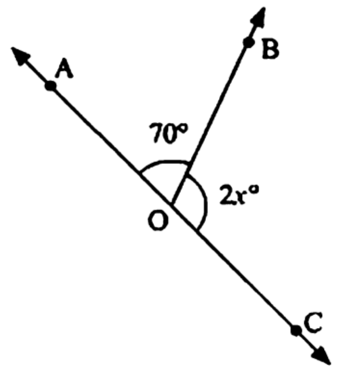
$7x = 360 - 150$

$7x = 210$

$x = \frac{210}{7} \Rightarrow x = 30$

Value of x is 30°

17. In the below figure, AOC is a line, find x.



Sol.17 Since ∠AOB and ∠BOC are linear pairs,

$\angle AOB + \angle BOC = 180°$

$70 + 2x = 180$

$2x = 180 - 70$

$2x = 110$

$x = \frac{110}{2}$

$x = 55$

Hence, the value of x is 55°

18. In the below figure, POS is a line, Find x ?

Sol.18 Since ∠POQ and ∠QOS = 180°

$\angle POQ + \angle QOR + \angle SOR = 180°$

$60 + 4x + 40 = 180$

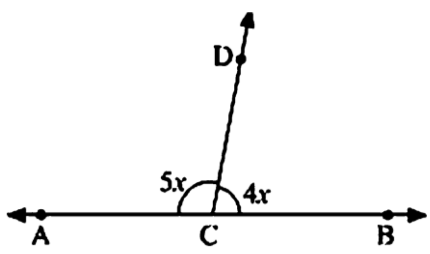
$4x = 180 - 100$

$4x = 80$

$x = 20$

Hence, value of x = 20

19. In the below figure, ACB is a line such that ∠DCA = 5x and ∠DCB = 4x. Find the value of x?



Sol.19 Here, $\angle ACD + \angle BCD = 180°$

[Since they are linear pairs]

$\angle DCA = 5x$ and $\angle DCB = 4x$

$5x + 4x = 180$

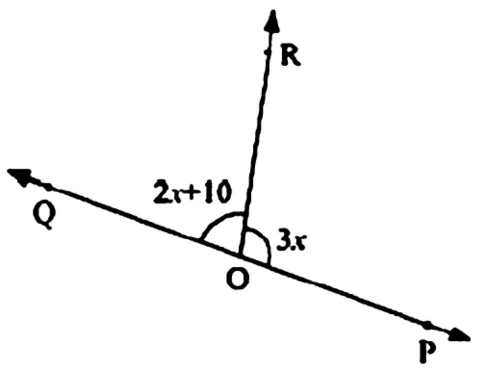
$9x = 180$

$x = 180$

$x = 20$

Hence, the value of x is $20°$

20. In the given figure, Given $\angle POR = 3x$ and $\angle QOR = 2x + 10$, Find the value of x for which POQ will be a line?



Sol.20 For the case that POR is a line

$\angle$**POR and** $\angle QOR = 180°$

$\angle POR + \angle QOR = 180°$

Also, given that,

$\angle PQR = 3x$ and $\angle QOR = 2x + 10$

$2x + 10 + 3x$ and $\angle QOR = 2x + 10$

$2x + 10 + 3x = 180$

$5x + 10 = 180$

$5x = 180 – 10$

$5x = 170$

$x = 34$

Hence the value of x is $34°$