

RATIO AND PROPORTION

EQUIVALENT RATIOS

Orange squash is to be mixed with water in a ratio of 1 : 6 ; this means that for every unit of orange squash, 6 units of water will be used. The table gives some examples:

Amount of Orange Squash (cm ³)	Amount of water (cm ³)
1	6
20	120
5	30

The ratios 1 : 6 and 20 : 120 and 5 : 30 are all equivalent ratios, but 1 : 6 is the simplest form.

Ratios can be simplified by dividing both sides by the same number : note the similarity to fractions.

An alternative method for some purpose, is to reduce to the form 1 : n or n : 1 by dividing both numbers by either the left-hand-side (LHS) or the right-hand-side (RHS). For example :

the ratio 4 : 10 may be simplified to

$$\frac{4}{4} : \frac{10}{4} \Rightarrow 1 : 2.5$$

the ratio 8 : 5 may be simplified to

$$\frac{8}{5} : \frac{5}{5} \Rightarrow 1.6 : 1$$

Ex.1 Write each of these ratios in its simplest form :

- (a) 7 : 14 (b) 15 : 25
(c) 10 : 4

Sol.

(a) Divide both sides by 7, giving

$$7 : 14 = \frac{7}{7} : \frac{14}{7} = 1 : 2$$

(b) Divide both sides by 5, giving

$$15 : 25 = \frac{15}{5} : \frac{25}{5} = 3 : 5$$

(c) Divide both sides by 2, giving

$$10 : 4 = \frac{10}{2} : \frac{4}{2} = 5 : 2$$

Ex.2 Write these ratios in the form 1 : n.

- (a) 3 : 12 (b) 5 : 6 (c) 10 : 42

Sol.

(a) Divide both sides by 3, giving

$$3 : 12 = 1 : 4$$

(b) Divide both sides by 5, giving

$$5 : 6 = 1 : \frac{6}{5} = 1 : 1.2$$

(c) Divide both sides by 10, giving

$$10 : 42 = 1 : \frac{42}{10} = 1 : 4.2$$

Ex.3 The scale on a map is 1 : 20000. What actual distance does a length of 8cm on the map represent ?

Sol. Actual distance = 8 × 20000
= 160 000 cm = 1600 m = 1.6 km

DIRECT PROPORTION

Direct proportion can be used to carry out calculations like the one below:

If 10 calculators cost £ 120,

then 1 calculator costs £ 12,

And 8 calculators cost £ 96.

Ex.4 If 6 copies of a book cost £ 9, calculate the cost of 8 books.

Sol. If 6 copies cost £ 9,
then 1 copy costs $\text{£ } \frac{9}{6} = \text{£ } 1.50$
and 8 copies cost $\text{£ } 1.50 \times 8 = \text{£ } 12$

Ex.5 If 25 floppy discs cost £ 5.50, calculate the cost of 11 floppy discs.

Sol. If 25 discs cost $\text{£ } 5.50 = 550\text{p}$
then 1 disc costs $= \frac{550}{25} = 22\text{p}$
so 11 discs cost $11 \times 22\text{p} = 242\text{p} = \text{£ } 2.42$

PROPORTIONAL DIVISION

Sometimes we need to divide something in a given ratio. Malcolm and Alison share the profits from their business in the ratio 2 : 3. This means that, out of every £ 5 profit. Malcolm gets £ 2 and Alison gets £ 3.

Ex.6 Julie and Jack run a stall at a car boot sale and take a total of £ 90. They share the money in the ratio 4 : 5. How much money does each receive.

Sol. As the ratio is 4 : 5, first add these numbers together to see by how many parts the £ 90 is to be divided.
 $4 + 5 = 9$, so 9 parts are needed.

Now divide the total by 9.

$\frac{90}{9} = 10$, so each part is £10.

Julie gets 4 parts at £ 10, giving $4 \times \text{£ } 10 = \text{£ } 40$.

Jack gets 5 parts at £ 10, giving $5 \times \text{£ } 10 = \text{£ } 50$.

Ex.7 Rachel, Ben and Emma are given £ 52. They decide to divide the money in the ratio of their ages, 10 : 9 : 7. How much does each receive ?

Sol. $10 + 9 + 7 = 26$ so 26 parts are needed.
Now divide the total by 26.

$\frac{52}{26} = 2$, so each part is £ 2.

Rachel gets 10 parts at £ 2, giving

$10 \times \text{£ } 2 = \text{£ } 20$

Ben gets 9 parts at £ 2, giving

$9 \times \text{£ } 2 = \text{£ } 18$

Emma gets 7 parts at £ 2, giving

$7 \times \text{£ } 2 = \text{£ } 14$

LINEAR CONVERSION

The ideas used in this unit can be used for converting masses, lengths and currencies.

Ex.8 If £ 1 is worth 9 French francs, convert :

(a) £ 22 to Ff, (b) 45 Ff to £

(c) 100 Ff to £.

Sol.

(a) $\text{£ } 22 = 22 \times 9 = 198\text{ Ff}$

(b) $1\text{ Ff} = \text{£ } \frac{1}{9}$ so $45\text{ Ff} = 45 \times \frac{1}{9} = \frac{45}{9} = \text{£ } 5$

(c) $100\text{ Ff} = 100 \times \frac{1}{9} = \frac{100}{9} = \text{£ } 11\frac{1}{9}$
 $= \text{£ } 11.11$ to the nearest pence.

Ex.9 Use the fact that 1 foot is approximately 30 cm to convert :

(a) 8 feet to cm

(b) 50 cm to feet

(c) 195 cm to feet

Sol.

(a) $8\text{ feet} = 8 \times 30 = 240\text{ cm}$

- (b) $1 \text{ cm} = \frac{1}{30} \text{ feet}$, so $50 \text{ cm} = 50 \times \frac{1}{30}$
 $= \frac{5}{3} = 1 \frac{2}{3} \text{ feet}$
- (c) $195 \text{ cm} = 195 \times \frac{1}{30} = \frac{195}{30} = \frac{13}{2} = 6 \frac{1}{2} \text{ feet}$

Ex.10 If £ 1 is worth \$ 1.60, convert :

- (a) £ 15 to dollars
 (b) \$ 8 to pounds

Sol.

- (a) £ 15 = $15 \times 1.60 = \$ 24$.
 (b) \$ 1 = £ $\frac{1}{1.60} = \frac{10}{16}$
 $\$ 8 = 8 \times \frac{10}{16} = \frac{80}{16} = \text{£ } 5$

INVERSE PROPORTION

Inverse proportion is when an increase in one quantity causes a decrease in another:

The relationship between speed and time is an example of inverse proportionality : as the speed increases, the journey time decreases, so the time for a journey can be found dividing the distance by the speed.

Ex.11

- (a) Ben rides his bike at a speed of 10 mph. How long does it takes him to cycle 40 miles ?
 (b) On another day he cycles the same route at a speed of 16 mph. How much time does this journey take ?

Sol.

(a) $\text{Time} = \frac{40}{10} = 4 \text{ hours}$

Note : Faster speed \Rightarrow shorter time.

(b) $\text{Time} = \frac{40}{16} = 2 \frac{1}{2} = 2 \frac{1}{2} \text{ hours}$.

Ex.11 Jai has to travel 280 miles. How long does it take if he travels at :

- (a) 50 mph
 (b) 60 mph
 (c) How much time does he save when he travels at the faster speed ?

Sol.

- (a) $\text{Time} = \frac{280}{50} = 5.6 \text{ hours} = 5 \text{ hours } 36 \text{ minutes}$.
 (b) $\text{Time} = \frac{280}{60} = 4 \frac{2}{3} \text{ hours} = 4 \text{ hours } 40 \text{ minutes}$
 (c) $\text{Time saved} = 5 \text{ hours } 36 \text{ mins} - 4 \text{ hours } 40 \text{ mins} = 56 \text{ minutes}$

Ex.12 In a factory, each employee make 40 chicken pies in one hour. How long will it take :

- (a) 6 people to make 40 pies,
 (b) 3 people to make 240 pies,
 (c) 10 people to make 600 pies ?

Sol.

- (a) 1 person makes 40 pies in 1 hour.
 6 people make 40 pies in $\frac{1}{6}$ hour (or 10 minutes).
 (b) 1 person makes 40 pies in 1 hour
 1 person makes 240 pies in $\frac{240}{40} = 6$ hours.
 3 people make 240 pies in $\frac{6}{3} = 2$ hours.
 (c) 1 person makes 40 pies in 1 hour.
 1 person makes 600 pies in $\frac{600}{40} = 15$ hours.

10 people make 600 pies in $\frac{15}{10} = 1\frac{1}{2}$ hours.

IMPORTANT FACTS & FORMULAE

1. **Ratio** : The ratio of two quantities a & b in the same units, is the fraction $\frac{a}{b}$ and we write it as a:b.

In the ratio a : b, we call a as the first term or antecedent and b, the second term or consequent.

Ex. The ratio 5 : 9 represents $\frac{5}{9}$ with antecedent = 5, consequent = 9.

Rule : The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

Ex. 4 : 5 = 8 : 10 = 12 : 15 etc. Also, 4 : 6 = 2:3.

2. **Proportion** : The equality of two ratios is called proportion.

If a : b = c : d, we write, a : b :: c : d and we say that a, b, c, d are in proportion. Here a and d are called extremes, while b and c are called mean terms.

Product of means : Product of extremes.

Thus, a : b :: c : d \Leftrightarrow (b \times c) = (a \times d).

Ex. 4 : 5 = 8 : 10 = 12 : 15 etc. Also, 4 : 6 = 2 : 3.

3. (i) **Fourth Proportional** : If a : b = c : d, then d is called the fourth proportional to a, b, c.

(ii) **Third Proportional** : If a : b = b : c, then c is called the third proportional to a and b.

(iii) **Mean Proportional** : Mean proportional between a and b is \sqrt{ab} .

4. (i) **Comparison of Ratios** :

We say that (a : b) > (c : d) $\Leftrightarrow \frac{a}{b} > \frac{c}{d}$

(ii) **Compounded Ratio** : The compounded ratio of the ratios (a : b), (c : d), (e : f) is (ace : bdf).

5. (i) **Duplicate ratio** of (a : b) is (a² : b²)

(ii) **Sub-duplicate ratio**:

(a : b) is (\sqrt{a} : \sqrt{b}).

(iii) **Triplicate ratio**: of (a : b) is (a³:b³).

(iv) **Sub-Triplicate ratio**: of (a : b) is (a^{1/3} : b^{1/3}).

(v) If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$.

(Componendo and Dividendo)

6. **Variation**

(i) We say that x is directly proportion to y, if x = ky for some constant k and we write, x \propto y.

(ii) We say that x is inversely proportional to y, if xy = k for some constant k and we write, x $\propto \frac{1}{y}$.

WORKSHEET

1. If $A : B = 5 : 7$ and $B : C = 6 : 11$, then $A : B : C$ is :
(a) $55 : 77 : 66$ (b) $30 : 42 : 77$
(c) $35 : 49 : 42$ (d) None of these
2. If $A : B = 3 : 4$ and $B : C = 8 : 9$, then $A : C$ is :
(a) $1 : 3$ (b) $3 : 2$ (c) $2 : 3$ (d) $1 : 2$
3. If $A : B = 8 : 15$, $B : C = 5 : 8$ and $C : D = 4 : 5$, then $A : D$ is equal to :
(a) $2 : 7$ (b) $4 : 15$ (c) $8 : 15$ (d) $15 : 4$
4. If $A : B : C = 2 : 3 : 4$, then $\frac{A}{B} : \frac{B}{C} : \frac{C}{A}$ is equal to :
(a) $4 : 9 : 16$ (b) $8 : 9 : 12$
(c) $8 : 9 : 16$ (d) $8 : 9 : 24$
5. If $A : B = \frac{1}{2} : \frac{3}{8}$, $B : C = \frac{1}{3} : \frac{5}{9}$ and $C : D = \frac{5}{6} : \frac{3}{4}$, then the ratio $A : B : C : D$ is :
(a) $4 : 6 : 8 : 10$ (b) $6 : 4 : 8 : 10$
(c) $6 : 8 : 9 : 10$ (d) $8 : 6 : 10 : 9$
6. If $A : B = 2 : 3$, $B : C = 4 : 5$ and $C : D = 6 : 7$, then $A : B : C : D$ is :
(a) $16 : 22 : 30 : 35$ (b) $16 : 24 : 15 : 35$
(c) $16 : 24 : 30 : 35$ (d) $18 : 24 : 30 : 35$
7. If $2A = 3B = 4C$, then $A : B : C$ is :
(a) $2 : 3 : 4$ (b) $4 : 3 : 2$
(c) $6 : 4 : 3$ (d) $20 : 15 : 2$
8. If $\frac{A}{3} : \frac{B}{4} : \frac{C}{5}$, then $A : B : C$ is :
(a) $4 : 3 : 5$ (b) $5 : 4 : 3$
(c) $3 : 4 : 5$ (d) $20 : 15 : 2$
9. If $2A = 3B$ and $4B = 5C$, then $A : C$ is
(a) $4 : 3$ (b) $8 : 15$ (c) $15 : 8$ (d) $3 : 4$
10. The ratio of $4^{3.5} : 2^5$ is same as :
(a) $2 : 1$ (b) $4 : 1$ (c) $7 : 5$ (d) $7 : 10$
11. If $\frac{1}{5} : \frac{1}{x} = \frac{1}{x} : \frac{1}{125}$, then the value of x is :
(a) 1.5 (b) 2 (c) 2.5 (d) 3.5
12. If $0.75 : x :: 5 : 8$, then x is equal to
(a) 1.12 (b) 1.20 (c) 1.25 (d) 1.30
13. If $x : y = 5 : 2$, then $(8x + 9y) : (8x + 2y)$ is :
(a) $22 : 29$ (b) $26 : 61$
(c) $29 : 22$ (d) $61 : 26$
14. If 15% of $x = 20\%$ of y , then $x : y$ is
(a) $3 : 4$ (b) $4 : 3$
(c) $17 : 16$ (d) $16 : 17$
15. If $(x : y) = 2 : 1$, then $(x^2 - y^2) : (x^2 + y^2)$ is :
(a) $3 : 5$ (b) $5 : 3$ (c) $1 : 3$ (d) $3 : 1$
16. If $(4x^2 - 3y^2) : (2x^2 + 5y^2) = 12 : 19$, then $(x : y)$ is :
(a) $2 : 3$ (b) $1 : 2$ (c) $3 : 2$ (d) $2 : 1$
17. If $x^2 + 4y^2 = 4xy$, then $x : y$ is
(a) $2 : 1$ (b) $1 : 2$ (c) $1 : 1$ (d) $1 : 4$

18. If $5x^2 - 13xy + 6y^2 = 0$, then $x : y$ is
 (a) (2 : 1) only (b) (3 : 5) only
 (c) (5 : 3) or (1 : 2) (d) (3 : 5) or (2 : 1)

19. If $\frac{x}{5} = \frac{y}{8}$, then $(x + 5) : (y + 8)$ is equal to :

- (a) 3 : 5 (b) 13 : 8 (c) 8 : 5 (d) 5 : 8

20. If $\frac{a}{3} = \frac{b}{4} = \frac{c}{7}$, then $\frac{a+b+c}{c}$ is equal to :

- (a) 7 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{1}{7}$

$$\Rightarrow A : B = 4 : 3, B : C = 3 : 5 \text{ and } C : D = 5 : \frac{9}{2}$$

$$\Rightarrow A : B : C : D = 4 : 3 : 5 : \frac{9}{2}$$

$$= 8 : 6 : 10 : 9$$

Sol.6 $A : B = 2 : 3, B : C = 4 : 5 = \left(4 \times \frac{3}{4}\right) :$

$$\left(5 \times \frac{3}{4}\right) = 3 : \frac{15}{4} \text{ and } C : D = 6 : 7$$

$$= \left(6 \times \frac{15}{24}\right) : \left(7 \times \frac{15}{24}\right) = \frac{15}{4} : \frac{35}{8}$$

$$\Rightarrow A : B : C : D = 2 : 3 : \frac{15}{4} : \frac{35}{8}$$

$$= 16 : 24 : 30 : 35.$$

HINT'S & SOLUTION

Sol.1 $A : B = 5 : 7, B : C = 6 : 11 = \left(6 \times \frac{7}{6}\right) :$

$$\left(11 \times \frac{7}{6}\right) = 7 : \frac{77}{6}.$$

$$\therefore A : B : C = 5 : 7 : \frac{77}{6} = 30 : 42 : 77.$$

Sol.2 $\left(\frac{A}{B} = \frac{3}{4}, \frac{B}{C} = \frac{8}{9}\right) \Rightarrow \frac{A}{C} = \left(\frac{A}{B} \times \frac{B}{C}\right)$

$$= \left(\frac{3}{4} \times \frac{8}{9}\right) = \frac{2}{3} \Rightarrow A : C = 2 : 3.$$

Sol.3 $\frac{A}{B} = \frac{8}{15}, \frac{B}{C} = \frac{5}{8} \text{ and } \frac{C}{D} = \frac{4}{5}$

$$\Rightarrow \frac{A}{D} = \left(\frac{A}{B} \times \frac{B}{C} \times \frac{C}{D}\right) = \left(\frac{8}{15} \times \frac{5}{8} \times \frac{4}{5}\right) = \frac{4}{15}$$

$$\Rightarrow A : D = 4 : 15.$$

Sol.4 Let $A = 2x, B = 3x$ and $C = 4x$. Then, $\frac{A}{B} =$

$$\frac{2x}{3x} = \frac{2}{3}, \frac{B}{C} = \frac{3x}{4x} = \frac{3}{4} \text{ and } \frac{C}{A} = \frac{4x}{2x} = \frac{2}{1}.$$

$$\Rightarrow \frac{A}{B} : \frac{B}{C} : \frac{C}{A} = \frac{2}{3} : \frac{3}{4} : \frac{2}{1} = 8 : 9 : 24.$$

Sol.5 $A : B = \frac{1}{2} : \frac{3}{8} = 4 : 3, B : C = \frac{1}{3} : \frac{5}{9} = 3 : 5,$

$$C : D = \frac{5}{6} : \frac{3}{4} = 10 : 9.$$

Sol.7 Let $2A = 3B = 4C = k$. Then, $A = \frac{k}{2}, b =$

$$\frac{k}{3}, c = \frac{k}{4}.$$

$$\Rightarrow A : B : C = \frac{k}{2} : \frac{k}{3} : \frac{k}{4} = 6 : 4 : 3.$$

Sol.8 Let $\frac{A}{3} = \frac{B}{4} = \frac{C}{5} = k$. Then, $A = 3k, B = 4k$

and $C = 5k$.

$$\Rightarrow A : B : C = 3k : 4k : 5k = 3 : 4 : 5.$$

Sol.9 $2A = 3B$ and $4B = 5C \Rightarrow$ and $\frac{A}{B} = \frac{3}{2}$ and

$$\frac{B}{C} = \frac{5}{4}$$

$$\Rightarrow \frac{A}{C} = \left(\frac{A}{B} \times \frac{B}{C}\right) = \left(\frac{3}{2} \times \frac{5}{4}\right) = \frac{15}{8}$$

$$= A : C = 15 : 8.$$

Sol.10 $\frac{4^{3.5}}{2^5} = \frac{(2^2)^{3.5}}{2^5} = \frac{2^{(2 \times 3.5)}}{2^5} = \frac{2^7}{2^5} = 2^2 = 4.$

$$\therefore \text{Required ratio is } 4 : 1.$$

Sol.11 $\frac{1}{5} : \frac{1}{x} = \frac{1}{x} : \frac{100}{125} \Rightarrow \frac{1}{x} \times \frac{1}{x} = \left(\frac{1}{5} \times \frac{100}{125}\right) = \frac{4}{25}$

$$\Rightarrow \frac{1}{x^2} = \frac{4}{25} \Rightarrow x^2 = \frac{25}{4} \Rightarrow x = \frac{5}{2} = 2.5.$$

Sol.12 $(x \times 5) = (0.75 \times 8) \Rightarrow x = \frac{6}{5} = 1.20.$

Sol.13 Let $x = 5k$ and $y = 2k$. Then, $\frac{8x+9y}{8x+2y}$
 $= \frac{(8 \times 5k) + (9 \times 2k)}{(8 \times 5k) + (2 \times 2k)} = \frac{58k}{44k} = \frac{29}{22}$
 $\Rightarrow (8x + 9y) : (8x + 2y) = 29 : 22.$

Sol.14 15% of $x = 20\%$ of $y \Rightarrow \frac{15x}{100} \Rightarrow \frac{20y}{100}$
 $= \frac{x}{y} = \left(\frac{20}{100} \times \frac{100}{15} \right) = \frac{4}{3}$
 $\Rightarrow x : y = 4 : 3.$

Sol.15 $\frac{x}{y} = \frac{2}{1} \Leftrightarrow \frac{x^2}{y^2} = \frac{4}{1}$
 $\Leftrightarrow \frac{x^2 + y^2}{x^2 - y^2} = \frac{4+1}{4-1}$
 [By componendo and dividendo]
 $\Leftrightarrow \frac{x^2 - y^2}{x^2 + y^2} = \frac{3}{5} \Leftrightarrow (x^2 - y^2) : (x^2 + y^2)$
 $= 3 : 5.$

Sol.16 $\frac{4x^2 - 3y^2}{2x^2 + 5y^2} = \frac{12}{19} \Leftrightarrow 19(4x^2 - 3y^2)$
 $= 12(2x^2 + 5y^2)$
 $\Leftrightarrow 52x^2 = 117y^2 \Leftrightarrow 4x^2 = 9y^2 \Leftrightarrow \frac{x^2}{y^2} = \frac{9}{4}$
 $\Leftrightarrow \frac{x}{y} = \frac{3}{2}$
 \therefore Required ratio is $3 : 2.$

Sol.17 $x^2 + 4y^2 = 4xy \Leftrightarrow x^2 - 4xy + 4y^2 = 0$
 $\Leftrightarrow (x - 2y)^2 = 0$
 $\Leftrightarrow (x - 2y) = 0 \Leftrightarrow x = 2y \Leftrightarrow \frac{x}{y} = \frac{2}{1}$

Sol.18 $5x^2 - 13xy + 6y^2 = 0$
 $\Leftrightarrow 5x^2 - 10xy - 3xy + 6y^2 = 0$
 $\Leftrightarrow 5x(x - 2y) - 3y(x - 2y) = 0$
 $\Leftrightarrow (x - 2y)(5x - 3y) = 0$
 $\Leftrightarrow x = 2y$ or $5x = 3y$
 $\Leftrightarrow \frac{x}{y} = \frac{2}{1}$ or $\frac{x}{y} = \frac{3}{5}$
 $\therefore (x : y) = (2 : 1)$ or $(3 : 5).$

Sol.19 Let $\frac{x}{5} = \frac{y}{8} = k$. Then, $x = 5k$ and $y = 8k$.

$\therefore \frac{x+5}{y+8} = \frac{5k+5}{8k+8} = \frac{5(k+1)}{8(k+1)} = \frac{5}{8}$
 $\Rightarrow (x + 5) : (y + 8) = 5 : 8.$

Sol.20 Let $\frac{a}{3} = \frac{b}{4} = \frac{c}{7} = k$. Then, $x = 5k$ and $y = 8k$.

$\therefore \frac{a+b+c}{c} = \frac{3k+4k+7k}{7k} = \frac{14k}{7k} = 2.$